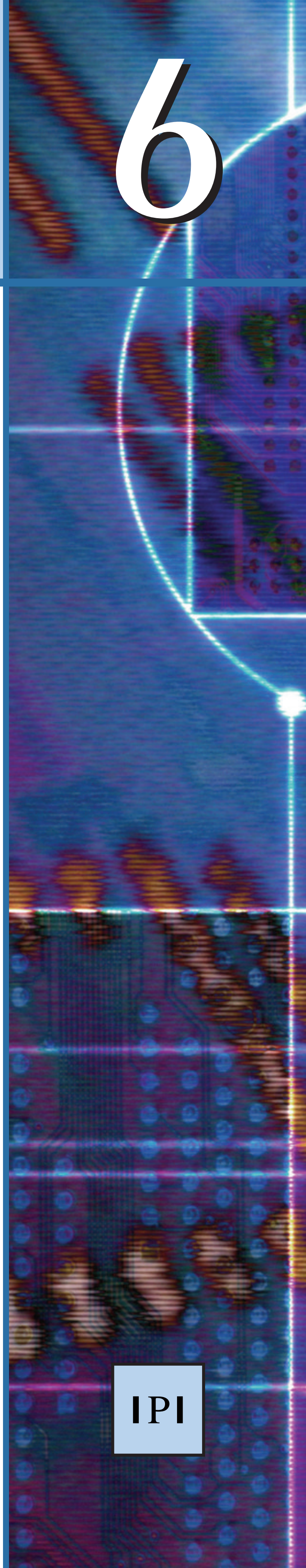
Parallel dc Circuits



Objectives

- *Become familiar with the characteristics of a parallel network and how to solve for the voltage, current, and power to each element.*
- *Develop a clear understanding of Kirchhoff's current law and its importance to the analysis of electric circuits.*
- *Become aware of how the source current will split between parallel elements and how to properly apply the current divider rule.*
- *Clearly understand the impact of open and short circuits on the behavior of a network.*
- *Learn how to use an ohmmeter, voltmeter, and ammeter to measure the important parameters of a parallel network.*

6.1 INTRODUCTION

Two network configurations, series and parallel, form the framework for some of the most complex network structures. A clear understanding of each will pay enormous dividends as more complex methods and networks are examined. The series connection was discussed in detail in the last chapter. We will now examine the **parallel circuit** and all the methods and laws associated with this important configuration.

6.2 PARALLEL RESISTORS

The term *parallel* is used so often to describe a physical arrangement between two elements that most individuals are aware of its general characteristics.

In general,

two elements, branches, or circuits are in parallel if they have two points in common.

For instance, in Fig. 6.1(a), the two resistors are in parallel because they are connected at points *a* and *b*. If both ends were *not* connected as shown, the resistors would not be in parallel. In Fig. 6.1(b), resistors R_1 and R_2 are in parallel because they again have points *a* and *b* in common. R_1 is not in parallel with R_3 because they are connected at only one point (*b*). Further, R_1 and R_3 are not in series because a third connection appears at point *b*. The same can be said for resistors R_2 and R_3. In Fig. 6.1(c), resistors R_1 and R_2 are in series because they have only one point in common that is not connected elsewhere in the network. Resistors R_1 and R_3 are not in parallel because they have only point *a* in common. In addition, they are not in series because of the third connection to point *a*. The same can be said for resistors R_2 and R_3. In a broader context, it can be said that the series combination of resistors R_1 and R_2 is in parallel with resistor R_3 (more will be said about this option in Chapter 7). Furthermore, even though the discussion above was only for resistors, it can be applied to any two-terminal elements such as voltage sources and meters.

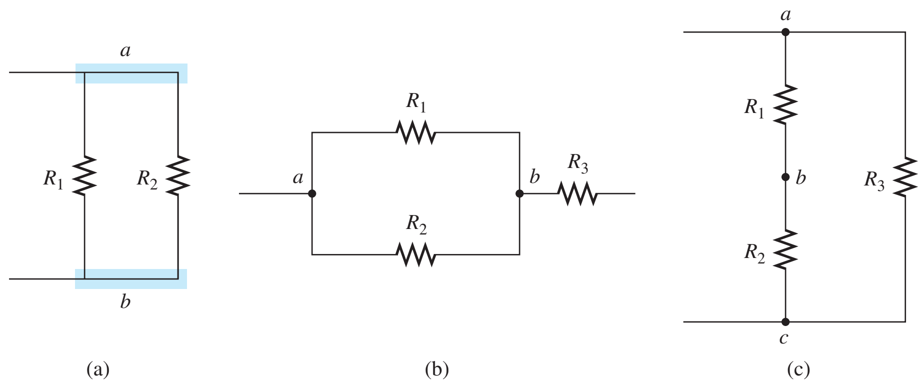


FIG. 6.1
(a) Parallel resistors; (b) R_1 and R_2 are in parallel; (c) R_3 is in parallel with the series combination of R_1 and R_2.

On schematics, the parallel combination can appear in a number of ways, as shown in Fig. 6.2. In each case, the three resistors are in parallel. They all have points *a* and *b* in common.

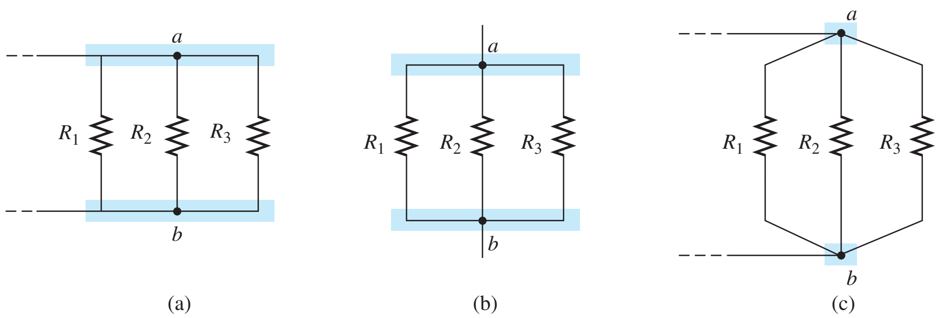


FIG. 6.2
Schematic representations of three parallel resistors.

For resistors in parallel as shown in Fig. 6.3, the total resistance is determined from the following equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \quad \textbf{(6.1)}$$

Since $G = 1/R$, the equation can also be written in terms of conductance levels as follows:

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N \quad \text{(siemens, S)} \quad \textbf{(6.2)}$$

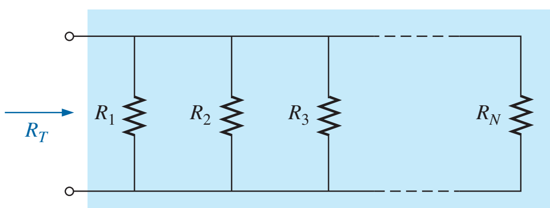


FIG. 6.3
Parallel combination of resistors.

which is an exact match in format with the equation for the total resistance of resistors in series: $R_T = R_1 + R_2 + R_3 + \cdots + R_N$. The result of this duality is that you can go from one equation to the other simply by interchanging R and G.

In general, however, when the total resistance is desired, the following format is applied:

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}} \quad \textbf{(6.3)}$$

Quite obviously, Eq. (6.3) is not as "clean" as the equation for the total resistance of series resistors. You must be careful when dealing with all the divisions into 1. The great feature about the equation, however, is that it can be applied to any number of resistors in parallel.

EXAMPLE 6.1

a. Find the total conductance of the parallel network in Fig. 6.4.
b. Find the total resistance of the same network using the results of part (a) and using Eq. (6.3).

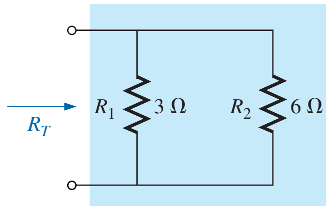


FIG. 6.4
Parallel resistors for Example 6.1.

Solutions:

a. $G_{1\pi} = \dfrac{1}{R_1} = \dfrac{1}{3\ \Omega} = 0.333\ \text{S}, \quad G_2 = \dfrac{1}{R_2} = \dfrac{1}{6\ \Omega} = 0.167\ \text{S}$

and $G_T = G_1 + G_2 = 0.333\ \text{S} + 0.167\ \text{S} = \mathbf{0.5\ S}$

b. $R_T = \dfrac{1}{G_T} = \dfrac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$

Applying Eq. (6.3)

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{1}{\dfrac{1}{3\ \Omega} + \dfrac{1}{6\ \Omega}}$$

$$= \frac{1}{0.333\ \text{S} + 0.167\ \text{S}} = \frac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$$

EXAMPLE 6.2

a. By inspection, which parallel element in Fig. 6.5 has the least conductance? Determine the total conductance of the network and note whether your conclusion was verified.

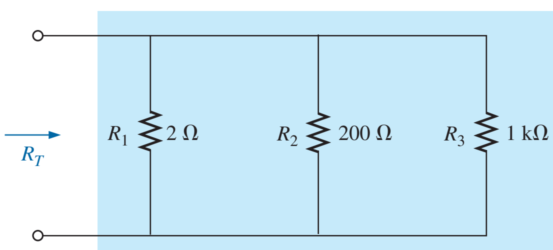


FIG. 6.5
Parallel resistors for Example 6.2.

b. Determine the total resistance from the results of part (a) and by applying Eq. (6.3).

Solutions:

a. Since the 1 kΩ resistor has the largest resistance and therefore the largest opposition to the flow of charge (level of conductivity), it will have the least level of conductance.

$$G_1 = \frac{1}{R_1} = \frac{1}{2\,\Omega} = 0.5\text{ S},\ G_2 = \frac{1}{R_2} + \frac{1}{200\,\Omega} = 0.005\text{ S} = 5\text{ mS}$$
$$G_3 = \frac{1}{R_3} = \frac{1}{1\text{ k}\Omega} = \frac{1}{1000\,\Omega} = 0.001\text{ S} = 1\text{ mS}$$
$$G_T = G_1 + G_2 + G_3 = 0.5\text{ S} + 5\text{ mS} + 1\text{ mS} = \mathbf{506\ mS}$$

Note the difference in conductance level between the 2 Ω (500 mS) and the 1 kΩ (1 mS) resistor.

b. $$R_T = \frac{1}{G_T} = \frac{1}{506\text{ mS}} = \mathbf{1.976\ \Omega}$$

Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{2\,\Omega}+\frac{1}{200\,\Omega}+\frac{1}{1\text{ k}\Omega}}$$
$$= \frac{1}{0.5\text{ S} + 0.005\text{ S} + 0.001\text{ S}} = \frac{1}{0.506\text{ S}} = \mathbf{1.98\ \Omega}$$

EXAMPLE 6.3 Find the total resistance of the configuration in Fig. 6.6.

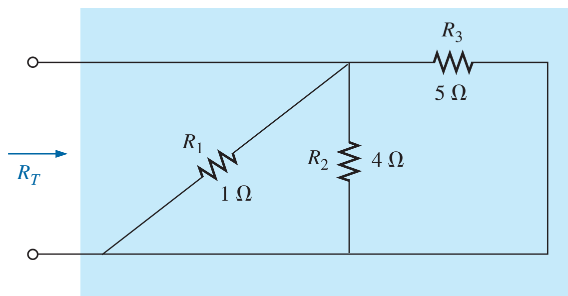


FIG. 6.6
Network to be investigated in Example 6.3.

Solution: First the network is redrawn as shown in Fig. 6.7 to clearly demonstrate that all the resistors are in parallel.

Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{1\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{5\,\Omega}}$$
$$= \frac{1}{1\text{ S} + 0.25\text{ S} + 0.2\text{ S}} = \frac{1}{1.45\text{ S}} \cong \mathbf{0.69\ \Omega}$$

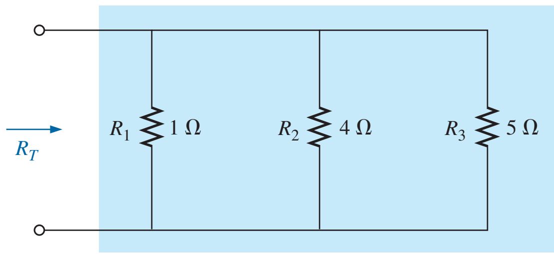


FIG. 6.7
Network in Fig. 6.6 redrawn.

If you review the examples above, you will find that the total resistance is less than the smallest parallel resistor. That is, in Example 6.1, 2 Ω is less than 3 Ω or 6 Ω. In Example 6.2, 1.976 Ω is less than 2 Ω, 100 Ω, or 1 kΩ; and in Example 6.3, 0.69 Ω is less than 1 Ω, 4 Ω, or 5 Ω. In general, therefore,

the total resistance of parallel resistors is always less than the value of the smallest resistor.

This is particularly important when you want a quick estimate of the total resistance of a parallel combination. Simply find the smallest value, and you know that the total resistance will be less than that value. It is also a great check on your calculations. In addition, you will find that

if the smallest resistor of a parallel combination is much smaller than the other parallel resistors, the total resistance will be very close to the smallest resistor value.

This fact is obvious in Example 6.2 where the total resistance of 1.976 Ω is very close to the smallest resistor of 2 Ω.

Another interesting characteristic of parallel resistors is demonstrated in Example 6.4.

EXAMPLE 6.4

a. What is the effect of adding another resistor of 100 Ω in parallel with the parallel resistors of Example 6.1 as shown in Fig. 6.8?
b. What is the effect of adding a parallel 1 Ω resistor to the configuration in Fig. 6.8?

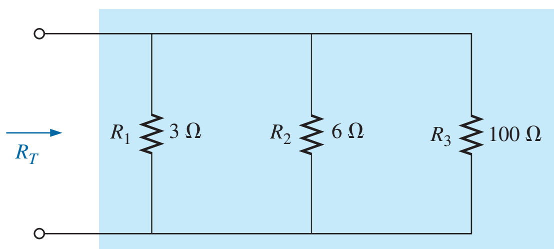


FIG. 6.8
Adding a parallel 100 Ω resistor to the network in Fig. 6.4.

Solutions:

a. Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{100\ \Omega}}$$

$$= \frac{1}{0.333\ \text{S} + 0.167\ \text{S} + 0.010\ \text{S}} = \frac{1}{0.510\ \text{S}} = \mathbf{1.96\ \Omega}$$

The parallel combination of the 3 Ω and 6 Ω resistors resulted in a total resistance of 2 Ω in Example 6.1. The effect of adding a resistor in parallel of 100 Ω had little effect on the total resistance because its resistance level is significantly higher (and conductance level significantly less) than the other two resistors. The total change in resistance was less than 2%. However, do note that the total resistance dropped with the addition of the 100 Ω resistor.

b. Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{100\ \Omega} + \frac{1}{1\ \Omega}}$$

$$= \frac{1}{0.333\ \text{S} + 0.167\ \text{S} + 0.010\ \text{S} + 1\ \text{S}} = \frac{1}{0.51\ \text{S}} = \mathbf{0.66\ \Omega}$$

The introduction of the 1 Ω resistor reduced the total resistance from 2 Ω to only 0.66 Ω—a decrease of almost 67%. The fact that the added resistor has a resistance level less than the other parallel elements and one-third that of the smallest contributed to the significant drop in resistance level.

In part (a) of Example 6.4, the total resistance dropped from 2 Ω to 1.96 Ω. In part (b), it dropped to 0.66 Ω. The results clearly reveal that

the total resistance of parallel resistors will always drop as new resistors are added in parallel, irrespective of their value.

Recall that this is the opposite of series resistors, where additional resistors of any value increase the total resistance.

For equal resistors in parallel, the equation for the total resistance becomes significantly easier to apply. For N equal resistors in parallel, Eq. (6.3) becomes

$$R_T = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R_N}}$$

$$= \frac{1}{N\left(\frac{1}{R}\right)} = \frac{1}{\frac{N}{R}}$$

and

$$\boxed{R_T = \frac{R}{N}} \qquad \textbf{(6.4)}$$

In other words,

the total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.

EXAMPLE 6.5 Find the total resistance of the parallel resistors in Fig. 6.9.

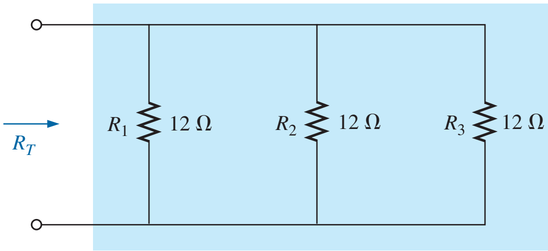


FIG. 6.9
Three equal parallel resistors to be investigated in Example 6.5.

Solution: Applying Eq. (6.4):

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

EXAMPLE 6.6 Find the total resistance for the configuration in Fig. 6.10.

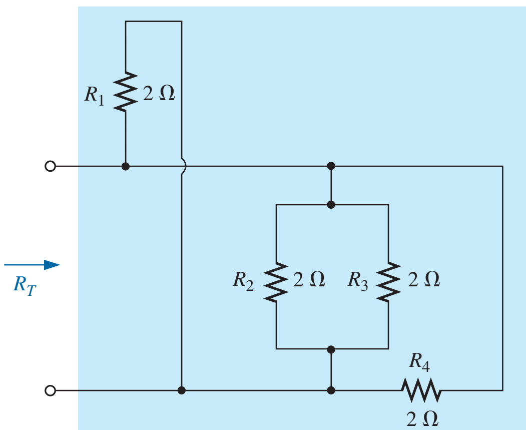


FIG. 6.10
Parallel configuration for Example 6.6.

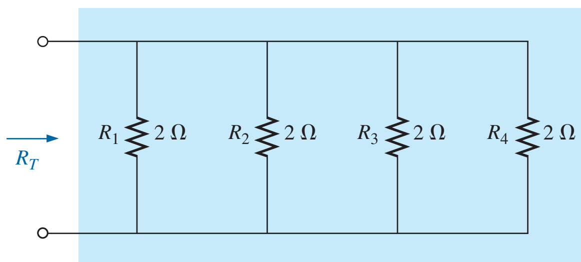


FIG. 6.11
Network in Fig. 6.10 redrawn.

Solution: Redrawing the network results in the parallel network in Fig. 6.11.

Applying Eq. (6.4):

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Special Case: Two Parallel Resistors

In the vast majority of cases, only two or three parallel resistors will have to be combined. With this in mind, an equation has been derived for two parallel resistors that is easy to apply and removes the need to continually worry about dividing into 1 and possibly misplacing a decimal point. For three parallel resistors, the equation to be derived here can be applied twice, or Eq. (6.3) can be used.

For two parallel resistors, the total resistance is determined by Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistor results in

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

and

$$R_T = \frac{R_1R_2}{R_1 + R_2} \qquad \textbf{(6.5)}$$

In words, the equation states that

the total resistance of two parallel resistors is simply the product of their values divided by their sum.

EXAMPLE 6.7 Repeat Example 6.1 using Eq. (6.5).

Solution: Eq. (6.5):

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = \frac{18}{9}\,\Omega = \mathbf{2\,\Omega}$$

which matches the earlier solution.

EXAMPLE 6.8 Determine the total resistance for the parallel combination in Fig. 6.7 using two applications of Eq. (6.5).

Solution: First the 1 Ω and 4 Ω resistors are combined using Eq. (6.5), resulting in the reduced network in Fig. 6.12.

Eq. (6.4): $R'_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(1\,\Omega)(4\,\Omega)}{1\,\Omega + 4\,\Omega} = \frac{4}{5}\,\Omega = 0.8\,\Omega$

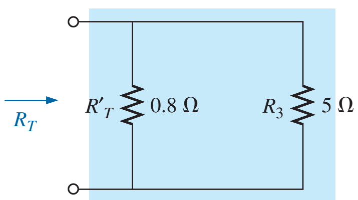


FIG. 6.12
Reduced equivalent in Fig. 6.7.

Then Eq. (6.5) is applied again using the equivalent value:

$$R_T = \frac{R'_T R_3}{R'_T + R_3} = \frac{(0.8\,\Omega)(5\,\Omega)}{0.8\,\Omega + 5\,\Omega} = \frac{4}{5.8}\,\Omega = \mathbf{0.69\,\Omega}$$

The result matches that obtained in Example 6.3.

Recall that series elements can be interchanged without affecting the magnitude of the total resistance. In parallel networks,

parallel resistors can be interchanged without affecting the total resistance.

The next example demonstrates this and reveals how redrawing a network can often define which operations or equations should be applied.

EXAMPLE 6.9 Determine the total resistance of the parallel elements in Fig. 6.13.

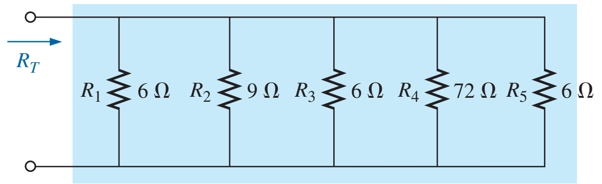


FIG. 6.13
Parallel network for Example 6.9.

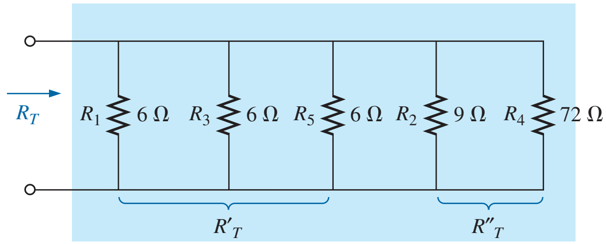


FIG. 6.14
Redrawn network in Fig. 6.13 (Example 6.9).

Solution: The network is redrawn in Fig. 6.14.

Eq. (6.4): $$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

Eq. (6.5): $$R''_T = \frac{R_2R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648}{81}\ \Omega = 8\ \Omega$$

Eq. (6.5): $$R_T = \frac{R'_TR''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16}{10}\ \Omega = \mathbf{1.6\ \Omega}$$

The preceding examples involve direct substitution; that is, once the proper equation has been defined, it is only a matter of plugging in the numbers and performing the required algebraic manipulations. The next two examples have a design orientation, in which specific network parameters are defined and the circuit elements must be determined.

EXAMPLE 6.10 Determine the value of R_2 in Fig. 6.15 to establish a total resistance of 9 kΩ.

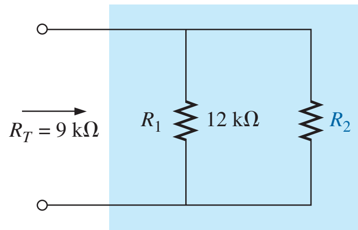


FIG. 6.15
Parallel network for Example 6.10.

Solution:

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$

$$R_T(R_1 + R_2) = R_1R_2$$

$$R_TR_1 + R_TR_2 = R_1R_2$$

$$R_TR_1 = R_1R_2 - R_TR_2$$

$$R_TR_1 = (R_1 - R_T)R_2$$

and $$R_2 = \frac{R_TR_1}{R_1 - R_T}$$

Substituting values:

$$R_2 = \frac{(9\ \text{k}\Omega)(12\ \text{k}\Omega)}{12\ \text{k}\Omega - 9\ \text{k}\Omega} = \frac{108}{3}\ \text{k}\Omega = \mathbf{36\ k\Omega}$$

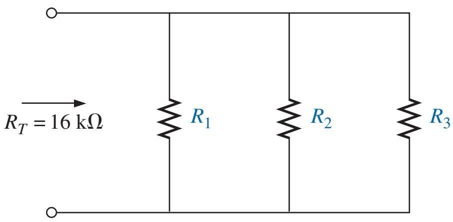


FIG. 6.16
Parallel network for Example 6.11.

EXAMPLE 6.11 Determine the values of R_1, R_2 and R_3 in Fig. 6.16 if $R_2 = 2R_1$, $R_3 = 2R_2$, and the total resistance is 16 kΩ.

Solution: Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However, $\quad R_2 = 2R_1 \quad$ and $\quad R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that
$$\frac{1}{16\ k\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and
$$\frac{1}{16\ k\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

or
$$\frac{1}{16\ k\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

resulting in $\quad R_1 = 1.75(16\ k\Omega) = \mathbf{28\ k\Omega}$

so that $\quad R_2 = 2R_1 = 2(28\ k\Omega) = \mathbf{56\ k\Omega}$

and $\quad R_3 = 2R_2 = 2(56\ k\Omega) = \mathbf{112\ k\Omega}$

Analogies

Analogies were effectively used to introduce the concept of parallel elements. They can also be used to help define *parallel configuration.* On a ladder, the rungs of the ladder form a parallel configuration. When ropes are tied between a grappling hook and a load, they effectively absorb the stress in a parallel configuration. The cables of a suspended roadway form a parallel configuration. There are numerous other analogies that demonstrate how connections between the same two points permit a distribution of stress between the parallel elements.

Instrumentation

As shown in Fig. 6.17, the total resistance of a parallel combination of resistive elements can be found by simply applying an ohmmeter. There is no polarity to resistance, so either lead of the ohmmeter can be connected to either side of the network. Although there are no supplies in Fig. 6.17, always keep in mind that ohmmeters can never be applied to a "live" circuit. It is not enough to set the supply to 0 V or to turn it off. It may still

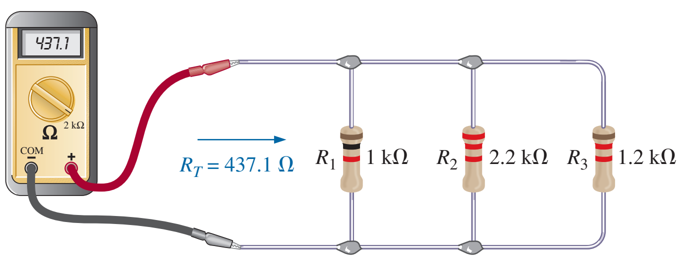


FIG. 6.17
Using an ohmmeter to measure the total resistance of a parallel network.

load down (change the network configuration of) the circuit and change the reading. It is best to remove the supply and apply the ohmmeter to the two resulting terminals. Since all the resistors are in the kilohm range, the 20 kΩ scale was chosen first. We then moved down to the 2 kΩ scale for increased precision. Moving down to the 200 Ω scale resulted in an "OL" indication since we were below the measured resistance value.

6.3 PARALLEL CIRCUITS

A **parallel circuit** can now be established by connecting a supply across a set of parallel resistors as shown in Fig. 6.18. The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor. Therefore, it should be quite clear that the applied voltage is the same across each resistor. In general,

the voltage is always the same across parallel elements.

Therefore, remember that

if two elements are in parallel, the voltage across them must be the same. However, if the voltage across two neighboring elements is the same, the two elements may or may not be in parallel.

The reason for this qualifying comment in the above statement is discussed in detail in Chapter 7.

For the voltages of the circuit in Fig. 6.18, the result is that

$$V_1 = V_2 = E \tag{6.6}$$

Once the supply has been connected, a source current is established through the supply that passes through the parallel resistors. The current that results is a direct function of the total resistance of the parallel circuit. The smaller the total resistance, the more the current, as occurred for series circuits also.

Recall from series circuits that the source does not "see" the parallel combination of elements. It reacts only to the total resistance of the circuit, as shown in Fig. 6.19. The source current can then be determined using Ohm's law:

$$I_s = \frac{E}{R_T} \tag{6.7}$$

Since the voltage is the same across parallel elements, the current through each resistor can also be determined using Ohm's law. That is,

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} \tag{6.8}$$

The direction for the currents is dictated by the polarity of the voltage across the resistors. Recall that for a resistor, current enters the positive side of a potential drop and leaves the negative. The result, as shown in Fig. 6.18, is that the source current enters point *a,* and currents I_1 and I_2 leave the same point. An excellent analogy for describing the flow of charge through the network of Fig. 6.18 is the flow of water through the parallel pipes of Fig. 6.20. The larger pipe with less "resistance" to the

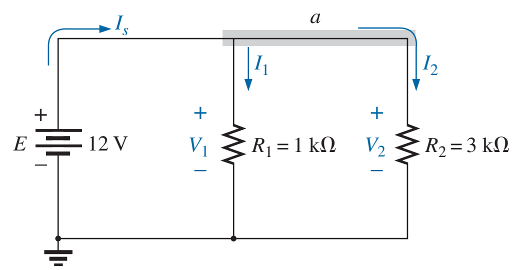


FIG. 6.18
Parallel network.

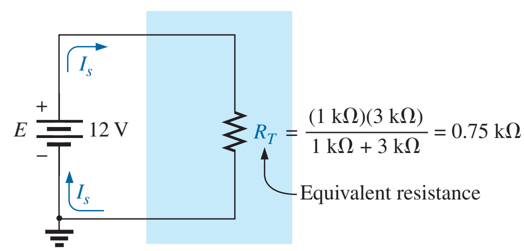


FIG. 6.19
Replacing the parallel resistors in Fig. 6.18 with the equivalent total resistance.

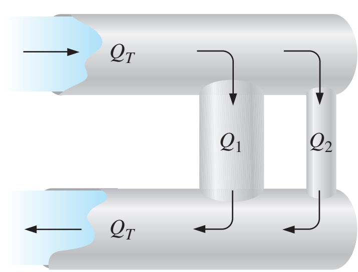


FIG. 6.20
Mechanical analogy for Fig. 6.18.

flow of water will have a larger flow of water through it. The thinner pipe with its increased "resistance" level will have less water through it. In any case, the total water entering the pipes at the top Q_T must equal that leaving at the bottom, with $Q_T = Q_1 + Q_2$.

The relationship between the source current and the parallel resistor currents can be derived by simply taking the equation for the total resistance in Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying both sides by the applied voltage:

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

resulting in

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

Then note that $E/R_1 = I_1$ and $E/R_2 = I_2$ to obtain

$$I_s = I_1 + I_2 \qquad \textbf{(6.9)}$$

The result reveals a very important property of parallel circuits:

For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

The duality that exists between series and parallel circuits continues to surface as we proceed through the basic equations for electric circuits. This is fortunate because it provides a way of remembering the characteristics of one using the results of another. For instance, in Fig. 6.21(a), we have a parallel circuit where it is clear that $I_T = I_1 + I_2$. By simply replacing the currents of the equation in Fig. 6.21(a) by a voltage level, as shown in Fig. 6.21(b), we have Kirchhoff's voltage law for a series circuit: $E = V_1 + V_2$. In other words,

for a parallel circuit, the source current equals the sum of the branch currents, while for a series circuit, the applied voltage equals the sum of the voltage drops.

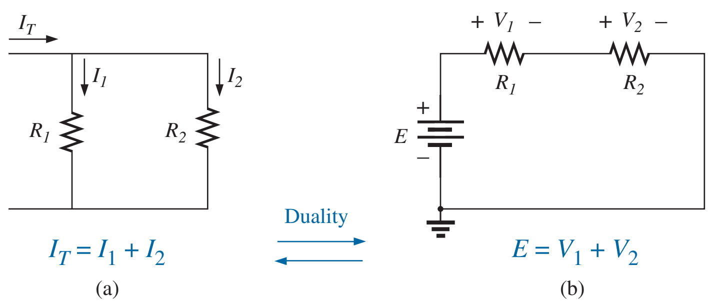


FIG. 6.21
Demonstrating the duality that exists between series and parallel circuits.

EXAMPLE 6.12 For the parallel network in Fig. 6.22:

a. Find the total resistance.
b. Calculate the source current.
c. Determine the current through each parallel branch.
d. Show that Eq. (6.9) is satisfied.

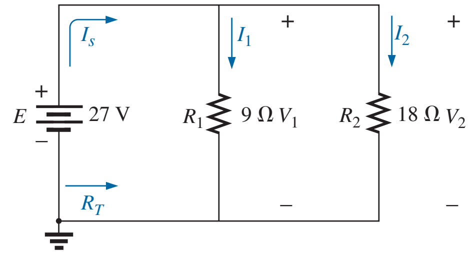


FIG. 6.22
Parallel network for Example 6.12.

Solutions:

a. Using Eq. (6.5):

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \frac{162}{27}\ \Omega = \mathbf{6\ \Omega}$$

b. Applying Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{27\ \text{V}}{6\ \Omega} = \mathbf{4.5\ A}$$

c. Applying Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27\ \text{V}}{9\ \Omega} = \mathbf{3\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27\ \text{V}}{18\ \Omega} = \mathbf{1.5\ A}$$

d. Substituting values from parts (b) and (c):

$$I_s = \mathbf{4.5\ A} = I_1 + I_2 = 3\ \text{A} + 1.5\ \text{A} = \mathbf{4.5\ A} \quad \text{(checks)}$$

EXAMPLE 6.13 For the parallel network in Fig. 6.23.

a. Find the total resistance.
b. Calculate the source current.
c. Determine the current through each branch.

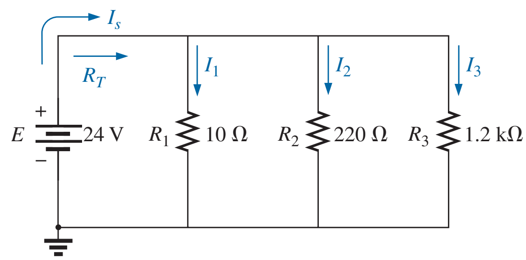


FIG. 6.23
Parallel network for Example 6.13.

Solutions:

a. Applying Eq. (6.3):

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = \frac{1}{\dfrac{1}{10\ \Omega} + \dfrac{1}{220\ \Omega} + \dfrac{1}{1.2\ \text{k}\Omega}}$$

$$= \frac{1}{100 \times 10^{-3} + 4.545 \times 10^{-3} + 0.833 \times 10^{-3}} = \frac{1}{105.38 \times 10^{-3}}$$

$$R_T = \mathbf{9.49\ \Omega}$$

Note that the total resistance is less than the smallest parallel resistor, and the magnitude is very close to the smallest resistor because the other resistors are larger by a factor greater than 10 : 1.

b. Using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{9.49\ \Omega} = \mathbf{2.53\ A}$$

c. Applying Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\text{ V}}{10\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\text{ V}}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\text{ V}}{1.2\text{ k}\Omega} = \mathbf{0.02\ A}$$

A careful examination of the results of Example 6.13 reveals that the larger the parallel resistor, the smaller the branch current. In general, therefore,

for parallel resistors, the greatest current will exist in the branch with the least resistance.

A more powerful statement is that

current always seeks the path of least resistance.

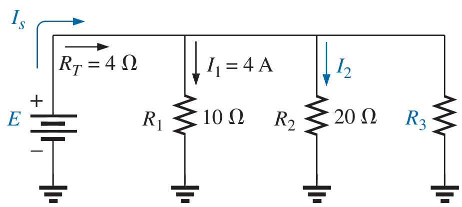


FIG. 6.24
Parallel network for Example 6.14.

EXAMPLE 6.14 Given the information provided in Fig. 6.24.

a. Determine R_3.
b. Find the applied voltage E.
c. Find the source current I_s.
d. Find I_2.

Solutions:

a. Applying Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Substituting: $$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$

so that $$0.25\text{ S} = 0.1\text{ S} + 0.05\text{ S} + \frac{1}{R_3}$$

and $$0.25\text{ S} = 0.15\text{ S} + \frac{1}{R_3}$$

with $$\frac{1}{R_3} = 0.1\text{ S}$$

and $$R_3 = \frac{1}{0.1\text{ S}} = \mathbf{10\ \Omega}$$

b. Using Ohm's law:

$$E = V_1 = I_1R_1 = (4\text{ A})(10\ \Omega) = \mathbf{40\ V}$$

c. $$I_s = \frac{E}{R_T} = \frac{40\text{ V}}{4\ \Omega} = \mathbf{10\ A}$$

d. Applying Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40\text{ V}}{20\ \Omega} = \mathbf{2\ A}$$

Mathcad Solution: This example provides an excellent opportunity to practice our skills using Mathcad. As shown in Fig. 6.25, the known parameters and quantities of the network are entered first, followed by an equation for the unknown resistor R_3. Note that after the first division operator is selected, a left bracket appears (to be followed eventually by a right enclosure bracket) to tell the computer that the mathematical operations in the denominator must be carried out first before the division into 1. In addition, each individual division into 1 is separated by brackets to ensure that the division operation is performed before each quantity is added to the neighboring factor. Finally, keep in mind that the Mathcad bracket must encompass each individual expression of the denominator before you place the right bracket in place.

This example shows that when units are added to all the parameters, the results appear with the proper unit of measurement. The units for each are entered by entering the magnitude of the quantity followed by the multiplication operation. Select the **Insert** option from the menu at the top of the screen and then select **Unit.** The **Unit** dialog box appears. For the resistor RT, select **Resistance** under the **Dimension** scroll list followed by Ohm (Ω) under the Unit listing. Click **OK,** and the unit of measurement appears beside the magnitude of the quantity.

After you enter the equation for R3, you can obtain the value of R3 by typing **R3** followed by the equal sign. As soon as you enter the equal sign,

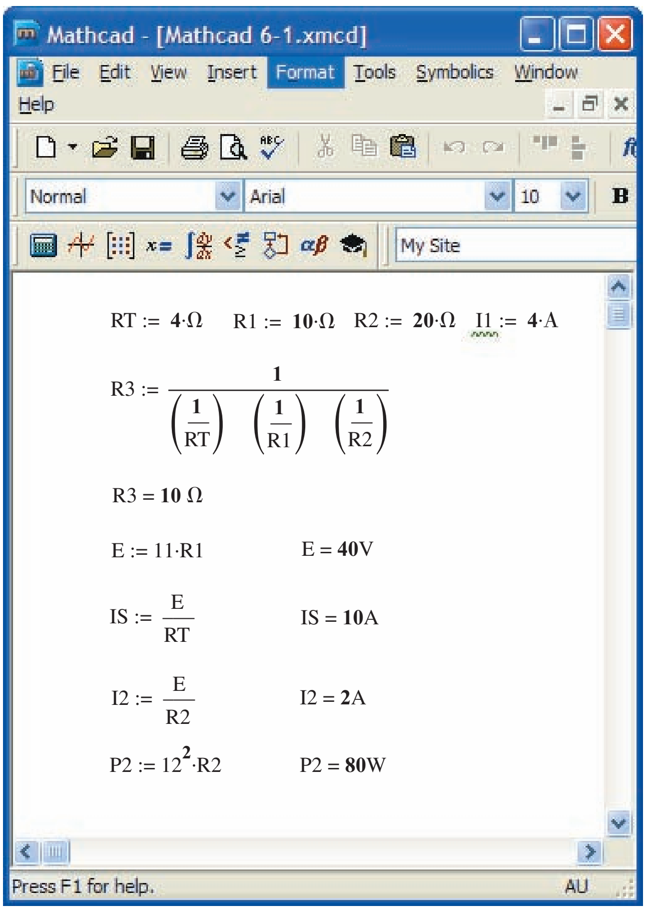


FIG. 6.25
Using Mathcad to confirm the results of Example 6.14.

the result of 10 Ω appears. Enter the equation for E. When you retype **E** followed by the equal sign, the result of 40 V appears. The remaining unknowns can then be found in the same manner.

One final note. You will find when you enter the variable **I1** and the equal sign, the word *function* appears. This is because the capital letter I is reserved for a defined Bessel function in Mathcad 12. However, you can override this result by using the sequence SHIFT-colon to enter the value of I1.

In each case, the quantity of interest was entered below the defining equation to obtain the numerical result by selecting an equal sign. As expected, all the results match the longhand solution.

Instrumentation

In Fig. 6.26, voltmeters have been connected to verify that the voltage across parallel elements is the same. Note that the positive or red lead of each voltmeter is connected to the high (positive) side of the voltage across each resistor to obtain a positive reading. The 20 V scale was used because the applied voltage exceeded the range of the 2 V scale.

In Fig. 6.27, an ammeter has been hooked up to measure the source current. First, the connection to the supply had to be broken at the positive terminal and the meter inserted as shown. Be sure to use ammeter terminals on your meter for such measurements. The red or positive lead of

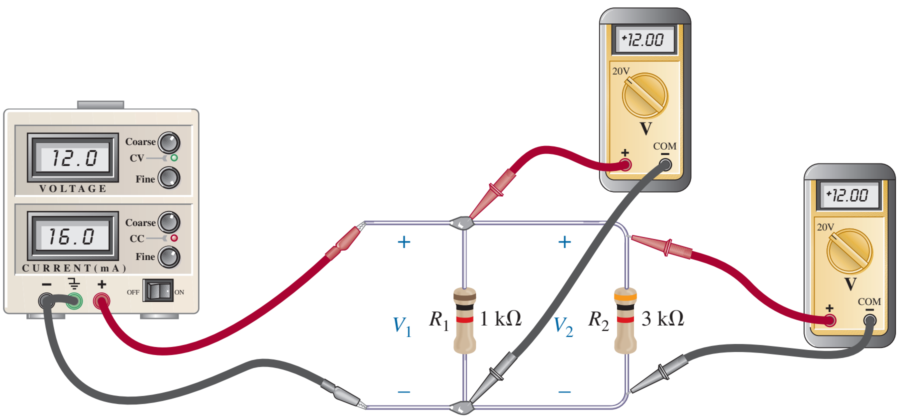


FIG. 6.26
Measuring the voltages of a parallel dc network.

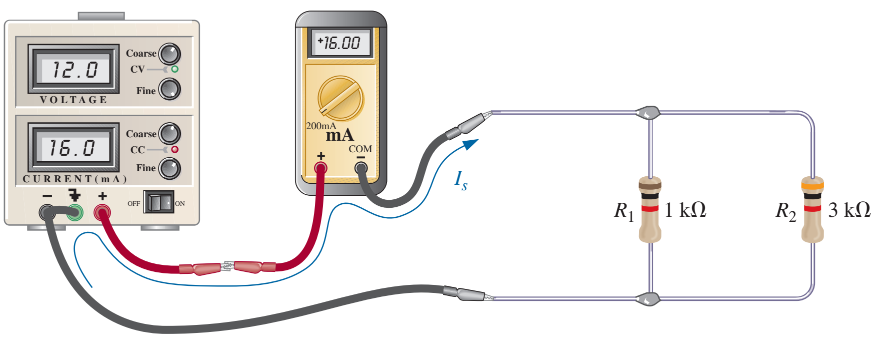


FIG. 6.27
Measuring the source current of a parallel network.

the meter is connected so that the source current enters that lead and leaves the negative or black lead to ensure a positive reading. The 200 mA scale was used because the source current exceeded the maximum value of the 2 mA scale. For the moment, we assume that the internal resistance of the meter can be ignored. Since the internal resistance of an ammeter on the 200 mA scale is typically only a few ohms, compared to the parallel resistors in the kilohm range, it is an excellent assumption.

A more difficult measurement is for the current through resistor R_1. This measurement often gives trouble in the laboratory session. First, as shown in Fig. 6.28(a), resistor R_1 must be disconnected from the upper connection point to establish an open circuit. The ammeter is then inserted between the resulting terminals so that the current enters the positive or red terminal, as shown in Fig. 6.28(b). Always remember: When using an ammeter, first establish an open circuit in the branch in which the current is to be measured, and then insert the meter.

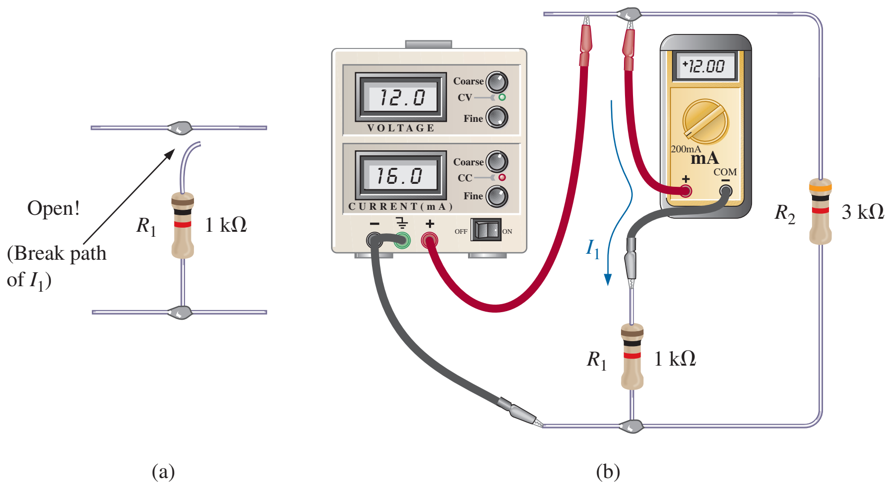


FIG. 6.28
Measuring the current through resistor R_1.

The easiest measurement is for the current through resistor R_2. Break the connection to R_2 above or below the resistor, and insert the ammeter with the current entering the positive or red lead to obtain a positive reading.

6.4 POWER DISTRIBUTION IN A PARALLEL CIRCUIT

Recall from the discussion of series circuits that the power applied to a series resistive circuit equals the power dissipated by the resistive elements. The same is true for parallel resistive networks. In fact,

for any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.

For the parallel circuit in Fig. 6.29:

$$P_E = P_{R_1} + P_{R_2} + P_{R_3} \qquad \textbf{(6.10)}$$

which is exactly the same as obtained for the series combination.

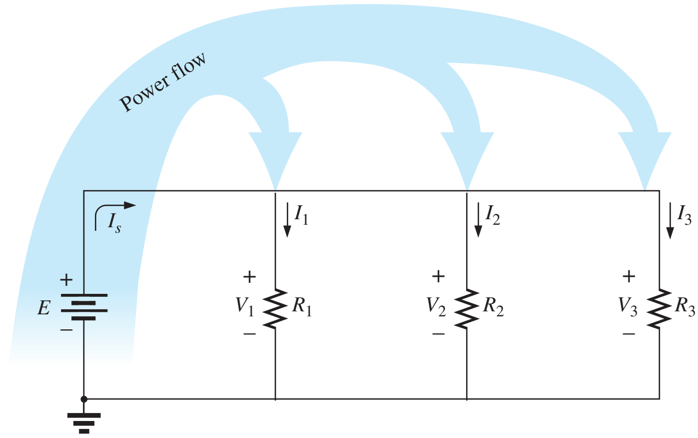


FIG. 6.29
Power flow in a dc parallel network.

The power delivered by the source in the same:

$$P_E = EI_s \qquad \text{(watts, W)} \tag{6.11}$$

as is the equation for the power to each resistor (shown for R_1 only):

$$P_1 = V_1I_1 = I_1^2R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)} \tag{6.12}$$

In the equation $P = V^2/R$, the voltage across each resistor in a parallel circuit will be the same. The only factor that changes is the resistance in the denominator of the equation. The result is that

in a parallel resistive network, the larger the resistor, the less the power absorbed.

EXAMPLE 6.15 For the parallel network in Fig. 6.30 (all standard values):

a. Determine the total resistance R_T.
b. Find the source current and the current through each resistor.
c. Calculate the power delivered by the source.

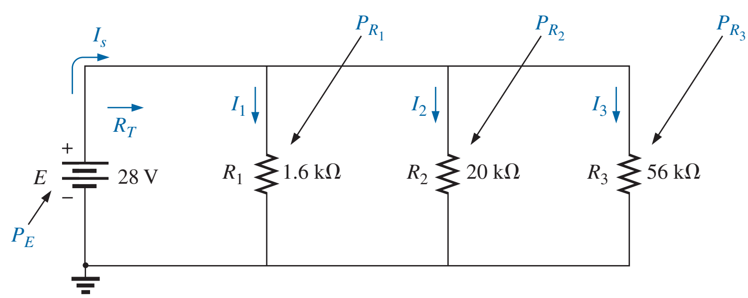


FIG. 6.30
Parallel network for Example 6.15.

d. Determine the power absorbed by each parallel resistor.
e. Verify Eq. (6.10).

Solutions:

a. Without making a single calculation, it should now be apparent from previous examples that the total resistance is less than 1.6 kΩ and very close to this value because of the magnitude of the other resistance levels.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1.6\text{ k}\Omega} + \frac{1}{20\text{ k}\Omega} + \frac{1}{56\text{ k}\Omega}}$$

$$= \frac{1}{625 \times 10^{-6} + 50 \times 10^{-6} + 17.867 \times 10^{-6}} = \frac{1}{692.867 \times 10^{-6}}$$

and $R_T = \mathbf{1.44\text{ k}\Omega}$

b. Applying Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{28\text{ V}}{1.44\text{ k}\Omega} = \mathbf{19.44\text{ mA}}$$

Recalling that current always seeks the path of least resistance immediately tells us that the current through the 1.6 kΩ resistor will be the largest and the current through the 56 kΩ resistor the smallest.

Applying Ohm's law again:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\text{ V}}{1.6\text{ k}\Omega} = \mathbf{17.5\text{ mA}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\text{ V}}{20\text{ k}\Omega} = \mathbf{1.4\text{ mA}}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\text{ V}}{56\text{ k}\Omega} = \mathbf{0.5\text{ mA}}$$

c. Applying Eq. (6.11):

$$P_E = EI_s = (28\text{ V})\,(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$$

d. Applying each form of the power equation:

$$P_1 = V_1I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$$

$$P_2 = I_2^2R_2 = (1.4\text{ mA})^2(20\text{ k}\Omega) = \mathbf{39.2\text{ mW}}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\text{ mW}}$$

A review of the results clearly substantiates the fact that the larger the resistor, the less the power absorbed.

e. $$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}} \quad \text{(checks)}$$

6.5 KIRCHHOFF'S CURRENT LAW

In the previous chapter, Kirchhoff's voltage law was introduced, providing a very important relationship between the voltages of a closed path.

Professor Gustav Kirchhoff is also credited with developing the following equally important relationship between the currents of a network, called **Kirchhoff's current law (KCL):**

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

The law can also be stated in the following way:

The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

In equation form, the above statement can be written as follows:

$$\boxed{\Sigma I_i = \Sigma I_o} \qquad \textbf{(6.13)}$$

with I_i representing the current entering, or "in," and I_o representing the current leaving, or "out."

In Fig. 6.31, for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving, as required by Eq. (6.13):

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A}} = \mathbf{12\text{ A}} \quad \text{(checks)}$$

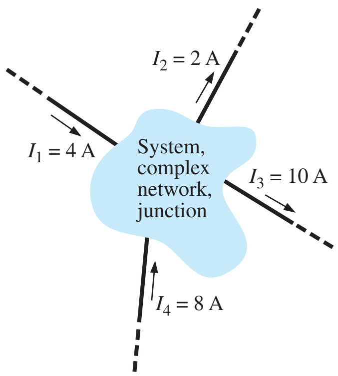


FIG. 6.31
Introducing Kirchhoff's current law.

The most common application of the law will be at a junction of two or more current paths, as shown in Fig. 6.32(a). Some students have difficulty initially determining whether a current is entering or leaving a junction. One approach that may help is to use the water analog in Fig. 6.32(b) where the junction in Fig. 6.32(a) is the small bridge across the stream. Simply relate the current of I_1 to the fluid flow of Q_1, the smaller branch current I_2 to the water flow Q_2, and the larger branch current I_3 to the flow Q_3. The water arriving at the bridge must equal the sum of that leaving the bridge so that $Q_1 = Q_2 + Q_3$. Since the current I_1 is pointing *at* the junction and the fluid flow Q_1 is *toward* the person on the bridge, both quantities are seen as approaching the junction and can be considered *entering* the junction. The currents I_2 and I_3 are both leaving the junction just as Q_2 and Q_3 are leaving the fork in the river. The quantities I_2, I_3, Q_2, and Q_3 are therefore all *leaving* the junction.

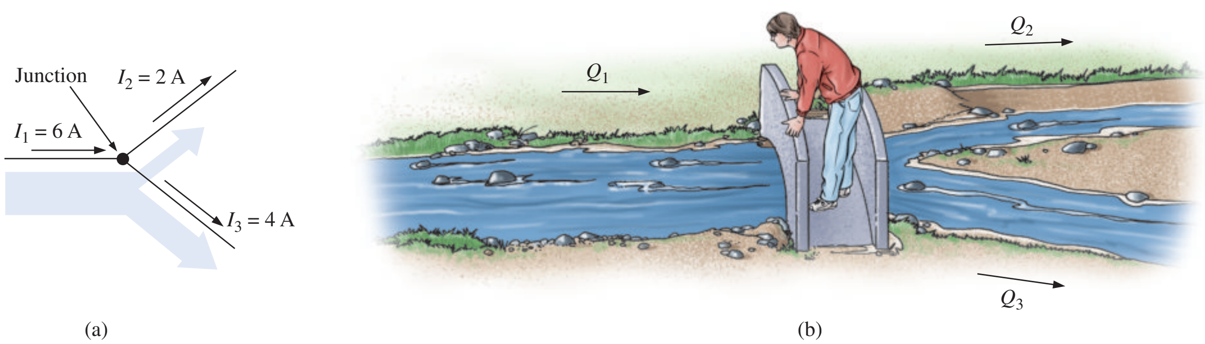


FIG. 6.32
(a) Demonstrating Kirchhoff's current law; (b) the water analogy for the junction in (a).

In the next few examples, unknown currents can be determined by applying Kirchhoff's current law. Remember to place all current levels entering the junction to the left of the equals sign and the sum of all currents leaving the junction to the right of the equals sign.

In technology, the term **node** is commonly used to refer to a junction of two or more branches. Therefore, this term is used frequently in the analyses to follow.

EXAMPLE 6.16 Determine currents I_3 and I_4 in Fig. 6.33 using Kirchhoff's current law.

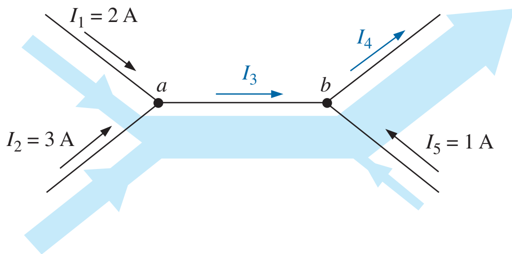


FIG. 6.33
Two-node configuration for Example 6.16.

Solution: There are two junctions or nodes in Fig. 6.33. Node *a* has only one unknown, while node *b* has two unknowns. Since a single equation can be used to solve for only one unknown, we must apply Kirchhoff's current law to node *a* first.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\text{ A}}$$

At node *b,* using the result just obtained:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\text{ A}}$$

Note that in Fig. 6.33, the width of the blue shaded regions matches the magnitude of the current in that region.

EXAMPLE 6.17 Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 6.34.

Solution: In this configuration, four nodes are defined. Nodes *a* and *c* have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$

and $$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\text{ A}}$$

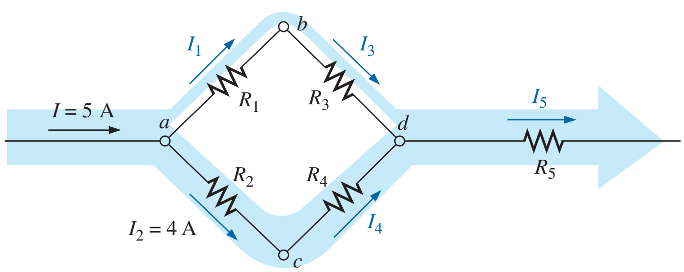


FIG. 6.34
Four-node configuration for Example 6.17.

At node *c:*

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$
and $$I_4 = I_2 = \mathbf{4\ A}$$

Using the above results at the other junctions results in the following.

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
and $$I_3 = I_1 = \mathbf{1\ A}$$

At node *d:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\ A}$$

If we enclose the entire network, we find that the current entering from the far left is $I = 5$ A, while the current leaving from the far right is $I_5 = 5$ A. The two must be equal since the net current entering any system must equal the net current leaving.

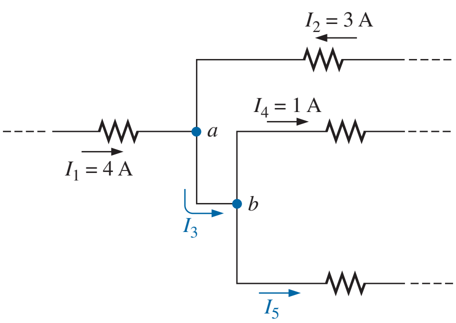


FIG. 6.35
Network for Example 6.18.

EXAMPLE 6.18 Determine currents I_3 and I_5 in Fig. 6.35 through applications of Kirchhoff's current law.

Solution: Note first that since node *b* has two unknown quantities (I_3 and I_5), and node *a* has only one, Kirchhoff's current law must first be applied to node *a.* The result is then applied to node *b.*

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$4\text{ A} + 3\text{ A} = I_3 = \mathbf{7\ A}$$

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 = I_4 + I_5$$
$$7\text{ A} = 1\text{ A} + I_5$$
and $$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\ A}$$

EXAMPLE 6.19 For the parallel dc network in Fig. 6.36.

a. Determine the source current I_s.
b. Find the source voltage *E.*

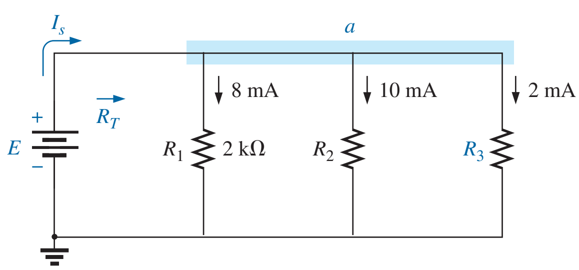


FIG. 6.36
Parallel network for Example 6.19.

c. Determine R_3.
d. Calculate R_T.

Solutions:

a. First apply Eq. (6.13) at node *a*. Although node *a* in Fig. 6.36 may not initially appear as a single junction, it can be redrawn as shown in Fig. 6.37, where it is clearly a common point for all the branches.

The result is

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$

Substituting values: $I_s = 8\text{ mA} + 10\text{ mA} + 2\text{ mA} = \mathbf{20\text{ mA}}$

Note in this solution that you do not need to know the resistor values or the voltage applied. The solution is determined solely by the current levels.

b. Applying Ohm's law:

$$E = V_1 = I_1R_1 = (8\text{ mA})(2\text{ k}\Omega) = \mathbf{16\text{ V}}$$

c. Applying Ohm's law in a different form:

$$R_3 = \frac{V_3}{I_3} = \frac{E}{I_3} = \frac{16\text{ V}}{2\text{ mA}} = \mathbf{8\text{ k}\Omega}$$

d. Applying Ohm's law again:

$$R_T = \frac{E}{I_s} = \frac{16\text{ V}}{20\text{ mA}} = \mathbf{0.8\text{ k}\Omega}$$

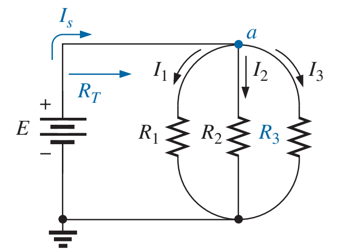


FIG. 6.37
Redrawn network in Fig. 6.36.

The application of Kirchhoff's current law is not limited to networks where all the internal connections are known or visible. For instance, all the currents of the integrated circuit in Fig. 6.38 are known except I_1. By treating the entire system (which could contain over a million elements) as a single node, we can apply Kirchhoff's current law as shown in Example 6.20.

Before looking at Example 6.20 in detail, note that the direction of the unknown current I_1 is not provided in Fig. 6.38. On many occasions, this will be true. With so many currents entering or leaving the system, it is difficult to know by inspection which direction should be assigned to I_1. *In such cases, simply make an assumption about the direction and then check out the result. If the result is negative, the wrong direction was assumed. If the result is positive, the correct direction was assumed. In either case, the magnitude of the current will be correct.*

EXAMPLE 6.20 Determine I_1 for the integrated circuit in Fig. 6.38.

Solution: Assuming that the current I_1 entering the chip results in the following when Kirchhoff's current law is applied:

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + 10\text{ mA} + 4\text{ mA} + 8\text{ mA} = 5\text{ mA} + 4\text{ mA} + 2\text{ mA} + 6\text{ mA}$$
$$I_1 + 22\text{ mA} = 17\text{ mA}$$
$$I_1 = 17\text{ mA} - 22\text{ mA} = \mathbf{-5\text{ mA}}$$

We find that the direction for I_1 is *leaving* the IC, although the magnitude of 5 mA is correct.

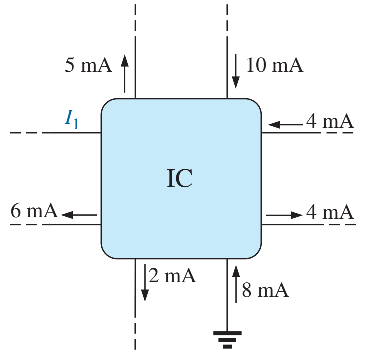


FIG. 6.38
Integrated circuit for Example 6.20.

As we leave this important section, be aware that Kirchhoff's current law will be applied in one form or another throughout the text. *Kirchhoff's laws are unquestionably two of the most important in this field because they are applicable to the most complex configurations in existence today.* They will not be replaced by a more important law or dropped for a more sophisticated approach.

6.6 CURRENT DIVIDER RULE

For series circuits we have the powerful voltage divider rule for finding the voltage across a resistor in a series circuit. We now introduce the equally powerful **current divider rule (CDR)** for finding the current through a resistor in a parallel circuit.

In Section 6.4, it was pointed out that current will always seek the path of least resistance. In Fig. 6.39, for example, the current of 9 A is faced with splitting between the three parallel resistors. Based on the previous sections, it should now be clear without a single calculation that the majority of the current will pass through the smallest resistor of 10 Ω, and the least current will pass through the 1 kΩ resistor. In fact, the current through the 100 Ω resistor will also exceed that through the 1 kΩ resistor. We can take it one step further by recognizing that the resistance of the 100 Ω resistor is 10 times that of the 10 Ω resistor. The result is a current through the 10 Ω resistor that is 10 times that of the 100 Ω resistor. Similarly, the current through the 100 Ω resistor is 10 times that through the 1 kΩ resistor.

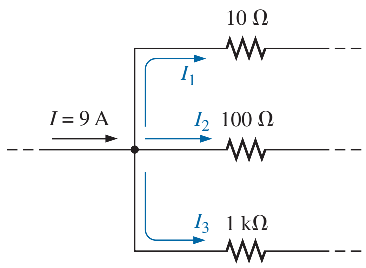


FIG. 6.39
Discussing the manner in which the current will split between three parallel branches of different resistive value.

In general,

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

EXAMPLE 6.21

a. Determine currents I_1 and I_3 for the network in Fig. 6.40.
b. Find the source current I_s.

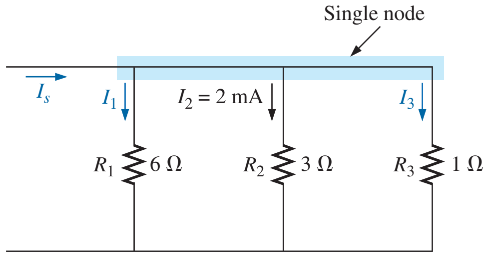


FIG. 6.40
Parallel network for Example 6.21.

Solutions:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\text{ mA}}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\text{ mA}}$$

b. Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\text{ mA}}$$

Although the above discussions and examples allowed us to determine the relative magnitude of a current based on a known level, they do not provide the magnitude of a current through a branch of a parallel network

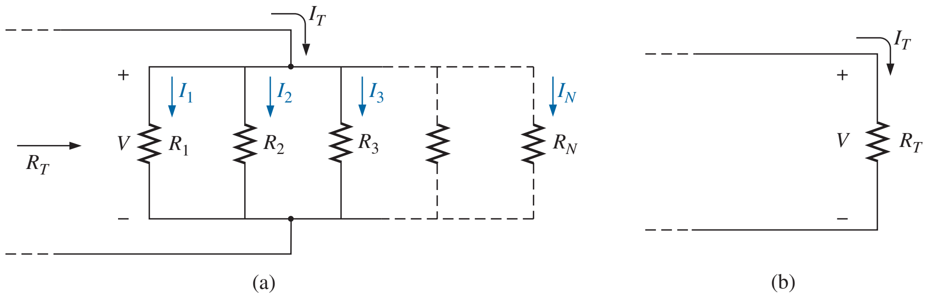


FIG. 6.41
Deriving the current divider rule: (a) parallel network of N parallel resistors; (b) reduced equivalent of part (a).

if only the total entering current is known. The result is a need for the current divider rule which will be derived using the parallel configuration in Fig. 6.41(a). The current I_T (using the subscript T to indicate the total entering current) splits between the N parallel resistors and then gathers itself together again at the bottom of the configuration. In Fig. 6.41(b), the parallel combination of resistors has been replaced by a single resistor equal to the total resistance of the parallel combination as determined in the previous sections.

The current I_T can then be determined using Ohm's law:

$$I_T = \frac{V}{R_T}$$

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

where the product I_xR_x refers to any combination in the series.

Substituting for V in the above equation for I_T, we have

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for I_x, the final result is the **current divider rule:**

$$I_x = \frac{R_T}{R_x}I_T \qquad \textbf{(6.14)}$$

which states that

the current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.

Since R_T and I_T are constants, for a particular configuration the larger the value of R_x (in the denominator), the smaller the value of I_x for that branch, confirming the fact that current always seeks the path of least resistance.

EXAMPLE 6.22 For the parallel network in Fig. 6.42, determine current I_1 using Eq. (6.14).

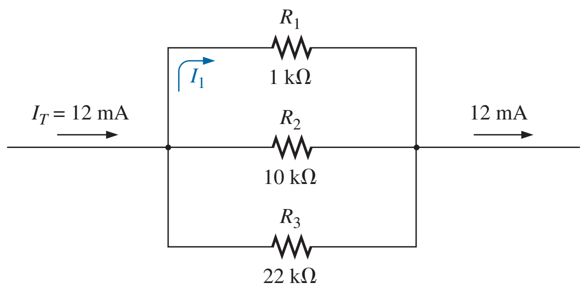


FIG. 6.42
Using the current divider rule to calculate current I_1 in Example 6.22.

Solution: Eq. (6.3):

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\dfrac{1}{1\text{ k}\Omega} + \dfrac{1}{10\text{ k}\Omega} + \dfrac{1}{22\text{ k}\Omega}}$$

$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$

$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

Eq. (6.14): $$I_1 = \frac{R_T}{R_1} I_T$$

$$= \frac{(873.01\ \Omega)}{1\text{ k}\Omega}(12\text{ mA}) = (0.873)(12\text{ mA}) = \mathbf{10.48\ mA}$$

and the smallest parallel resistor receives the majority of the current.

Note also that

for a parallel network, the current through the smallest resistor will be very close to the total entering current if the other parallel elements of the configuration are much larger in magnitude.

In Example 6.22, the current through R_1 is very close to the total current because R_1 is 10 times less than the next smallest resistor.

Special Case: Two Parallel Resistors

For the case of two parallel resistors as shown in Fig 6.43, the total resistance is determined by

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Substituting R_T into Eq. (6.14) for current I_1 results in

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

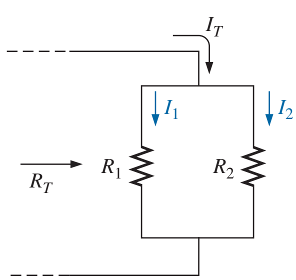


FIG. 6.43
Deriving the current divider rule for the special case of only two parallel resistors.

and $$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T \qquad \textbf{(6.15a)}$$

Similarly, for I_2,

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T \qquad \textbf{(6.15b)}$$

Eq. (6.15) states that

for two parallel resistors, the current through one is equal to the other resistor times the total entering current divided by the sum of the two resistors.

Since the combination of two parallel resistors is probably the most common parallel configuration, the simplicity of the format for Eq. (6.15) suggests that it is worth memorizing. Take particular note, however, that the denominator of the equation is simply the *sum,* not the total resistance, of the combination.

EXAMPLE 6.23 Determine current I_2 for the network in Fig. 6.44 using the current divider rule.

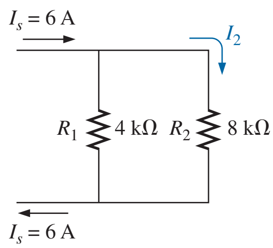


FIG. 6.44
Using the current divider rule to determine current I_2 in Example 6.23.

Solution: Using Eq. (6.15b):

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T$$
$$= \left(\frac{4\text{ k}\Omega}{4\text{ k}\Omega + 8\text{ k}\Omega}\right)6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

Using Eq. (6.14):

$$I_2 = \frac{R_T}{R_2}I_T$$

with $$R_T = 4\text{ k}\Omega \parallel 8\text{ k}\Omega = \frac{(4\text{ k}\Omega)(8\text{ k}\Omega)}{4\text{ k}\Omega + 8\text{ k}\Omega} = 2.667\text{ k}\Omega$$

and $$I_2 = \left(\frac{2.667\text{ k}\Omega}{8\text{ k}\Omega}\right)6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

matching the above solution.

It would appear that the solution with Eq. 6.15(b) is more direct in Example 6.23. However, keep in mind that Eq. (6.14) is applicable to any parallel configuration, removing the necessity to remember two equations.

Now we present a design-type problem.

EXAMPLE 6.24 Determine resistor R_1 in Fig. 6.45 to implement the division of current shown.

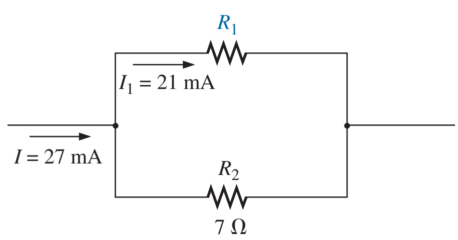


FIG. 6.45
A design-type problem for two parallel resistors (Example 6.24).

Solution: There are essentially two approaches to this type of problem. One involves the direct substitution of known values into the current divider rule equation followed by a mathematical analysis. The other is the sequential application of the basic laws of electric circuits. First we will use the latter approach.

Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$27 \text{ mA} = 21 \text{ mA} + I_2$$

and $$I_2 = 27 \text{ mA} - 21 \text{ mA} = 6 \text{ mA}$$

The voltage V_2: $$V_2 = I_2R_2 = (6 \text{ mA})(7 \ \Omega) = 42 \text{ mV}$$

so that $$V_1 = V_2 = 42 \text{ mV}$$

Finally, $$R_1 = \frac{V_1}{I_1} = \frac{42 \text{ mV}}{21 \text{ mA}} = \mathbf{2 \ \Omega}$$

Now for the other approach using the current divider rule:

$$I_1 = \frac{R_2}{R_1 + R_2} I_T$$
$$21 \text{ mA} = \left(\frac{7 \ \Omega}{R_1 + 7 \ \Omega}\right) 27 \text{ mA}$$
$$(R_1 + 7 \ \Omega)(21 \text{ mA}) = (7 \ \Omega)(27 \text{ mA})$$
$$(21 \text{ mA})R_1 + 147 \text{ mV} = 189 \text{ mV}$$
$$(21 \text{ mA})R_1 = 189 \text{ mV} - 147 \text{ mV} = 42 \text{ mV}$$

and $$R_1 = \frac{42 \text{ mV}}{21 \text{ mA}} = \mathbf{2 \ \Omega}$$

In summary, therefore, remember that current always seeks the path of least resistance, and the ratio of the resistor values is the inverse of the resulting current levels, as shown in Fig 6.46. The thickness of the blue bands in Fig. 6.46 reflects the relative magnitude of the current in each branch.

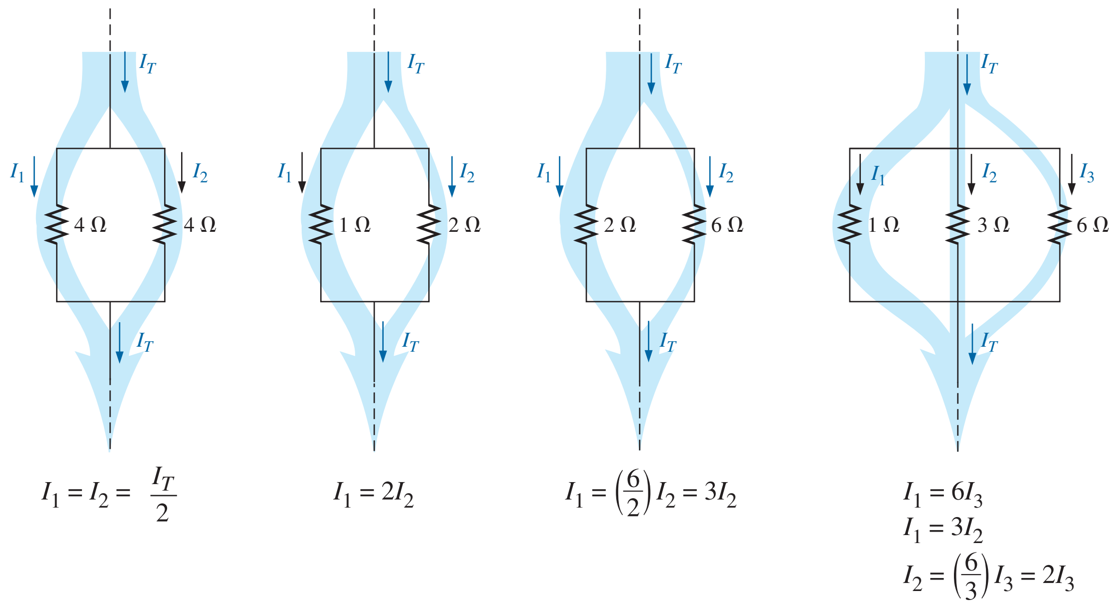


FIG. 6.46
Demonstrating how current divides through equal and unequal parallel resistors.

6.7 VOLTAGE SOURCES IN PARALLEL

Because the voltage is the same across parallel elements,

voltage sources can be placed in parallel only if they have the same voltage.

The primary reason for placing two or more batteries or supplies in parallel is to increase the current rating above that of a single supply. For example, in Fig 6.47, two ideal batteries of 12 V have been placed in parallel. The total source current using Kirchhoff's current law is now the sum of the rated currents of each supply. The resulting power available will be twice that of a single supply if the rated supply current of each is the same. That is,

with $\quad I_1 = I_2 = I$

then $\quad P_T = E(I_1 + I_2) = E(I + I) = E(2I) = 2(EI) = 2P_{(\text{one supply})}$

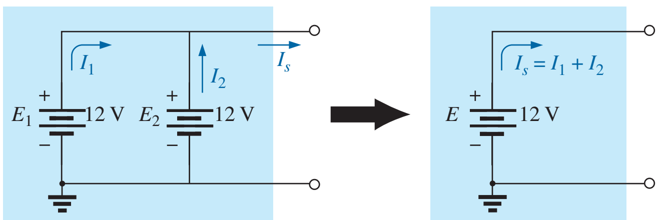


FIG. 6.47
Demonstrating the effect of placing two ideal supplies of the same voltage in parallel.

If for some reason two batteries of different voltages are placed in parallel, both will become ineffective or damaged because the battery with the larger voltage rapidly discharges through the battery with the smaller terminal voltage. For example, consider two lead-acid batteries of different terminal voltages placed in parallel as shown in Fig 6.48. It makes no sense to talk about placing an ideal 12 V battery in parallel with a 6 V battery, because Kirchhoff's voltage law would be violated. However, we can examine the effects if we include the internal resistance levels as shown in Fig. 6.48.

The only current-limiting resistors in the network are the internal resistances, resulting in a very high discharge current for the battery with the larger supply voltage. The resulting current for the case in Fig. 6.48 would be

$$I = \frac{E_1 - E_2}{R_{\text{int}_1} + R_{\text{int}_2}} = \frac{12\text{ V} - 6\text{ V}}{0.03\ \Omega + 0.02\ \Omega} = \frac{6\text{ V}}{0.05\ \Omega} = 120\text{ A}$$

This value far exceeds the rated drain current of the 12 V battery, resulting in rapid discharge of E_1 and a destructive impact on the smaller supply due to the excessive currents. This type of situation did arise on occasion when some cars still had 6 V batteries. Some people thought, "If I have a 6 V battery, a 12 V battery will work twice as well"—not true!

In general,

it is always recommended that when you are replacing batteries in series or parallel, all the batteries be replaced.

A fresh battery placed in parallel with an older battery probably has a higher terminal voltage and immediately starts discharging through the older battery. In addition, the available current is less for the older battery,

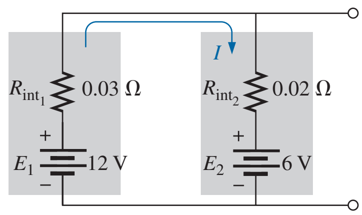


FIG. 6.48
Examining the impact of placing two lead-acid batteries of different terminal voltages in parallel.

resulting in a higher-than-rated current drain from the newer battery when a load is applied.

6.8 OPEN AND SHORT CIRCUITS

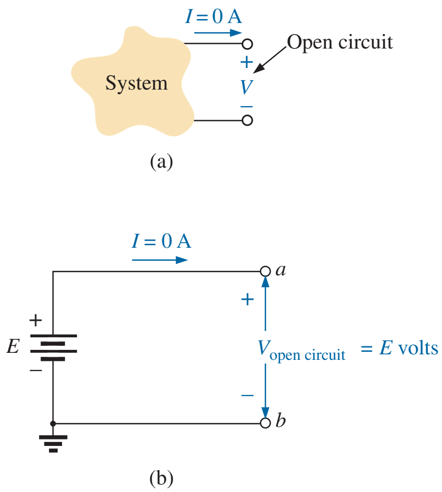


FIG. 6.49
Defining an open circuit.

Open circuits and short circuits can often cause more confusion and difficulty in the analysis of a system than standard series or parallel configurations. This will become more obvious in the chapters to follow when we apply some of the methods and theorems.

An **open circuit** is two isolated terminals not connected by an element of any kind, as shown in Fig. 6.49(a). Since a path for conduction does not exist, the current associated with an open circuit must always be zero. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. In summary, therefore,

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

In Fig. 6.49(b), an open circuit exists between terminals a and b. The voltage across the open-circuit terminals is the supply voltage, but the current is zero due to the absence of a complete circuit.

Some practical examples of open circuits and their impact are provided in Fig. 6.50. In Fig. 6.50(a), the excessive current demanded by the circuit

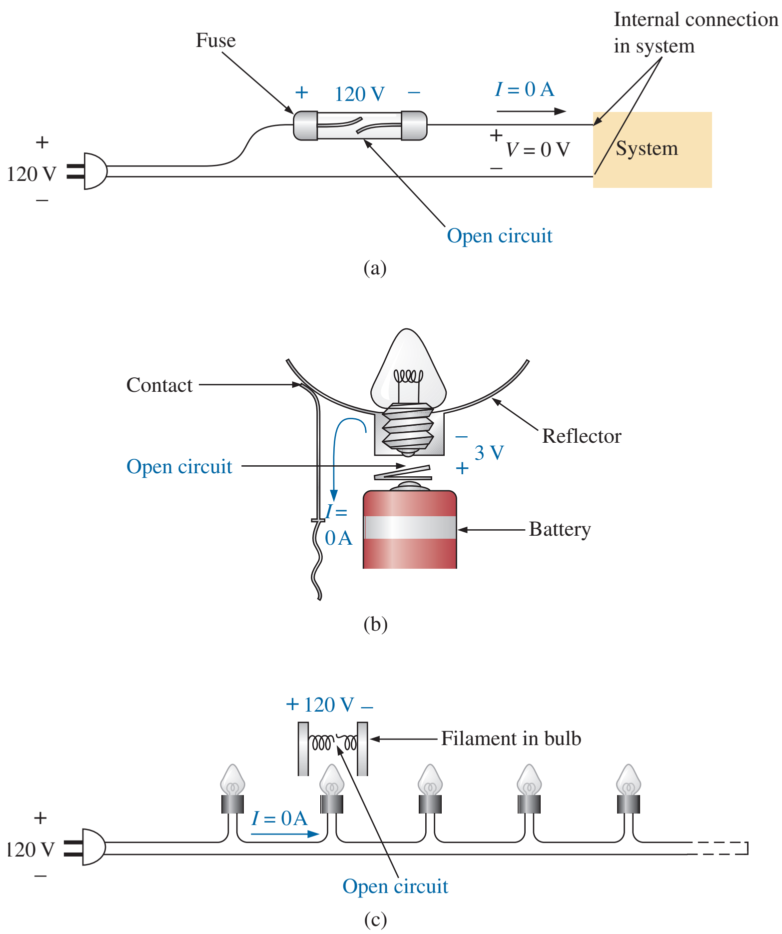


FIG. 6.50
Examples of open circuits.

caused a fuse to fail, creating an open circuit that reduced the current to zero amperes. However, it is important to note that *the full applied voltage is now across the open circuit,* so you must be careful when changing the fuse. If there is a main breaker ahead of the fuse, throw it first to remove the possibility of getting a shock. This situation clearly reveals the benefit of circuit breakers: You can reset the breaker without having to get near the hot wires.

In Fig. 6.50(b), the pressure plate at the bottom of the bulb cavity in a flashlight was bent when the flashlight was dropped. An open circuit now exists between the contact point of the bulb and the plate connected to the batteries. The current has dropped to zero amperes, but the 3 V provided by the series batteries appears across the open circuit. The situation can be corrected by placing a flat-edge screwdriver under the plate and bending it toward the bulb.

Finally, in Fig. 6.50(c), the filament in a bulb in a series connection has opened due to excessive current or old age, creating an open circuit that knocks out all the bulbs in the series configuration. Again, the current has dropped to zero amperes, but the full 120 V will appear across the contact points of the bad bulb. For situations such as this, *you should remove the plug from the wall before changing the bulb.*

A **short circuit** is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 6.51. The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit is always zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and $V = IR = I(0\ \Omega) = 0$ V.

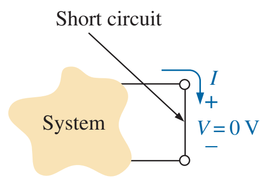


FIG. 6.51
Defining a short circuit.

In summary, therefore,

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

In Fig. 6.52(a), the current through the 2 Ω resistor is 5 A. If a short circuit should develop across the 2 Ω resistor, the total resistance of the parallel combination of the 2 Ω resistor and the short (of essentially zero ohms) will be

$$2\ \Omega \parallel 0\ \Omega = \frac{(2\ \Omega)(0\ \Omega)}{2\ \Omega + 0\ \Omega} = 0\ \Omega$$

as indicated in Fig. 6.52(b), and the current will rise to very high levels, as determined by Ohm's law:

$$I = \frac{E}{R} = \frac{10\text{ V}}{0\ \Omega} \rightarrow \infty\text{ A}$$

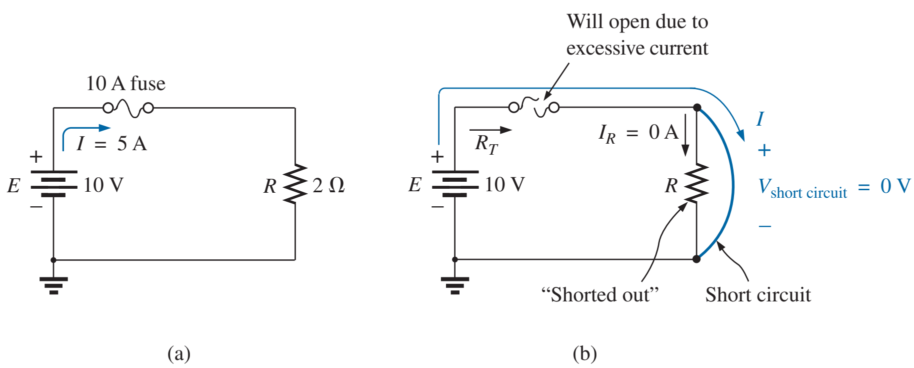


FIG. 6.52
Demonstrating the effect of a short circuit on current levels.

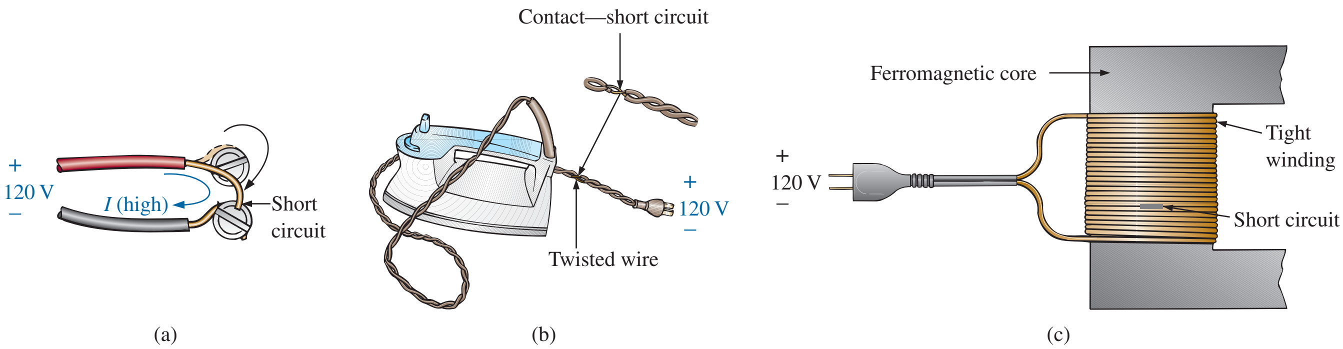


FIG. 6.53
Examples of short circuits.

The effect of the 2 Ω resistor has effectively been "shorted out" by the low-resistance connection. The maximum current is now limited only by the circuit breaker or fuse in series with the source.

Some practical examples of short circuits and their impact are provided in Fig. 6.53. In Fig. 6.53(a), a hot (the feed) wire wrapped around a screw became loose and is touching the return connection. A short-circuit connection between the two terminals has been established that could result in a very heavy current and a possible fire hazard. Hopefully, the breaker will "pop," and the circuit will be deactivated. Problems such as this is one of the reasons aluminum wires (cheaper and lighter than copper) are not permitted in residential or industrial wiring. Aluminum is more sensitive to temperature than copper and will expand and contract due to the heat developed by the current passing through the wire. Eventually, this expansion and contraction can loosen the screw, and a wire under some torsional stress from the installation can move and make contact as shown in Fig. 6.53(a). Aluminum is still used in large panels as a bus-bar connection, but it is bolted down.

In Fig. 6.53(b), the wires of an iron have started to twist and crack due to excessive currents or long-term use of the iron. Once the insulation breaks down, the twisting can cause the two wires to touch and establish a short circuit. Hopefully, a circuit breaker or fuse will quickly disconnect the circuit. Often, it is not the wire of the iron that causes the problem but a cheap extension cord with the wrong gage wire. Be aware that you cannot tell the capacity of an extension cord by its outside jacket. It may have a thick orange covering but have a very thin wire inside. Check the gage on the wire the next time you buy an extension cord, and be sure that it is at least #14 gage, with #12 being the better choice for high-current appliances.

Finally, in Fig. 6.53(c), the windings in a transformer or motor for residential or industrial use are illustrated. The windings are wound so tightly together with such a very thin coating of insulation that it is possible with age and use for the insulation to break down and short out the windings. In many cases, shorts can develop, but a short will simply reduce the number of effective windings in the unit. The tool or appliance may still work, but with less strength or rotational speed. If you notice such a change in the response, you should check the windings because a short can lead to a dangerous situation. In many cases, the state of the windings can be checked with a simple ohmmeter reading. If a short has occurred, the length of usable wire in the winding has been reduced, and the resistance drops. If you know what the resistance normally is, you can compare and make a judgment.

For the layperson, the terminology *short circuit* or *open circuit* is usually associated with dire situations such as power loss, smoke, or fire. However, in network analysis, both can play an integral role in determining specific parameters about a system. Most often, however, if a short-circuit condition is to be established, it is accomplished with a *jumper*—a lead of negligible resistance to be connected between the points of interest. Establishing an open circuit just requires making sure that the terminals of interest are isolated from each other.

EXAMPLE 6.25 Determine voltage V_{ab} for the network in Fig. 6.54.

Solution: The open circuit requires that I be zero amperes. The voltage drop across both resistors is therefore zero volts since $V = IR = (0)R = 0$ V. Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = \mathbf{20\ V}$$

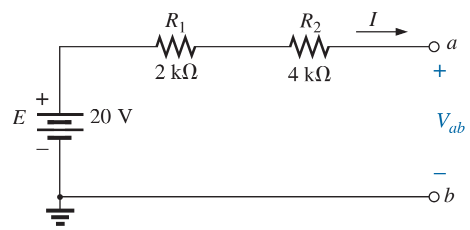


FIG. 6.54
Network for Example 6.25.

EXAMPLE 6.26 Determine voltages V_{ab} and V_{cd} for the network in Fig. 6.55.

Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0 V drop across each resistor. Both resistors can therefore be replaced by short circuits, as shown in Fig. 6.56. Voltage V_{ab} is then directly across the 10 V battery, and

$$V_{ab} = E_1 = \mathbf{10\ V}$$

Voltage V_{cd} requires an application of Kirchhoff's voltage law:

$$+E_1 - E_2 - V_{cd} = 0$$

or
$$V_{cd} = E_1 - E_2 = 10\text{ V} - 30\text{ V} = \mathbf{-20\ V}$$

The negative sign in the solution indicates that the actual voltage V_{cd} has the opposite polarity of that appearing in Fig. 6.55.

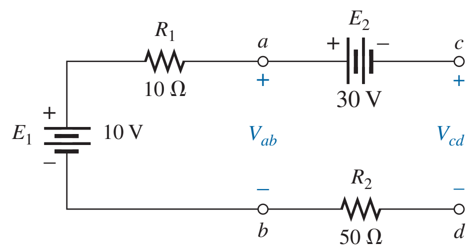


FIG. 6.55
Network for Example 6.26.

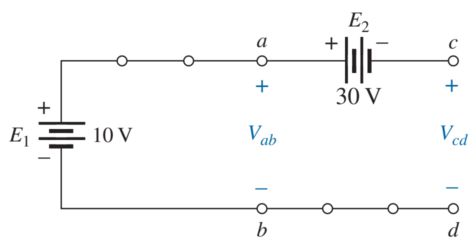


FIG. 6.56
Circuit in Fig. 6.55 redrawn.

EXAMPLE 6.27 Determine the unknown voltage and current for each network in Fig. 6.57.

Solution: For the network in Fig. 6.57(a), the current I_T will take the path of least resistance, and since the short-circuit condition at the end of the network is the least-resistance path, all the current will pass through the short circuit. This conclusion can be verified using the current divider rule. The voltage across the network is the same as that across the short circuit and will be zero volts, as shown in Fig. 6.58(a).

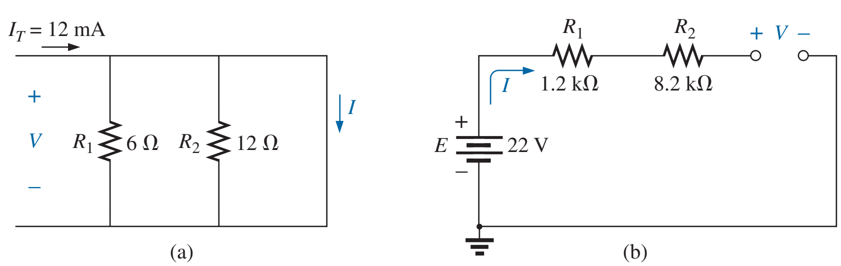


FIG. 6.57
Networks for Example 6.27.

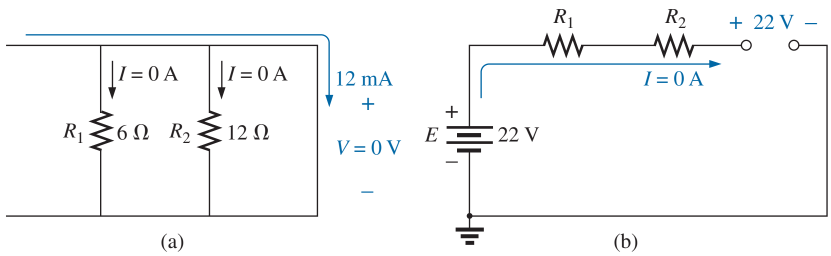


FIG. 6.58
Solutions to Example 6.27.

For the network in Fig. 6.57(b), the open-circuit condition requires that the current be zero amperes. The voltage drops across the resistors must therefore be zero volts, as determined by Ohm's law [$V_R = IR = (0)R = 0$ V], with the resistors acting as a connection from the supply to the open circuit. The result is that the open-circuit voltage is $E = 22$ V, as shown in Fig. 6.58(b).

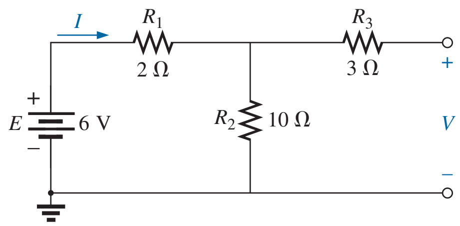


FIG. 6.59
Network for Example 6.28.

EXAMPLE 6.28 Determine V and I for the network in Fig. 6.59 if resistor R_2 is shorted out.

Solution: The redrawn network appears in Fig. 6.60. The current through the 3 Ω resistor is zero due to the open circuit, causing all the current I to pass through the jumper. Since $V_{3\Omega} = IR = (0)R = 0$ V, the voltage V is directly across the short, and

$$V = \mathbf{0\ V}$$

with $$I = \frac{E}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

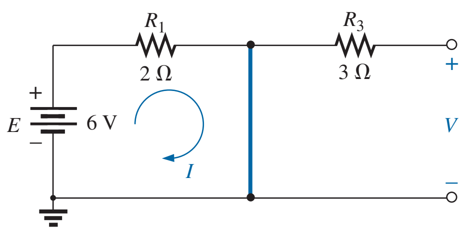


FIG. 6.60
Network in Fig. 6.59 with R_2 replaced by a jumper.

6.9 VOLTMETER LOADING EFFECTS

In previous chapters, we learned that ammeters are not ideal instruments. When you insert an ammeter, you actually introduce an additional resistance in series with the branch in which you are measuring the current. Generally, this is not a serious problem, but it can have a troubling effect on your readings, so it is important to be aware of it.

Voltmeters also have an internal resistance that appears between the two terminals of interest when a measurement is being made. While an ammeter places an additional resistance in series with the branch of interest, a voltmeter places an additional resistance *across* the element, as shown in Fig. 6.61. Since it appears in parallel with the element of interest, *the ideal level for the internal resistance of a voltmeter would be infinite ohms, just as zero ohms would be ideal for an ammeter.* Unfortunately, the internal resistance of any voltmeter is not infinite and changes from one type of meter to another.

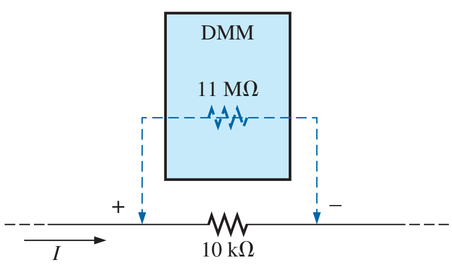


FIG. 6.61
Voltmeter loading.

Most digital meters have a fixed internal resistance level in the megohm range that remains the same *for all its scales.* For example, the meter in Fig. 6.61 has the typical level of 11 MΩ for its internal resistance, no matter which voltage scale is used. When the meter is placed across the 10 kΩ resistor, the total resistance of the combination is

$$R_T = 10\text{ k}\Omega \parallel 11\text{ M}\Omega = \frac{(10^4\,\Omega)(11\times 10^6\,\Omega)}{10^4\,\Omega + (11\times 10^6)} = 9.99\text{ k}\Omega$$

and the behavior of the network is not seriously affected. The result, therefore, is that

most digital voltmeters can be used in circuits with resistances up to the high-kilohm range without concern for the effect of the internal resistance on the reading.

However, if the resistances are in the megohm range, you should investigate the effect of the internal resistance.

An analog VOM is a different matter, however, because the internal resistance levels are much lower and the internal resistance levels are a function of the scale used. If a VOM on the 2.5 V scale were placed across the 10 kΩ resistor in Fig. 6.61, the internal resistance might be 50 kΩ, resulting in a combined resistance of

$$R_T = 10\text{ k}\Omega \parallel 50\text{ k}\Omega = \frac{(10^4\,\Omega)(50\times 10^3\,\Omega)}{10^4\,\Omega + (50\times 10^3\,\Omega)} = 8.33\text{ k}\Omega$$

and the behavior of the network would be affected because the 10 kΩ resistor would appear as an 8.33 kΩ resistor.

To determine the resistance R_m of any scale of a VOM, simply multiply the **maximum voltage** of the chosen scale by the **ohm/volt (Ω/V) rating** normally appearing at the bottom of the face of the meter. That is,

$$R_m\text{ (VOM)} = (\text{scale})(\Omega/\text{V rating})$$

For a typical Ω/V rating of 20,000, the 2.5 V scale would have an internal resistance of

$$(2.5\text{ V})(20{,}000\ \Omega/\text{V}) = \mathbf{50\ k\Omega}$$

whereas for the 100 V scale, the internal resistance of the VOM would be

$$(100\text{ V})(20{,}000\ \Omega/\text{V}) = \mathbf{2\ M\Omega}$$

and for the 250 V scale,

$$(250\text{ V})(20{,}000\ \Omega/\text{V}) = \mathbf{5\ M\Omega}$$

EXAMPLE 6.29 For the relatively simple circuit in Fig. 6.62(a):

a. What is the open-circuit voltage V_{ab}?
b. What will a DMM indicate if it has an internal resistance of 11 MΩ? Compare your answer to that of part (a).
c. Repeat part (b) for a VOM with an Ω/V rating of 20,000 on the 100 V scale.

Solutions:

a. Due to the open circuit, the current is zero, and the voltage drop across the 1 MΩ resistor is zero volts. The result is that the entire source voltage appears between points *a* and *b*, and

$$V_{ab} = \mathbf{20\ V}$$

b. When the meter is connected as shown in Fig. 6.62(b), a complete circuit has been established, and current can pass through the circuit.

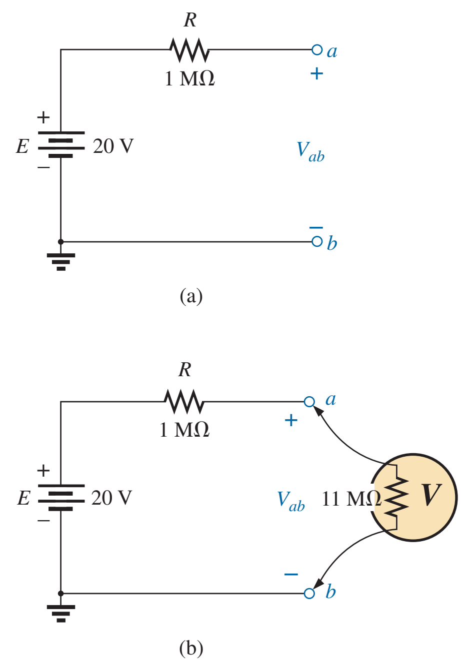


FIG. 6.62
(a) Measuring an open-circuit voltage with a voltmeter; (b) determining the effect of using a digital voltmeter with an internal resistance of 11 MΩ on measuring an open-circuit voltage (Example 6.29).

The voltmeter reading can be determined using the voltage divider rule as follows:

$$V_{ab} = \frac{(11\ \text{M}\Omega)(20\ \text{V})}{11\ \text{M}\Omega + 1\ \text{M}\Omega)} = \mathbf{18.33\ V}$$

and the reading is affected somewhat.

c. For the VOM, the internal resistance of the meter is

$$R_m = (100\ \text{V})\ (20{,}000\ \Omega/\text{V}) = 2\ \text{M}\Omega$$

and
$$V_{ab} = \frac{(2\ \text{M}\Omega)(20\ \text{V})}{2\ \text{M}\Omega + 1\ \text{M}\Omega)} = \mathbf{13.33\ V}$$

which is considerably below the desired level of 20 V.

6.10 SUMMARY TABLE

Now that the series and parallel configurations have been covered in detail, we will review the salient equations and characteristics of each. The equations for the two configurations have a number of similarities. In fact, the equations for one can often be obtained directly from the other by simply applying the **duality** principle. Duality between equations means that the format for an equation can be applied to two different situations by just changing the variable of interest. For instance, the equation for the total resistance of a series circuit is the sum of the resistances. By changing the resistance parameters to conductance parameters, you can obtain the equation for the total conductance of a parallel network—an easy way to remember the two equations. Similarly, by starting with the total conductance equation, you can easily write the total resistance equation for series circuits by replacing the conductance parameters by resistance parameters. Series and parallel networks share two important dual relationships: (1) between resistance of series circuits and conductance of parallel circuits and (2) between the voltage or current of a series circuit and the current or voltage, respectively, of a parallel circuit. Table 6.1 summarizes this duality.

The format for the total resistance for a series circuit has the same format as the total conductance of a parallel network as shown in Table 6.1. All that is required to move back and forth between the series and parallel headings is to interchange the letters *R* and *G*. For the special case of two elements, the equations have the same format, but the equation applied for the total resistance of the parallel configuration has changed. In the series configuration, the total resistance increases with each added resistor. For parallel networks, the total conductance increases with each additional conductance. The result is that the total conductance of a series circuit drops with added resistive elements while the total resistance of parallel networks decreases with added elements.

In a series circuit, the current is the same everywhere. In a parallel network, the voltage is the same across each element. The result is a duality between voltage and current for the two configurations. What is true for one in one configuration is true for the other in the other configuration. In a series circuit, the applied voltage divides between the series elements. In a parallel network, the current divides between parallel elements. For series circuits, the largest resistor captures the largest share of the applied voltage. For parallel networks, the branch with the highest conductance captures the greater share of the incoming current. In addition, for series circuits, the applied voltage equals the sum of the voltage drops across the

TABLE 6.1
Summary table.

Series and Parallel Circuits		
Series	**Duality**	**Parallel**
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftarrows G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftarrows G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements $R_T = R_1 + R_2$	$R \rightleftarrows G$	$G_T = G_1 + G_2$ and $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$
I the same through series elements	$I \rightleftarrows V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftarrows I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \rightleftarrows I$ and $R \rightleftarrows G$	Greatest I through largest G (smallest R)
$V_x = \dfrac{R_x E}{R_T}$	$E, V \rightleftarrows I$ and $R \rightleftarrows G$	$I_x = \dfrac{G_x I_T}{G_T} = \dfrac{R_T I_T}{R_x}$ with $I_1 = \dfrac{R_2 I_T}{R_1 + R_2}$ and $I_2 = \dfrac{R_1 I_T}{R_1 + R_2}$
$P = EI_T$	$E \rightleftarrows I$ and $I \rightleftarrows E$	$P = I_T E$
$P = I^2 R$	$I \rightleftarrows V$ and $R \rightleftarrows G$	$P = V^2 G = V^2/R$
$P = V^2/R$	$V \rightleftarrows I$ and $R \rightleftarrows G$	$P = I^2/G = I^2 R$

series elements of the circuit, while the source current for parallel branches equals the sum of the currents through all the parallel branches.

The total power delivered to a series or parallel network is determined by the product of the applied voltage and resulting source current. The power delivered to each element is also the same for each configuration. Duality can be applied again, but the equation $P = EI$ results in the same result as $P = IE$. Also, $P = I^2R$ can be replaced by $P = V^2G$ for parallel elements, but essentially each can be used for each configuration. The duality principle can be very helpful in the learning process. Remember this as you progress through the next few chapters. You will find in the later chapters that this duality can also be applied between two important elements—inductors and capacitors.

6.11 TROUBLESHOOTING TECHNIQUES

The art of *troubleshooting* is not limited solely to electrical or electronic systems. In the broad sense,

troubleshooting is a process by which acquired knowledge and experience are used to localize a problem and offer or implement a solution.

There are many reasons why the simplest electrical circuit does not operate correctly. A connection may be open; the measuring instruments may need calibration; the power supply may not be on or may have been connected incorrectly to the circuit; an element may not be performing correctly due to earlier damage or poor manufacturing; a fuse may have blown; and so on. Unfortunately, a defined sequence of steps does not

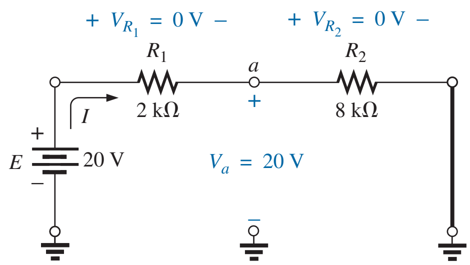


FIG. 6.63
A malfunctioning network.

exist for identifying the wide range of problems that can surface in an electrical system. It is only through experience and a clear understanding of the basic laws of electric circuits that you can become proficient at quickly locating the cause of an erroneous output.

It should be fairly obvious, however, that the first step in checking a network or identifying a problem area is to have some idea of the expected voltage and current levels. For instance, the circuit in Fig. 6.63 should have a current in the low milliampere range, with the majority of the supply voltage across the 8 kΩ resistor. However, as indicated in Fig. 6.63, $V_{R_1} = V_{R_2} = 0$ V and $V_a = 20$ V. Since $V = IR$, the results immediately suggest that $I = 0$ A and an open circuit exists in the circuit. The fact that $V_a = 20$ V immediately tells us that the connections are true from the ground of the supply to point *a*. The open circuit must therefore exist between R_1 and R_2 or at the ground connection of R_2. An open circuit at either point results in $I = 0$ A and the readings obtained previously. Keep in mind that, even though $I = 0$ A, R_1 does form a connection between the supply and point *a*. That is, if $I = 0$ A, $V_{R_1} = IR_2 = (0)R_2 = 0$ V, as obtained for a short circuit.

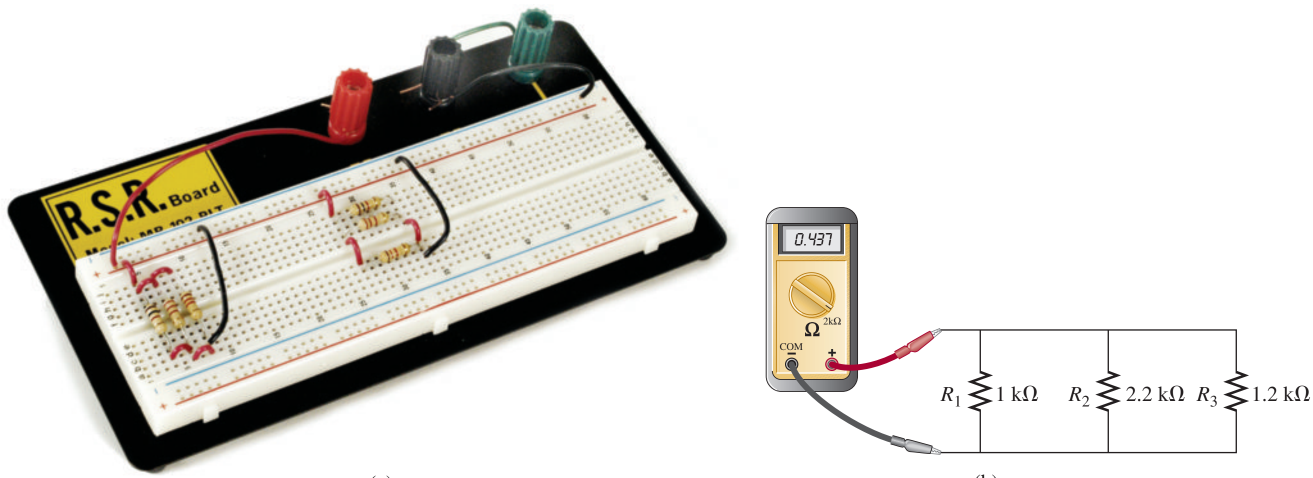
In Fig. 6.63, if $V_{R_1} \cong 20$ V and V_{R_2} is quite small ($\cong 0.08$ V), it first suggests that the circuit is complete, a current does exist, and a problem surrounds the resistor R_2. R_2 is not shorted out since such a condition would result in $V_{R_2} = 0$ V. A careful check of the inserted resistor reveals that an 8 Ω resistor was used rather than the 8 kΩ resistor specified—an incorrect reading of the color code. To avoid this, an ohmmeter should be used to check a resistor to validate the color-code reading or to ensure that its value is still in the prescribed range set by the color code.

Occasionally, the problem may be difficult to diagnose. You've checked all the elements, and all the connections appear tight. The supply is on and set at the proper level; the meters appear to be functioning correctly. In situations such as this, experience becomes a key factor. Perhaps you can recall when a recent check of a resistor revealed that the internal connection (not externally visible) was a "make or break" situation or that the resistor was damaged earlier by excessive current levels, so its actual resistance was much lower than called for by the color code. Recheck the supply! Perhaps the terminal voltage was set correctly, but the current control knob was left in the zero or minimum position. Is the ground connection stable? The questions that arise may seem endless. However, as you gain experience, you will be able to localize problems more rapidly. Of course, the more complicated the system, the longer the list of possibilities, but it is often possible to identify a particular area of the system that is behaving improperly before checking individual elements.

6.12 PROTOBOARDS (BREADBOARDS)

In Section 5.12, the protoboard was introduced with the connections for a simple series circuit. To continue the development, the network in Fig. 6.17 was set up on the board in Fig. 6.64(a) using two different techniques. The possibilities are endless, but these two solutions use a fairly straightforward approach.

First, note that the supply lines and ground are established across the length of the board using the horizontal conduction zones at the top and bottom of the board through the connections to the terminals. The network to the left on the board was used to set up the circuit in much the same manner as it appears in the schematic of Fig. 6.64(b). This approach required that the resistors be connected between two vertical conducting



(a) (b)

FIG. 6.64
Using a protoboard to set up the circuit in Fig. 6.17.

strips. If placed perfectly vertical in a single conducting strip, the resistors would have shorted out. Often, setting the network up in a manner that best copies the original can make it easier to check and make measurements. The network to the right in part (a) used the vertical conducting strips to connect the resistors together at each end. Since there wasn't enough room for all three, a connection had to be added from the upper vertical set to the lower set. The resistors are in order R_1, R_2, and R_3 from the top-down. For both configurations, the ohmmeter can be connected to the positive lead of the supply terminal and the negative or ground terminal.

Take a moment to review the connections and think of other possibilities. Improvements can often be made, and it can be satisfying to find the most effective setup with the least number of connecting wires.

6.13 APPLICATIONS

One of the most important advantages of the parallel configuration is that

if one branch of the configuration should fail (open circuit), the remaining branches will still have full operating power.

In a home, the parallel connection is used throughout to ensure that if one circuit has a problem and opens the circuit breaker, the remaining circuits still have the full 120 V. The same is true in automobiles, computer systems, industrial plants, and wherever it would be disastrous for one circuit to control the total power distribution.

Another important advantage is that

branches can be added at any time without affecting the behavior of those already in place.

In other words, unlike the series connection where an additional component reduces the current level and perhaps affects the response of some of the existing components, an additional parallel branch will not affect the current level in the other branches. Of course, the current demand from the supply increases as determined by Kirchhoff's current law, so you must be aware of the limitations of the supply.

The following are some of the most common applications of the parallel configuration.

Car System

As you begin to examine the electrical system of an automobile, the most important thing to understand is that the entire electrical system of a car is run as a *dc system.* Although the generator produces a varying ac signal, rectification converts it to one having an average dc level for charging the battery. In particular, note the filter capacitor in the alternator branch in Fig. 6.65 to smooth out the rectified ac waveform and to provide an improved dc supply. The charged battery must therefore provide the required direct current for the entire electrical system of the car. Thus, the power demand on the battery at any instant is the product of the terminal voltage and the current drain of the total load of every operating system of the car. This certainly places an enormous burden on the battery and its internal chemical reaction and warrants all the battery care we can provide.

Since the electrical system of a car is essentially a parallel system, the total current drain on the battery is the sum of the currents to all the parallel branches of the car connected directly to the battery. In Fig. 6.65, a few branches of the wiring diagram for a car have been sketched to provide some background information on basic wiring, current levels, and fuse configurations. Every automobile has fuse links and fuses, and some also have circuit breakers, to protect the various components of the car and to ensure that a dangerous fire situation does not develop. Except for a few branches that may have series elements, the operating voltage for most components of a car is the terminal voltage of the battery which we will designate as 12 V even though it will typically vary between 12 V and the charging level of 14.6 V. In other words, each component is connected to the battery at one end and to the ground or chassis of the car at the other end.

Referring to Fig. 6.65, note that the alternator or charging branch of the system is connected directly across the battery to provide the charging current as indicated. Once the car is started, the rotor of the alternator turns,

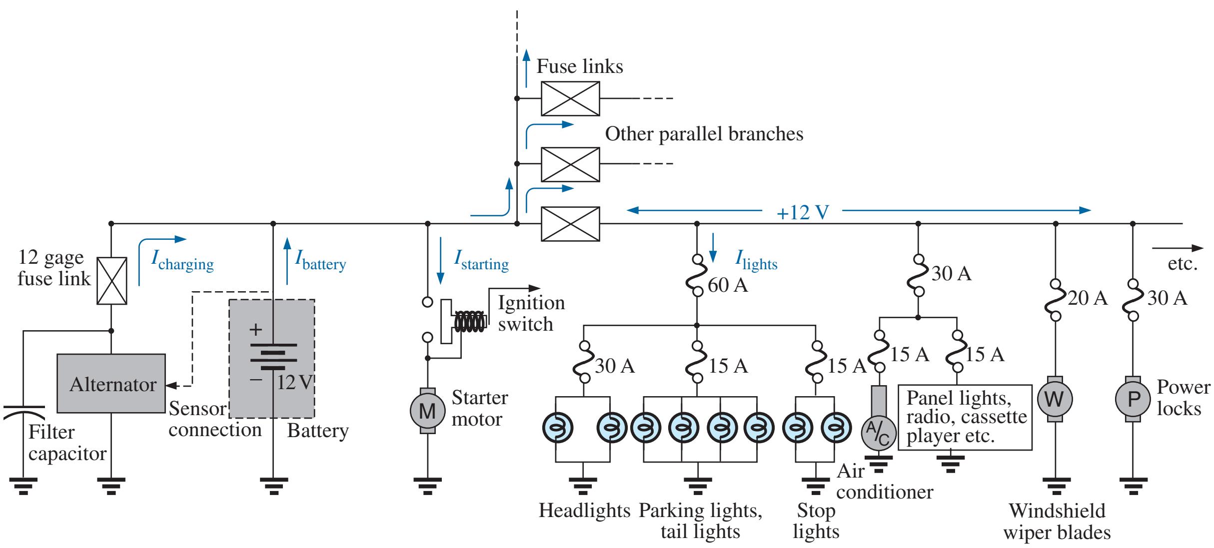


FIG. 6.65
Expanded view of an automobile's electrical system.

generating an ac varying voltage which then passes through a rectifier network and filter to provide the dc charging voltage for the battery. Charging occurs only when the sensor, connected directly to the battery, signals that the terminal voltage of the battery is too low. Just to the right of the battery the starter branch was included to demonstrate that there is no fusing action between the battery and starter when the ignition switch is activated. The lack of fusing action is provided because enormous starting currents (hundreds of amperes) flow through the starter to start a car that has not been used for days and/or has been sitting in a cold climate—and high friction occurs between components until the oil starts flowing. The starting level can vary so much that it would be difficult to find the right fuse level, and frequent high currents may damage the fuse link and cause a failure at expected levels of current. When the ignition switch is activated, the starting relay completes the circuit between the battery and starter, and hopefully the car starts. If a car fails to start, the first thing to check is the connections at the battery, starting relay, and starter to be sure that they are not providing an unexpected open circuit due to vibration, corrosion, or moisture.

Once the car has started, the starting relay opens, and the battery begins to activate the operating components of the car. Although the diagram in Fig. 6.65 does not display the switching mechanism, the entire electrical network of the car, except for the important external lights, is usually disengaged so that the full strength of the battery can be dedicated to the starting process. The lights are included for situations where turning the lights off, even for short periods of time, could create a dangerous situation. If the car is in a safe environment, it is best to leave the lights off when starting to save the battery an additional 30 A of drain. If the lights are on, they dim because of the starter drain, which may exceed 500 A. Today, batteries are typically rated in cranking (starting) current rather than ampere-hours. Batteries rated with cold cranking ampere ratings between 700 A and 1000 A are typical today.

Separating the alternator from the battery and the battery from the numerous networks of the car are fuse links such as shown in Fig. 6.66. Fuse links are actually wires of a specific gage designed to open at fairly high current levels of 100 A or more. They are included to protect against those situations where there is an unexpected current drawn from the many circuits to which they are connected. That heavy drain can, of course, be from a short circuit in one of the branches, but in such cases the fuse in that branch will probably release. The fuse link is an additional protection for the line if the total current drawn by the parallel-connected branches begins to exceed safe levels. The fuses following the fuse link have the appearance shown in Fig. 6.66(b), where a gap between the legs of the fuse

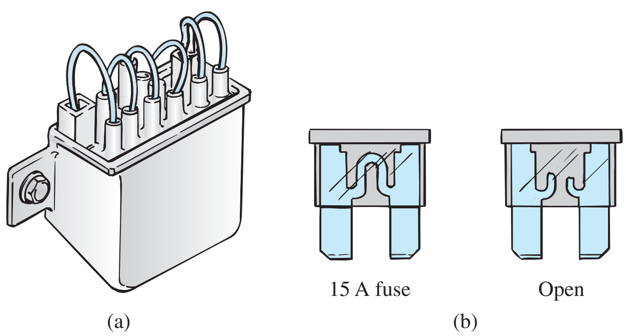


FIG. 6.66
Car fuses: (a) fuse link; (b) plug-in.

indicates a blown fuse. As shown in Fig. 6.65, the 60 A fuse (often called a *power distribution fuse*) for the lights is a second-tier fuse sensitive to the total drain from the three light circuits. Finally, the third fuse level is for the individual units of a car such as the lights, air conditioner, and power locks. In each case, the fuse rating exceeds the normal load (current level) of the operating component, but the level of each fuse does give some indication of the demand to be expected under normal operating conditions. For instance, headlights typically draw more than 10 A, tail lights more than 5 A, air conditioner about 10 A (when the clutch engages), and power windows 10 A to 20 A depending on how many are operated at once.

Some details for only one section of the total car network are provided in Fig. 6.65. In the same figure, additional parallel paths with their respective fuses have been provided to further reveal the parallel arrangement of all the circuits.

In most vehicles the return path to the battery through the ground connection is through the chassis of the car. That is, there is only one wire to each electrical load, with the other end simply grounded to the chassis. The return to the battery (chassis to negative terminal) is therefore a heavy-gage wire matching that connected to the positive terminal. In some cars constructed of a mixture of materials such as metal, plastic, and rubber, the return path through the metallic chassis may be lost, and two wires must be connected to each electrical load of the car.

House Wiring

In Chapter 4, the basic power levels of importance were discussed for various services to the home. We are now ready to take the next step and examine the actual connection of elements in the home.

First, it is important to realize that except for some very special circumstances, the basic wiring is done in a parallel configuration. Each parallel branch, however, can have a combination of parallel and series elements. Every full branch of the circuit receives the full 120 V or 208 V, with the current determined by the applied load. Figure 6.67(a) provides the detailed wiring of a single circuit having a light bulb and two outlets. Figure 6.67(b) shows the schematic representation. Note that although each load is in parallel with the supply, switches are always connected in series with the load. The power is transmitted to the lamp only when the switch is closed and the full 120 V appears across the bulb. The connection point for the two outlets is in the ceiling box holding the light bulb. Since a switch is not present, both outlets are always "hot" unless the circuit breaker in the main panel is opened. This is important to understand in case you are tempted to change the light fixture by simply turning off the wall switch. True, if you're very careful, you can work with one line at a time (being sure that you don't touch the other line at any time), but it is much safer to throw the circuit breaker on the panel whenever working on a circuit. Note in Fig. 6.67(a) that the *feed* wire (black) into the fixture from the panel is connected to the switch and both outlets at one point. It is not connected directly to the light fixture because the lamp would be on all the time. Power to the light fixture is made available through the switch. The continuous connection to the outlets from the panel ensures that the outlets are "hot" whenever the circuit breaker in the panel is on. Note also how the *return* wire (white) is connected directly to the light switch and outlets to provide a return for each component. There is no need for the white wire to go through the switch since an applied voltage is a two-point connection and the black wire is controlled by the switch.

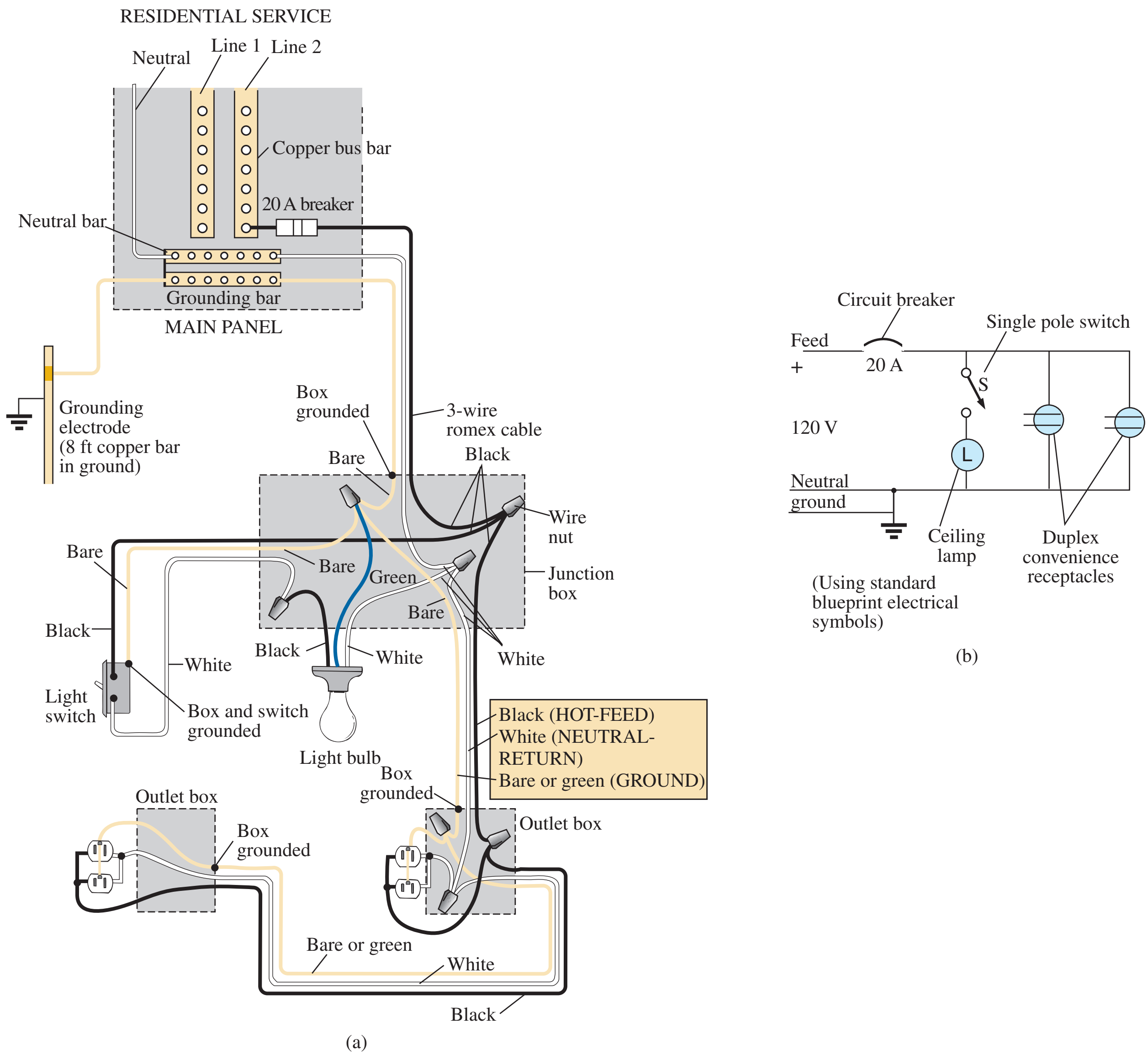


FIG. 6.67
Single phase of house wiring: (a) physical details; (b) schematic representation.

Proper grounding of the system in total and of the individual loads is one of the most important facets in the installation of any system. There is a tendency at times to be satisfied that the system is working and to pay less attention to proper grounding technique. Always keep in mind that a properly grounded system has a direct path to ground if an undesirable situation should develop. The absence of a direct ground causes the system to determine its own path to ground, and you could be that path if you happened to touch the wrong wire, metal box, metal pipe, and so on. In Fig. 6.67(a), the connections for the ground wires have been included. For the romex (plastic-coated wire) used in Fig. 6.67(a), the ground wire is provided as a bare copper wire. Note that it is connected to the panel which in turn is directly connected to the grounded 8-ft copper rod. In addition, note that the ground connection is carried through the entire circuit, including the switch, light fixture, and outlets. It is one continuous connection. If the outlet box, switch box, and housing for the light fixture

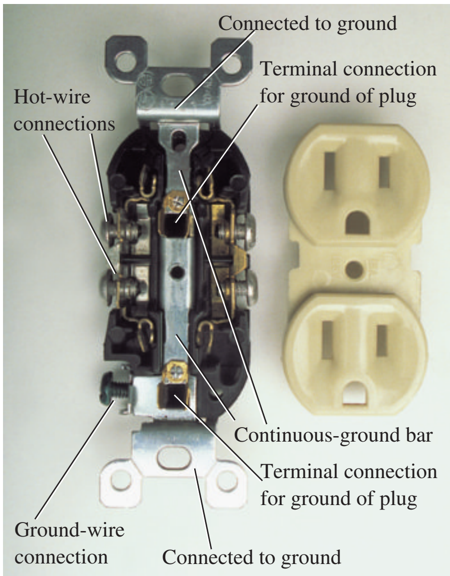


FIG. 6.68
Continuous ground connection in a duplex outlet.

are made of a conductive material such as metal, the ground will be connected to each. If each is plastic, there is no need for the ground connection. However, the switch, both outlets, and the fixture itself are connected to ground. For the switch and outlets, there is usually a green screw for the ground wire which is connected to the entire framework of the switch or outlet as shown in Fig. 6.68, including the ground connection of the outlet. For both the switch and the outlet, even the screw or screws used to hold the outside plate in place are grounded since they are screwed into the metal housing of the switch or outlet. When screwed into a metal box, the ground connection can be made by the screws that hold the switch or outlet in the box as shown in Fig. 6.68. *Always pay strict attention to the grounding process whenever installing any electrical equipment.*

On the practical side, whenever hooking up a wire to a screw-type terminal, always wrap the wire around the screw in the clockwise manner so that when you tighten the screw, it grabs the wire and turns it in the same direction. An expanded view of a typical house-wiring arrangement appears in Chapter 15.

Parallel Computer Bus Connections

The internal construction (hardware) of large mainframe computers and personal computers is set up to accept a variety of adapter cards in the slots appearing in Fig. 6.69(a). The primary board (usually the largest), commonly called the *motherboard,* contains most of the functions required for full computer operation. Adapter cards are normally added to expand the memory, set up a network, add peripheral equipment, and so on. For instance, if you decide to add another hard drive to your computer, you can simply insert the card into the proper channel of Fig. 6.69(a). The bus connectors are connected in parallel with common connections to the power supply, address and data buses, control signals, ground, and so on. For instance, if the bottom connection of each bus connector is a ground connection, that ground connection carries through each bus connector and is immediately connected to any adapter card installed. Each card has a slot connector that fits directly into the bus connector without the need for any soldering or construction. The pins of the adapter card are then designed to provide a path between the motherboard and its components to support the desired function. Note in Fig. 6.69(b), which is a back view of the region identified in Fig. 6.69(a), that if you follow the path of the second pin from the top on the far left, you will see that it is connected to the same pin on the other three bus connectors.

Most small laptop computers today have all the options already installed, thereby bypassing the need for bus connectors. Additional memory and other upgrades are added as direct inserts into the motherboard.

6.14 COMPUTER ANALYSIS

PSpice

Parallel dc Network The computer analysis coverage for parallel dc circuits is very similar to that for series dc circuits. However, in this case the voltage is the same across all the parallel elements, and the current through each branch changes with the resistance value. The parallel network to be analyzed will have a wide range of resistor values to demonstrate the effect on the resulting current. The following is

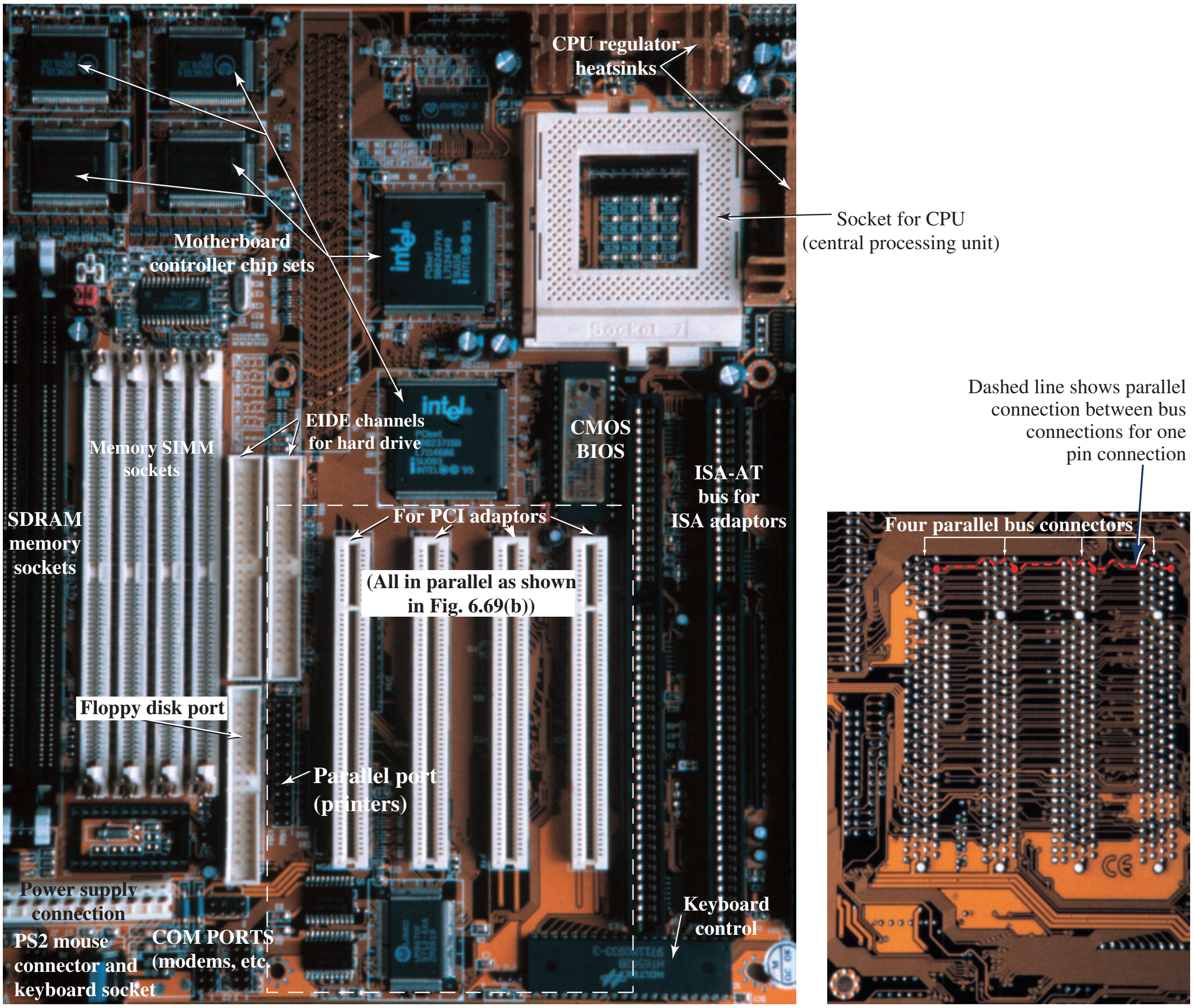


FIG. 6.69
(a) Motherboard for a desktop computer; (b) the printed circuit board connections for the region indicated in part (a).

a list of abbreviations for any parameter of a network when using PSpice:

$$\begin{aligned}
\mathbf{f} &= 10^{-15}\\
\mathbf{p} &= 10^{-12}\\
\mathbf{n} &= 10^{-9}\\
\mathbf{u} &= 10^{-6}\\
\mathbf{m} &= 10^{-3}\\
\mathbf{k} &= 10^{+3}\\
\mathbf{MEG} &= 10^{+6}\\
\mathbf{G} &= 10^{+9}\\
\mathbf{T} &= 10^{+12}
\end{aligned}$$

In particular, note that **m** (or **M**) is used for "milli," and **MEG** for "megohms." Also, PSpice does not distinguish between upper- and lowercase units, but certain parameters typically use either the upper- or lowercase abbreviation as shown above.

Since the details of setting up a network and going through the simulation process were covered in detail in Sections 4.9 and 5.14 for dc circuits, the coverage here is limited solely to the various steps required. These steps will help you learn how to "draw" a circuit and then run a simulation fairly quickly and easily.

After selecting the **Create document** key (the top left of the screen), the following sequence opens the **Schematic** window: **PSpice 6-1-OK-Create a blank project-OK-PAGE1** (if necessary).

Add the voltage source and resistors as described in detail in earlier sections, but now you need to turn the resistors 90°. You do this by right-clicking before setting a resistor in place. Choose **Rotate** from the list of options, which turns the resistor counterclockwise 90°. It can also be rotated by simultaneously selecting **Ctrl-R.** The resistor can then be placed in position by a left click. An additional benefit of this maneuver is that the remaining resistors to be placed will already be in the vertical position. The values selected for the voltage source and resistors appear in Fig. 6.70.

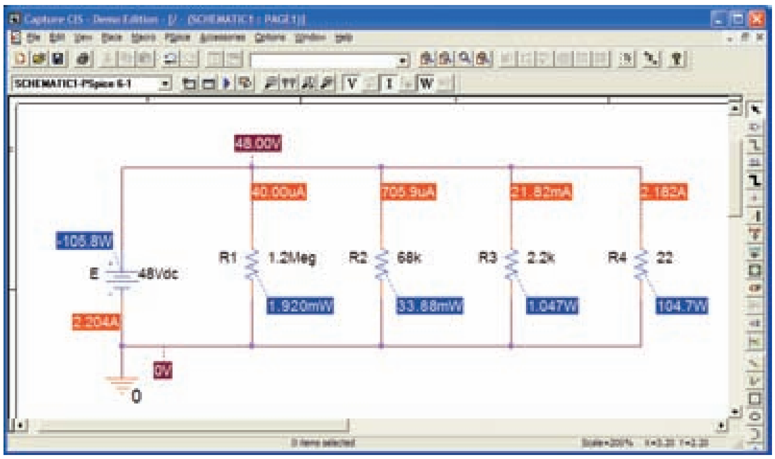


FIG. 6.70
Applying PSpice to a parallel network.

Once the network is complete, you can obtain the simulation and the results through the following sequence: **Select New Simulation Profile** key-**Bias Point-Create-Analysis-Bias Point-OK-Run PSpice** key-**Exit(X).**

The result, shown in Fig. 6.70, reveals that the voltage is the same across all the parallel elements and the current increases significantly with decrease in resistance. The range in resistor values suggests, by inspection, that the total resistance is just less than the smallest resistance of 22 Ω. Using Ohm's law and the source current of 2.204 A results in a total resistance of $R_T = E/I_s = 48\text{ V}/2.204\text{ A} = 21.78\ \Omega$, confirming the above conclusion.

Multisim

Parallel dc Network For comparison purposes with the PSpice approach, the same parallel network in Fig. 6.70 is now analyzed using Multisim. The source and ground are selected and placed as shown in Fig. 6.71 using the procedure defined in previous chapters. For the resistors, choose **VIRTUAL_RESISTOR,** but you must rotate it 90° to match the configuration of Fig. 6.70. You do this by first clicking on the resistor symbol to place it in the active state. (Be sure that the resulting small black squares surround the symbol, label, and value; otherwise, you may have activated only the label or value.) Then right-click inside the rectangle. Select **90° Clockwise,** and the resistor is turned automatically. Unfortunately, there is no continuum here, so the next resistor has to be turned using the same procedure. The values of each resistor are set by double-clicking on the resistor symbol to obtain the **BASIC-VIRTUAL** dialog box. Remember that the unit of measurement is controlled by the scrolls at the right of the unit of measurement. For Multisim, unlike PSpice, megohm uses capital **M** and milliohm uses lowercase **m.**

This time, rather than using meters to make the measurements, you will use indicators. The **Indicators** key pad is the tenth down on the left toolbar. It has the appearance of an LCD display with the number 8. Select it, and eight possible indicators appear. For this example, the **AMMETER** indicator, representing an ammeter, is used since we are interested only in the current levels. When you select **AMMETER,** a **Component** listing appears with four choices, with each option referring to a position for the ammeter. The **H** means "horizontal" as shown in the picture window when the dialog box is first opened. The **HR** means "horizontal," but with the polarity reversed. The **V** is for a vertical configuration with the positive sign at the top, and the **VR** is the vertical position with the positive sign at the bottom. Select the one you want followed by an **OK,** and your choice appears in that position on the screen. Click it into position, and you can return for the next indicator. Once all the elements are in place and their values set, initiate simulation with the sequence **Simulate-Run.** The results shown in Fig. 6.71 appear.

Note that all the results appear with the indicator boxes. All are positive results because the ammeters were all entered with a configuration that would result in conventional current entering the positive current. Also note that as was true for inserting the meters, the indicators are placed in series with the branch in which the current is to be measured.

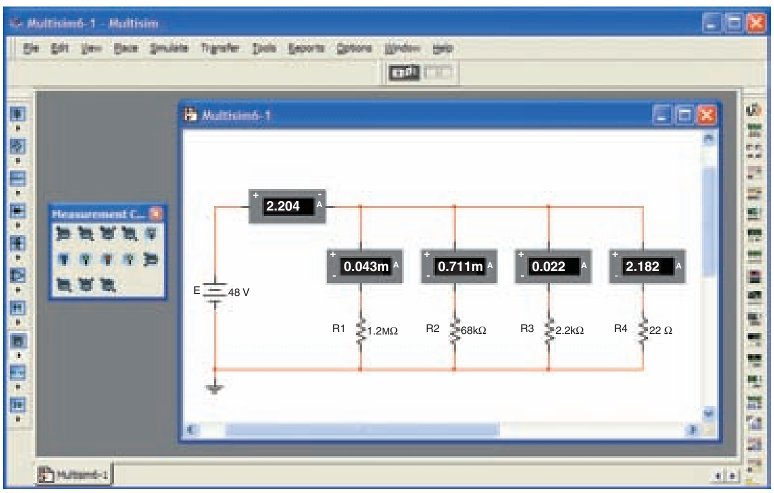


FIG. 6.71

Using the indicators of Multisim to display the currents of a parallel dc network.

PROBLEMS

SECTION 6.2 Parallel Resistors

1. For each configuration in Fig. 6.72, find the voltage sources and/or resistors elements (individual elements, not combinations of elements) that are in parallel. Remember that elements in parallel have the same voltage.

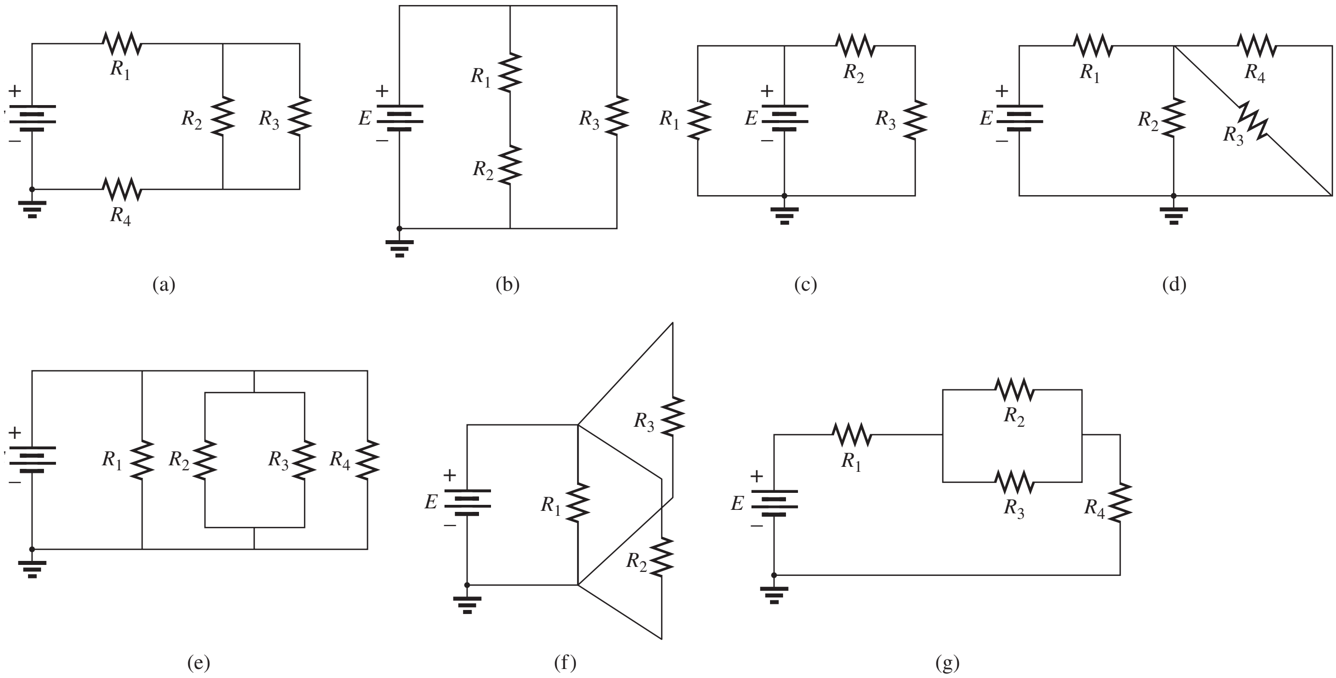


FIG. 6.72
Problem 1.

2. For the network in Fig. 6.73:
   - **a.** Find the elements (voltage sources and/or resistors) that are in parallel.
   - **b.** Find the elements (voltage sources and/or resistors) that are in series.
3. Find the total resistance for each configuration in Fig. 6.74. Note that only standard value resistors were used.
4. For each circuit board in Fig. 6.75, find the total resistance between connection tabs 1 and 2.
5. The total resistance of each of the configurations in Fig. 6.76 is specified. Find the unknown resistance.

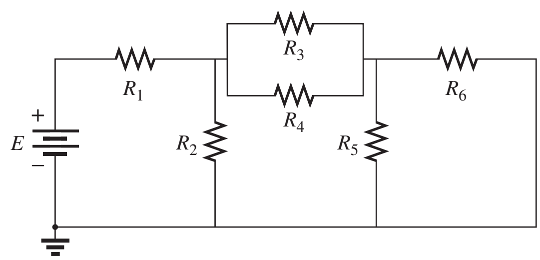


FIG. 6.73
Problem 2.

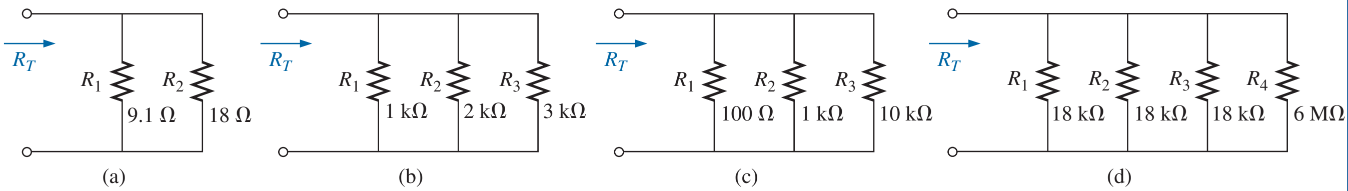


FIG. 6.74
Problem 3.

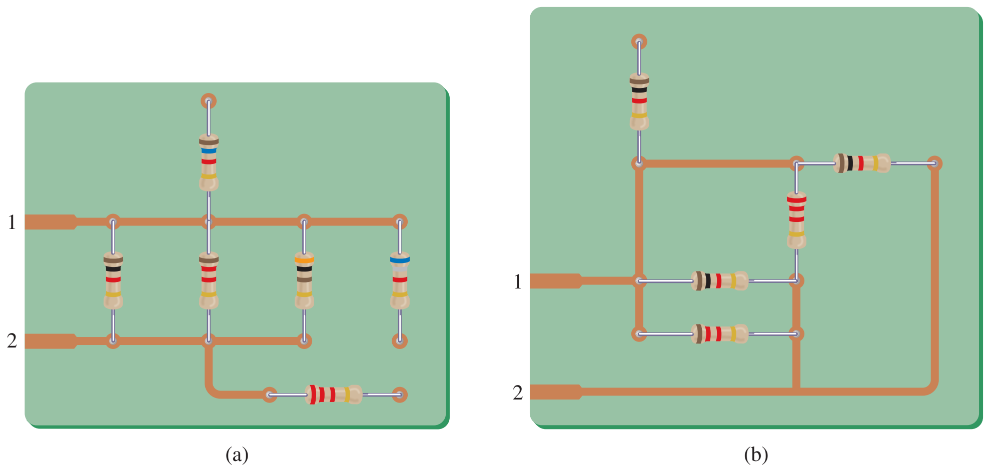


FIG. 6.75
Problem 4.

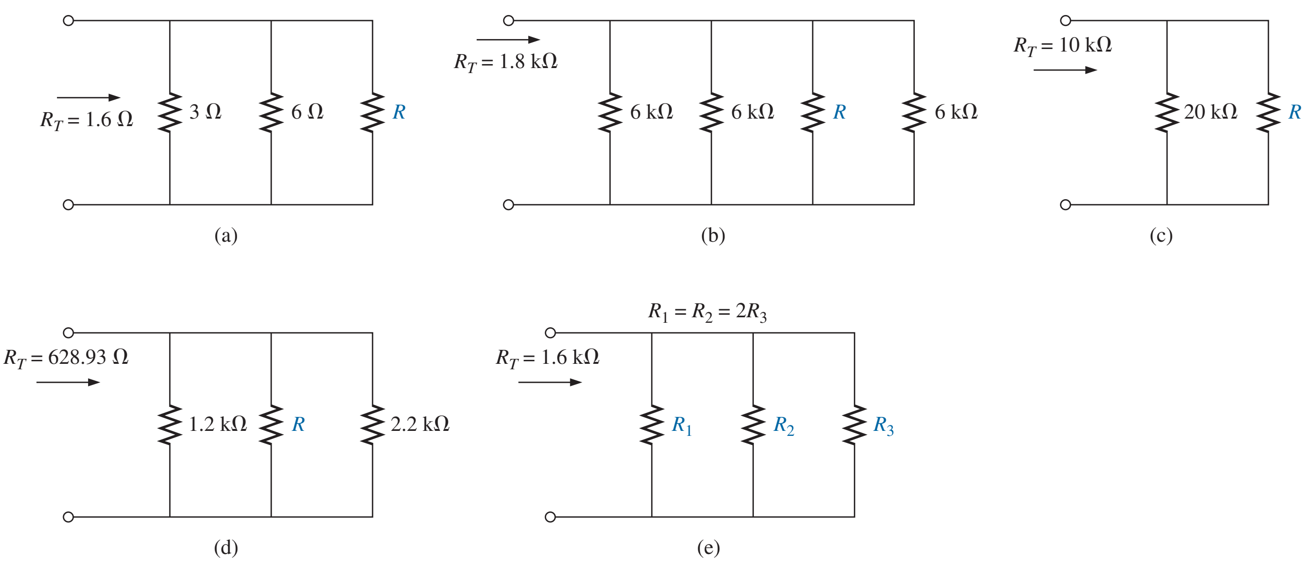


FIG. 6.76
Problem 5.

6. For the parallel network in Fig. 6.77, composed of standard values:

a. Which resistor has the most impact on the total resistance?

b. Without making a single calculation, what is an approximate value for the total resistance?

c. Calculate the total resistance and comment on your response to part (b).

d. On an approximate basis, which resistors can be ignored when determining the total reistance?

e. If we add another parallel resistor of any value to the network, what is the impact on the total resistance?

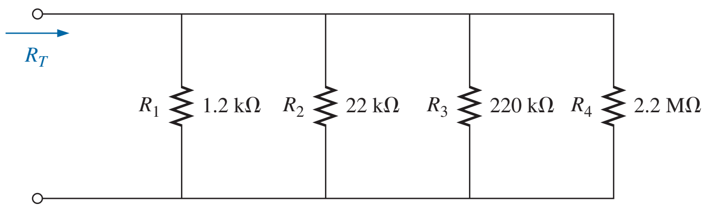


FIG. 6.77
Problem 6.

7. What is the ohmmeter reading for each configuration in Fig. 6.78?

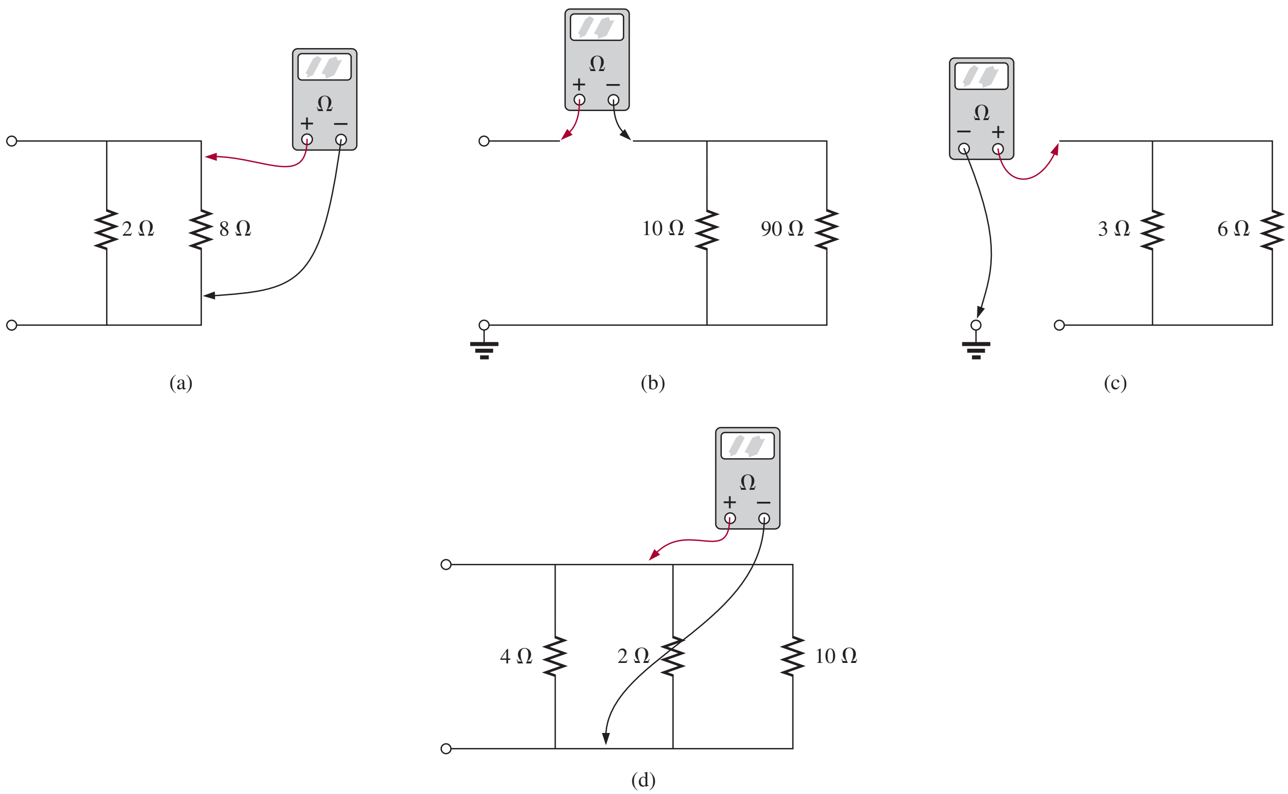


FIG. 6.78
Problem 7.

***8.** Determine the unknown resistors in Fig. 6.79 given the fact that $R_2 = 5R_1$ and $R_3 = (1/2)R_1$.

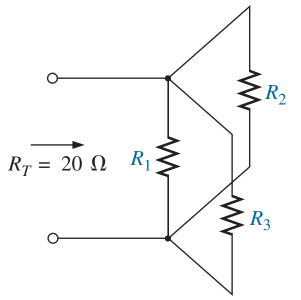


FIG. 6.79
Problem 8.

***9.** Determine R_1 for the network in Fig. 6.80.

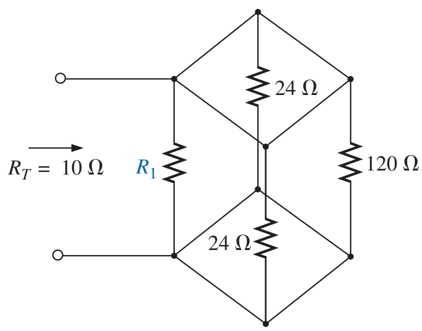


FIG. 6.80
Problem 9.

SECTION 6.3 Parallel Circuits

10. For the parallel network in Fig. 6.81:

a. Find the total resistance.
b. What is the voltage across each branch?
c. Determine the source current and the current through each branch.
d. Verify that the source current equals the sum of the branch currents.

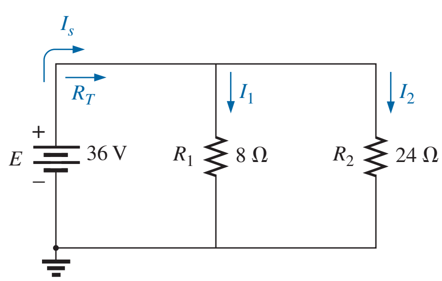


FIG. 6.81
Problem 10.

11. Repeat the analysis of Problem 10 for the network in Fig. 6.82.

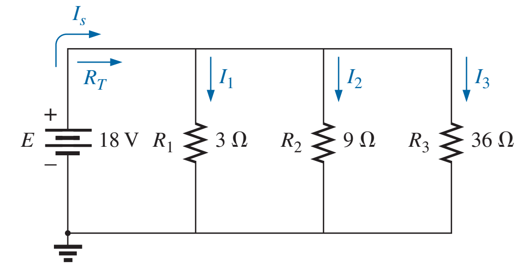


FIG. 6.82
Problems 11 and 14.

12. Repeat the analysis of Problem 10 for the network in Fig. 6.83, constructed of standard value resistors.

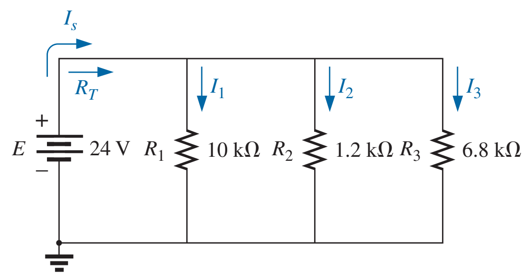


FIG. 6.83
Problem 12.

13. For the parallel network in Fig. 6.84:

a. Without making a single calculation, make a guess on the total resistance.
b. Calculate the total resistance and compare it to your guess in part (a).
c. Without making a single calculation, which branch will have the most current? Which will have the least?
d. Calculate the current through each branch, and compare your results to the assumptions of part (c).
e. Find the source current and test whether it equals the sum of the branch currents.
f. How does the magnitude of the source current compare to that of the branch currents?

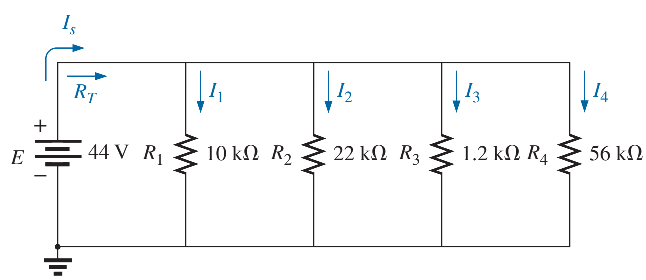


FIG. 6.84
Problem 13.

14. For the network in Fig. 6.82:

a. Redraw the network and insert ammeters to measure the source current and the current through each branch.

b. Connect a voltmeter to measure the source voltage and the voltage across resistor, R_3. Is there any difference in the connections? Why?

15. What is the response of the voltmeter and ammeters connected in Fig. 6.85?

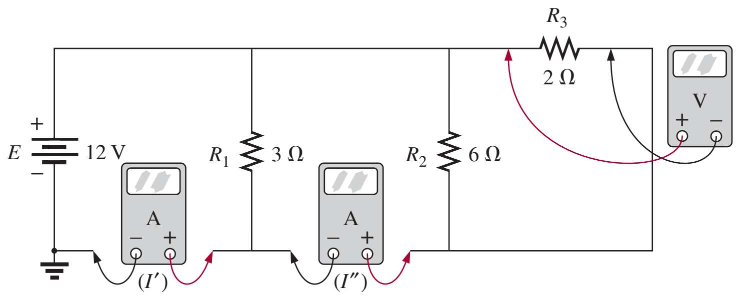


FIG. 6.85
Problem 15.

16. Given the information provided in Fig. 6.86, find the unknown quantities: E, R_1, and I_3.

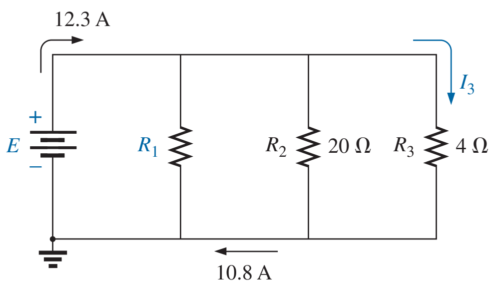


FIG. 6.86
Problem 16.

17. Determine the currents I_1 and I_s for the networks in Fig. 6.87.

***18.** For the network in Fig. 6.88:

a. Find the current I.

b. Determine the voltage V.

c. Calculate the source current I_s.

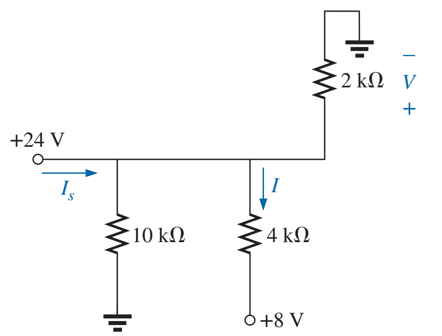


FIG. 6.88
Problem 18.

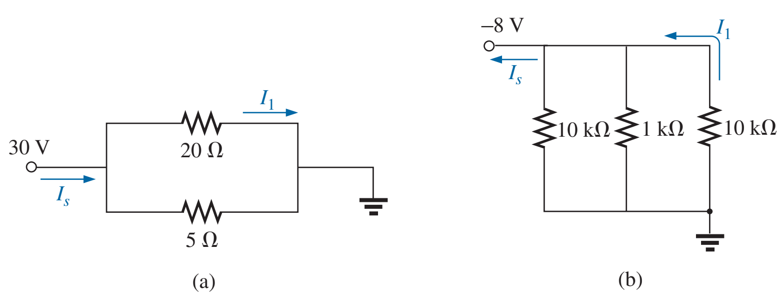


FIG. 6.87
Problem 17.

SECTION 6.4 Power Distribution in a Parallel Circuit

19. For the configuration in Fig. 6.89:

a. Find the total resistance and the current through each branch.

b. Find the power delivered to each resistor.

c. Calculate the power delivered by the source.

d. Compare the power delivered by the source to the sum of the powers delivered to the resistors.

e. Which resistor received the most power? Why?

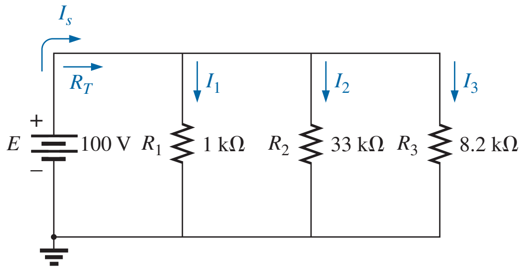


FIG. 6.89
Problem 19.

20. Eight holiday lights are connected in parallel as shown in Fig. 6.90.

a. If the set is connected to a 120 V source, what is the current through each bulb if each bulb has an internal resistance of 1.8 kΩ?

b. Determine the total resistance of the network.

c. Find the current drain from the supply.

d. What is the power delivered to each bulb?

e. Using the results of part (d), what is the power delivered by the source?

f. If one bulb burns out (that is, the filament opens up), what is the effect on the remaining bulbs? What is the effect on the source current? Why?

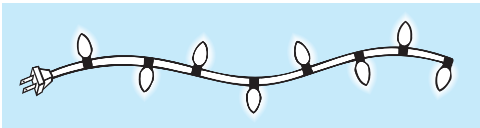


FIG. 6.90
Problem 20.

21. Determine the power delivered by the dc battery in Fig. 6.91.

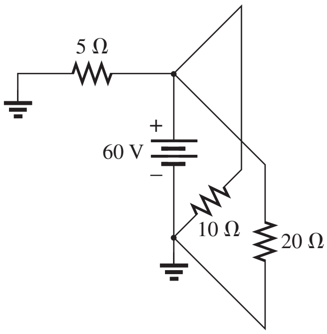


FIG. 6.91
Problem 21.

22. A portion of a residential service to a home is depicted in Fig. 6.92.

a. Determine the current through each parallel branch of the system.

b. Calculate the current drawn from the 120 V source. Will the 20 A breaker trip?

c. What is the total resistance of the network?

d. Determine the power delivered by the source. How does it compare to the sum of the wattage ratings appearing in Fig. 6.92?

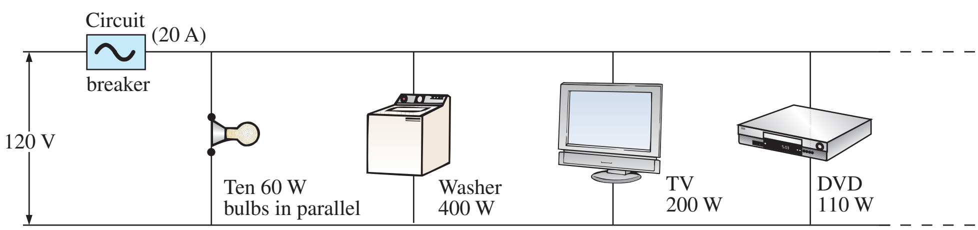


FIG. 6.92
Problem 22.

***23.** For the network in Fig. 6.93:

a. Find the current I_1.

b. Calculate the power dissipated by the 4 Ω resistor.

c. Find the current I_2.

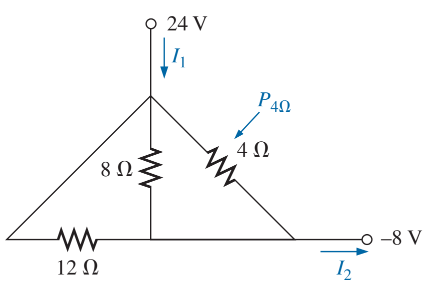


FIG. 6.93
Problem 23.

SECTION 6.5 Kirchhoff's Current Law

24. Using Kirchhoff's current law, determine the unknown currents for the parallel network in Fig. 6.94.

25. Using Kirchoff's current law, find the unknown currents for the complex configurations in Fig. 6.95.

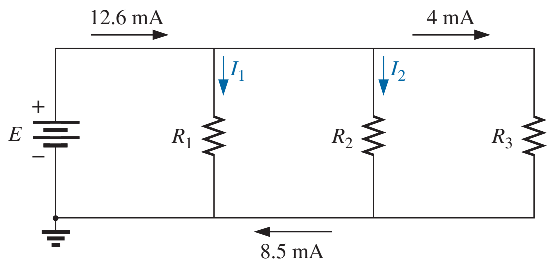


FIG. 6.94
Problem 24.

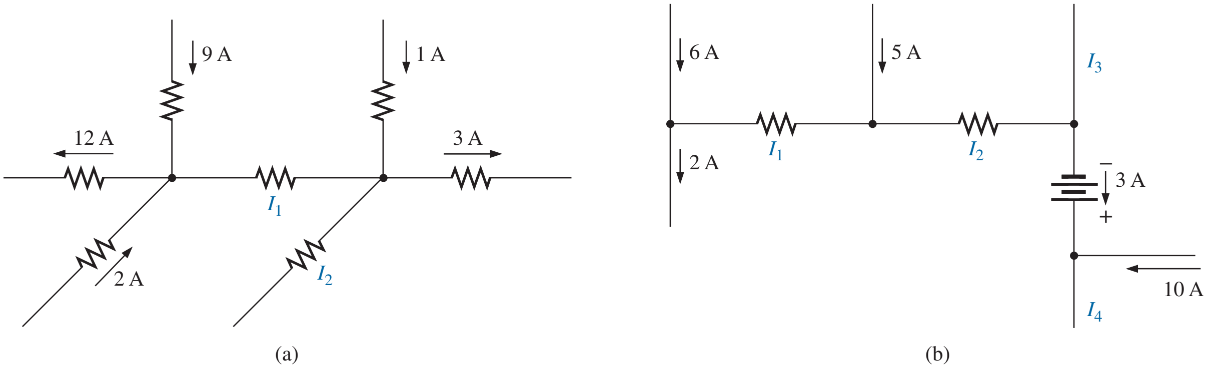


FIG. 6.95
Problem 25.

26. Using Kirchhoff's current law, determine the unknown currents for the networks in Fig. 6.96.

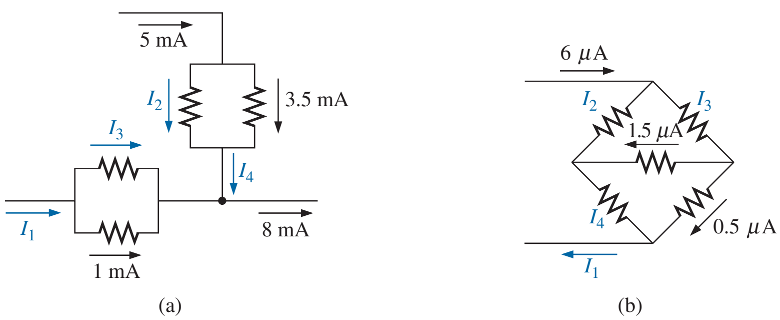


FIG. 6.96
Problem 26.

27. Using the information provided in Fig. 6.97, find the branch resistors R_1 and R_3, the total resistance R_T, and the voltage source E.

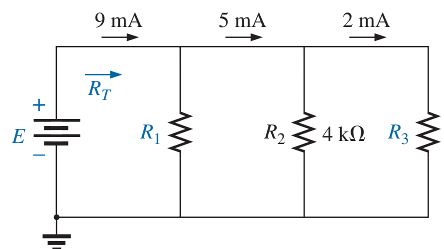


FIG. 6.97
Problem 27.

28. Find the unknown quantities for the networks in Fig. 6.98 using the information provided.

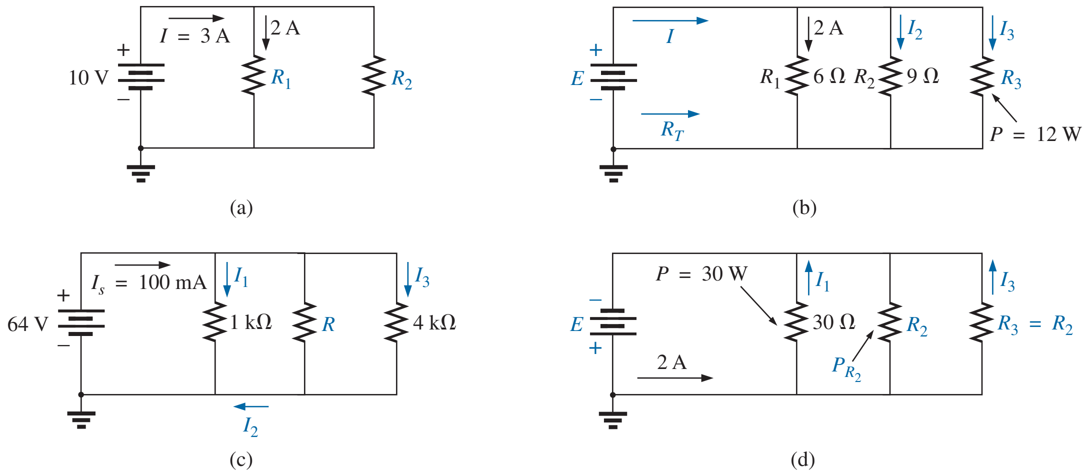


FIG. 6.98
Problem 28.

SECTION 6.6 Current Divider Rule

29. Based solely on the resistor values, determine all the currents for the configuration in Fig. 6.99. Do not use Ohm's law.

30. Determine the currents for the configurations in Fig. 6.100.

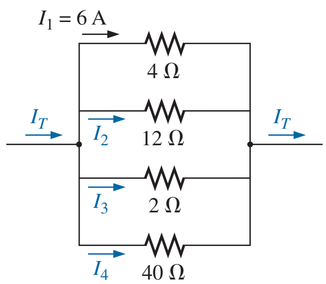


FIG. 6.99
Problem 29.

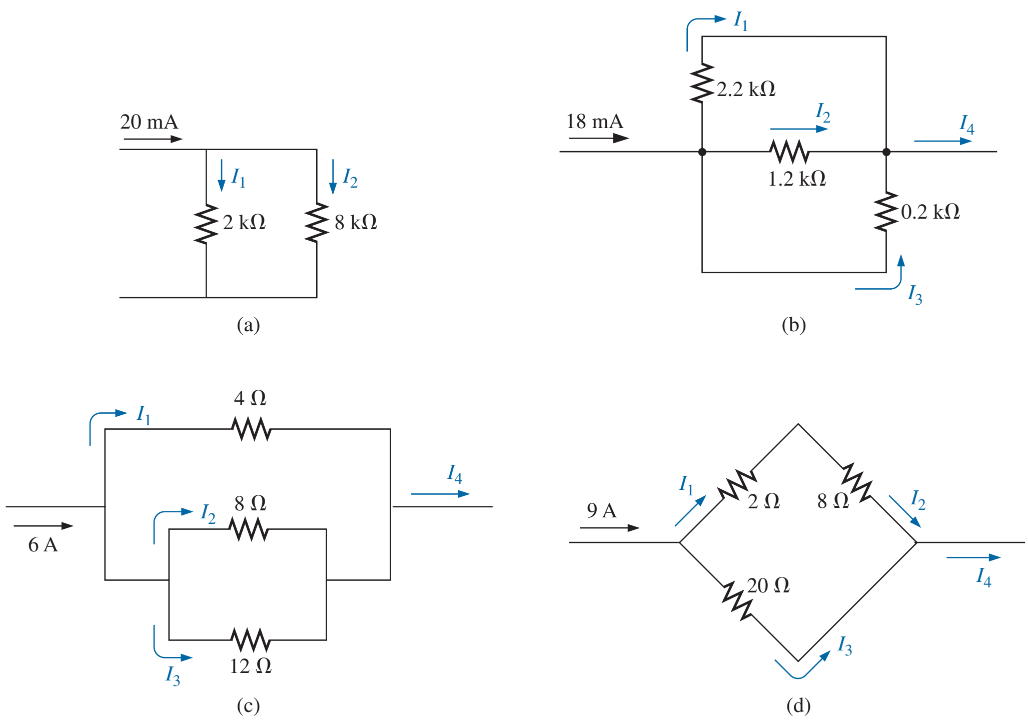


FIG. 6.100
Problem 30.

31. Parts (a) through (e) of this problem should be done by inspection—that is, mentally. The intent is to obtain an approximate solution without a lengthy series of calculations. For the network in Fig. 6.101:

- **a.** What is the approximate value of I_1 considering the magnitude of the parallel elements?
- **b.** What is the ratio I_1/I_2? Using the result of part (a), what is an approximate value of I_2?
- **c.** What is the ratio I_1/I_3? Using the result, what is an approximate value of I_3?
- **d.** What is the ratio I_1/I_4? Using the result, what is an approximate value of I_4?
- **e.** What is the effect of the parallel 100 kΩ resistor on the above calculations? How much smaller will the current I_4 be than the current I_1?
- **f.** Calculate the current through the 1 Ω resistor using the current divider rule. How does it compare to the result of part (a)?
- **g.** Calculate the current through the 10 Ω resistor. How does it compare to the result of part (b)?
- **h.** Calculate the current through the 1 kΩ resistor. How does it compare to the result of part (c)?
- **i.** Calculate the current through the 100 kΩ resistor. How does it compare to the solutions to part (e)?

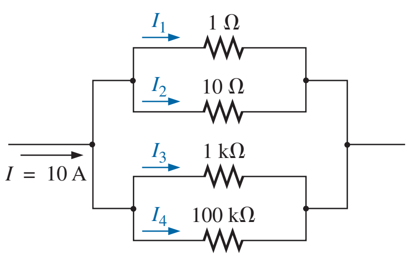


FIG. 6.101
Problem 31.

32. Find the unknown quantities for the networks in Fig. 6.102 using the information provided.

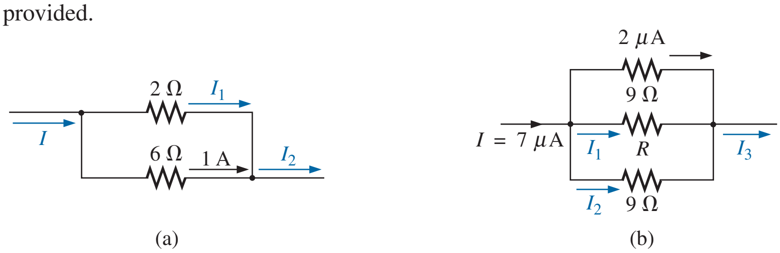


FIG. 6.102
Problem 32.

33. **a.** Find resistor R for the network in Fig. 6.103 that will ensure that $I_1 = 3I_2$.
b. Find I_1 and I_2.

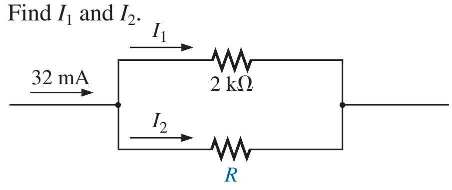


FIG. 6.103
Problem 33.

34. Design the network in Fig. 6.104 such that $I_2 = 2I_1$ and $I_3 = 2I_2$.

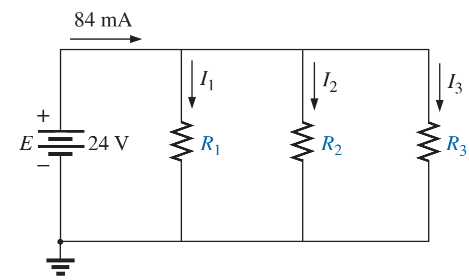


FIG. 6.104
Problem 34.

SECTION 6.7 Voltage Source in Parallel

35. Assuming identical supplies in Fig. 6.105.

- **a.** Find the indicated currents.
- **b.** Find the power delivered by each source.
- **c.** Find the total power delivered by both sources, and compare it to the power delivered to the load R_L.
- **d.** If only source current were available, what would the current drain be to supply the same power to the load? How does the current level compare to the calculated level of part (a)?

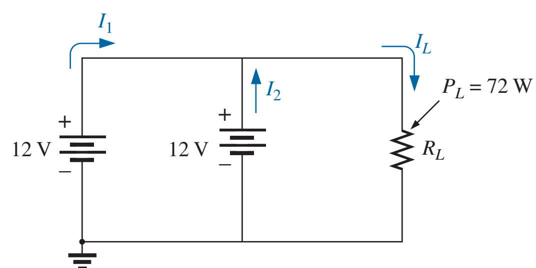


FIG. 6.105
Problem 35.

36. Assuming identical supplies, determine currents I_1, I_2, and I_3 for the configuration in Fig. 6.106.

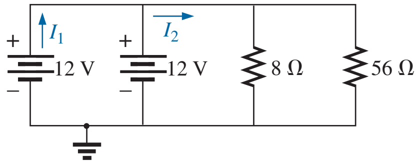


FIG. 6.106
Problem 36.

37. Assuming identical supplies, determine the current I and resistance R for the parallel network in Fig. 6.107.

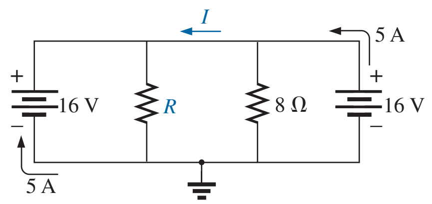


FIG. 6.107
Problem 37.

SECTION 6.8 Open and Short Circuits

38. For the network in Fig. 6.108:
- **a.** Determine I_s and V_L.
- **b.** Determine I_s if R_L is shorted out.
- **c.** Determine V_L if R_L is replaced by an open circuit.

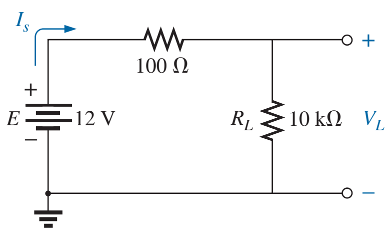


FIG. 6.108
Problem 38.

39. For the network in Fig. 6.109:
- **a.** Determine the open-circuit voltage V_L.
- **b.** If the 2.2 kΩ resistor is short circuited, what is the new value of V_L?
- **c.** Determine V_L if the 4.7 kΩ resistor is replaced by an open circuit.

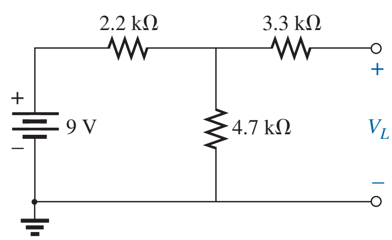


FIG. 6.109
Problem 39.

***40.** For the network in Fig. 6.110, determine
- **a.** the short-circuit currents I_1 and I_2.
- **b.** the voltages V_1 and V_2.
- **c.** the source current I_s.

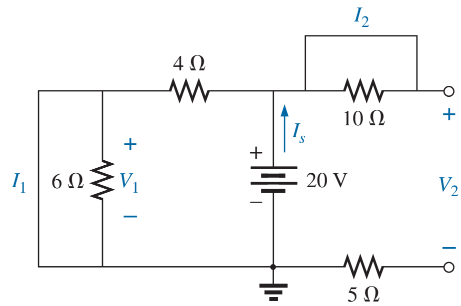


FIG. 6.110
Problem 40.

SECTION 6.9 Voltmeter Loading Effects

41. For the simple series configuration in Fig. 6.111:
- **a.** Determine voltage V_2.
- **b.** Determine the reading of a DMM having an internal resistance of 11 MΩ when used to measure V_2.
- **c.** Repeat part (b) with a VOM having an Ω/V rating of 20,000 using the 20 V scale. Compare the results of parts (b) and (c). Explain any differences.
- **d.** Repeat parts (a) through (c) with $R_1 = 100$ kΩ and $R_2 = 200$ kΩ.
- **e.** Based on the above, what general conclusions can you make about the use of a DMM or a VOM in the voltmeter mode?

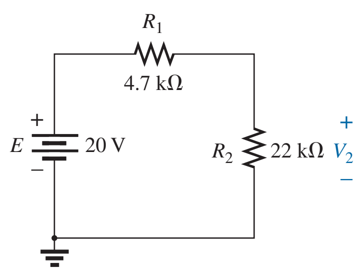


FIG. 6.111
Problem 41.

42. Given the configuration in Fig. 6.112:
   a. What is the voltage between points *a* and *b?*
   b. What will the reading of a DMM be when placed across terminals *a* and *b* if the internal resistance of the meter is 11 MΩ?
   c. Repeat part (b) if a VOM having an Ω/V rating of 20,000 using the 200 V scale is used. What is the reading using the 20 V scale? Is there a difference? Why?

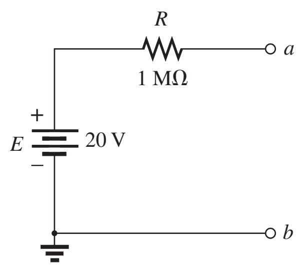


FIG. 6.112
Problem 42.

SECTION 6.10 Troubleshooting Techniques

43. Based on the measurements of Fig. 6.113, determine whether the network is operating correctly. If not, try to determine why.

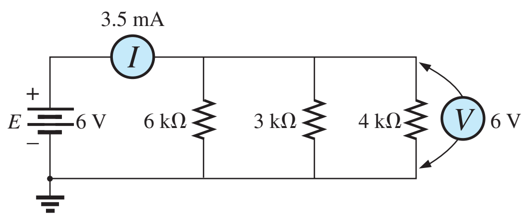


FIG. 6.113
Problem 43.

44. Referring to Fig. 6.114, find the voltage V_{ab} without the meter in place. When the meter is applied to the active network, it reads 8.8 V. If the measured value does not equal the theoretical value, which element or elements may have been connected incorrectly?

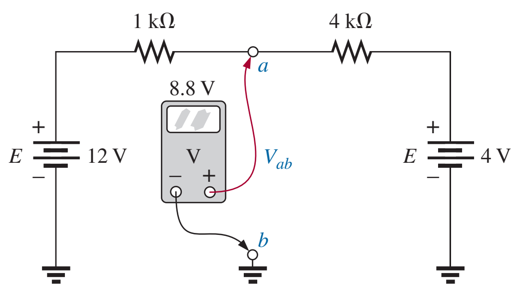


FIG. 6.114
Problem 44.

45. a. The voltage V_a for the network in Fig. 6.115 is −1 V. If it suddenly jumped to 20 V, what could have happened to the circuit structure? Localize the problem area.
   b. If the voltage V_a is 6 V rather than −1 V, try to explain what is wrong about the network construction.

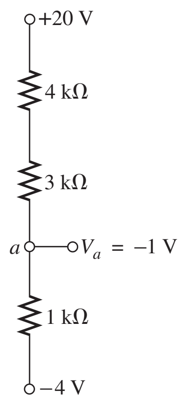


FIG. 6.115
Problem 45.

SECTION 6.14 Computer Analysis

46. Using PSpice or Multisim, verify the results of Example 6.13.
47. Using PSpice or Multisim, determine the solution to Problem 10, and compare your answer to the longhand solution.
48. Using PSpice or Multisim, determine the solution to Problem 12, and compare your answer to the longhand solution.

GLOSSARY

Current divider rule (CDR) A method by which the current through parallel elements can be determined without first finding the voltage across those parallel elements.

Kirchhoff's current law (KCL) The algebraic sum of the currents entering and leaving a node is zero.

Node A junction of two or more branches.

Ohm/volt (Ω/V) rating A rating used to determine both the current sensitivity of the movement and the internal resistance of the meter.

Open circuit The absence of a direct connection between two points in a network.

Parallel circuit A circuit configuration in which the elements have two points in common.

Short circuit A direct connection of low resistive value that can significantly alter the behavior of an element or system.