3

RESISTANCE

Objectives

- ***Become familiar with the parameters that determine the resistance of an element and be able to calculate the resistance from the given dimensions and material characteristics.***
- ***Understand the effects of temperature on the resistance of a material and how to calculate the change in resistance with temperature.***
- ***Develop some understanding of superconductors and how they can affect future development in the industry.***
- ***Become familiar with the broad range of commercially available resistors available today and how to read the value of each from the color code or labeling.***
- ***Become aware of a variety of elements such as thermistors, photoconductive cells, and varistors and how their terminal resistance is controlled.***

3.1 INTRODUCTION

In the previous chapter, we found that placing a voltage across a wire or simple circuit results in a flow of charge or current through the wire or circuit. The question remains, however, What determines the level of current that results when a particular voltage is applied? Why is the current heavier in some circuits than in others? The answers lie in the fact that there is an opposition to the flow of charge in the system that depends on the components of the circuit. This opposition to the flow of charge through an electrical circuit, called **resistance,** has the units of **ohms** and uses the Greek letter *omega* (Ω) as its symbol. The graphic symbol for resistance, which resembles the cutting edge of a saw, is provided in Fig. 3.1.

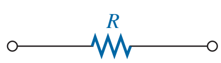


FIG. 3.1
Resistance symbol and notation.

This opposition, due primarily to collisions and friction between the free electrons and other electrons, ions, and atoms in the path of motion, converts the supplied electrical energy into **heat** that raises the temperature of the electrical component and surrounding medium. The heat you feel from an electrical heater is simply due to passing current through a high-resistance material.

Each material with its unique atomic structure reacts differently to pressures to establish current through its core. Conductors that permit a generous flow of charge with little external pressure have low resistance levels, while insulators have high resistance characteristics.

3.2 RESISTANCE: CIRCULAR WIRES

The resistance of any material is due primarily to four factors:

1. ***Material***
2. ***Length***
3. ***Cross-sectional area***
4. ***Temperature of the material***

As noted in Section 3.1, the atomic structure determines how easily a free electron will pass through a material. The longer the path through which the free electron must pass, the greater the resistance factor. Free electrons pass more easily through conductors with larger cross-sectional areas. In addition, the higher the temperature of the conductive materials, the greater the internal vibration and motion of the components that make up the atomic structure of the wire, and the more difficult it is for the free electrons to find a path through the material.

The first three elements are related by the following basic equation for resistance:

$$R = \rho \frac{l}{A} \qquad \begin{aligned} \rho &= \text{CM-}\Omega/\text{ft at } T = 20^\circ\text{C} \\ l &= \text{feet} \\ A &= \text{area in circular mils (CM)} \end{aligned} \qquad \textbf{(3.1)}$$

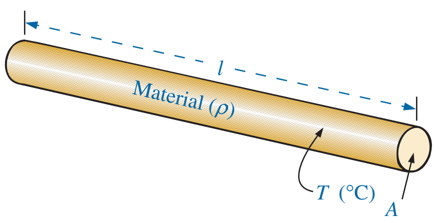


FIG. 3.2
Factors affecting the resistance of a conductor.

with each component of the equation defined by Fig. 3.2.

The material is identified by a factor called the **resistivity,** which uses the Greek letter *rho* (ρ) as its symbol and is measured in CM-Ω/ft. Its value at a temperature of 20°C (room temperature = 68°F) is provided in Table 3.1 for a variety of common materials. Since the larger the resistivity, the greater the resistance to setting up a flow of charge, it appears as a multiplying factor in Eq. (3.1); that is, it appears in the numerator of the equation. It is important to realize at this point that since the resistivity is provided at a particular temperature, *Eq. (3.1) is applicable only at room temperature.* The effect of higher and lower temperatures is considered in Section 3.4.

TABLE 3.1
Resistivity (ρ) of various materials.

Material	ρ (CM · Ω/ft)@20°C
Silver	9.9
Copper	**10.37**
Gold	14.7
Aluminum	17.0
Tungsten	33.0
Nickel	47.0
Iron	74.0
Constantan	295.0
Nichrome	600.0
Calorite	720.0
Carbon	21,000.0

Since the resistivity is in the numerator of Eq. (3.1),

the higher the resistivity, the greater the resistance of a conductor

as shown for two conductors of the same length in Fig. 3.3(a).

Further,

the longer the conductor, the greater the resistance

since the length also appears in the numerator of Eq. (3.1). Note Fig. 3.3(b). Finally,

the greater the area of a conductor, the less the resistance

because the area appears in the denominator of Eq. (3.1). Note Fig. 3.3(c).

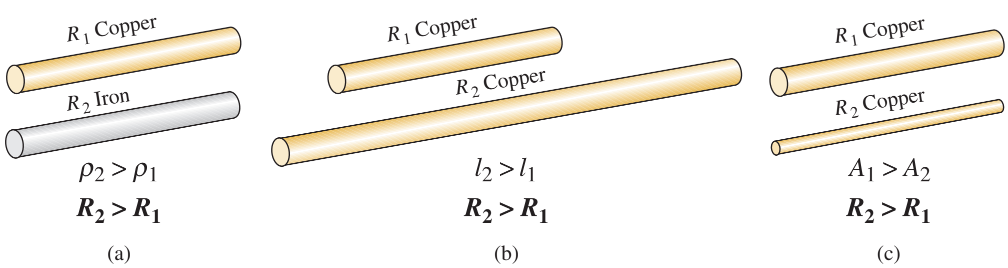


FIG. 3.3
Cases in which $R_2 > R_1$. For each case, all remaining parameters that control the resistance level are the same.

Circular Mils (CM)

In Eq. (3.1), the area is measured in a quantity called **circular mils** (CM). It is the quantity used in most commercial wire tables, and thus it needs to be carefully defined. The *mil* is a unit of measurement for length and is related to the inch by

$$\mathbf{1\ mil} = \frac{\mathbf{1}}{\mathbf{1000}}\ \mathbf{in.}$$

or

$$\mathbf{1000\ mils = 1\ in.}$$

In general, therefore, the mil is a very small unit of measurement for length. There are 1000 mils in an inch, or 1 mil is only 1/1000 of an inch. It is a length that is not visible with the naked eye although it can be measured with special instrumentation. The phrase *milling* used in steel factories is derived from the fact that a few mils of material are often removed by heavy machinery such as a lathe, and the thickness of steel is usually measured in mils.

By definition,

a wire with a diameter of 1 mil has an area of 1 CM.

as shown in Fig. 3.4.

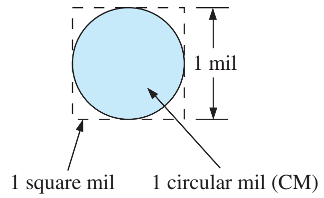


FIG. 3.4
Defining the circular mil (CM).

An interesting result of such a definition is that the area of a circular wire in circular mils can be defined by the following equation:

$$A_{CM} = (d_{mils})^2 \qquad \textbf{(3.2)}$$

Verification of this equation appears in Fig. 3.5 which shows that a wire with a diameter of 2 mils has a total area of 4 CM and a wire with a diameter of 3 mils has a total area of 9 CM.

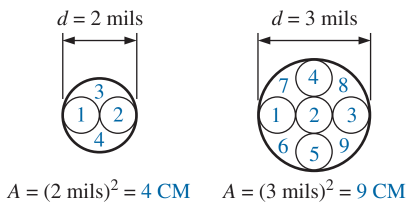


FIG. 3.5
Verification of Eq. (3.2): $A_{CM} = (d_{mils})^2$.

Remember, to compute the area of a wire in circular mils when the diameter is given in inches, first convert the diameter to mils by simply writing the diameter in decimal form and moving the decimal point three places to the right. For example,

$$\frac{1}{8}\text{ in.} = 0.\underset{\text{3 places}}{125}\text{ in.} = 125\text{ mils}$$

Then the area is determined by

$$A_{CM} = (d_{mils})^2 = (125\text{ mils})^2 = \mathbf{15{,}625\ CM}$$

Sometimes when you are working with conductors that are not circular, you will need to convert square mils to circular mils, and vice versa. Applying the basic equation for the area of a circle and substituting a diameter of 1 mil results in

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(1\text{ mil})^2 = \frac{\pi}{4}\text{ sq mils} \overset{\text{by definition}}{\equiv} 1\text{ CM}$$

from which we can conclude the following:

$$1\text{ CM} = \frac{\pi}{4}\text{ sq mils} \qquad \textbf{(3.3)}$$

or

$$1\text{ sq mil} = \frac{4}{\pi}\text{ CM} \qquad \textbf{(3.4)}$$

EXAMPLE 3.1 What is the resistance of a 100 ft length of copper wire with a diameter of 0.020 in. at 20°C?

Solution:

$$\rho = 10.37\,\frac{\text{CM-}\Omega}{\text{ft}} \qquad 0.020\text{ in.} = 20\text{ mils}$$

$$A_{CM} = (d_{\text{mils}})^2 = (20\text{ mils})^2 = 400\text{ CM}$$

$$R = \rho\frac{l}{A} = \frac{(10.37\text{ CM-}\Omega/\text{ft})(100\text{ ft})}{400\text{ CM}}$$

$$R = \mathbf{2.59\ \Omega}$$

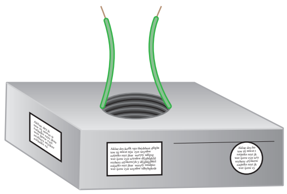


FIG. 3.6
Example 3.2.

EXAMPLE 3.2 An undetermined number of feet of wire have been used from the carton in Fig. 3.6. Find the length of the remaining copper wire if it has a diameter of 1/16 in. and a resistance of 0.5 Ω.

Solution:

$$\rho = 10.37\text{ CM-}\Omega/\text{ft} \qquad \frac{1}{16}\text{ in.} = 0.0625\text{ in.} = 62.5\text{ mils}$$

$$A_{\text{CM}} = (d_{\text{mils}})^2 = (62.5\text{ mils})^2 = 3906.25\text{ CM}$$

$$R = \rho\frac{l}{A} \Rightarrow l = \frac{RA}{\rho} = \frac{(0.5\ \Omega)(3906.25\text{ CM})}{10.37\,\dfrac{\text{CM-}\Omega}{\text{ft}}} = \frac{1953.125}{10.37}$$

$$l = \mathbf{188.34\ ft}$$

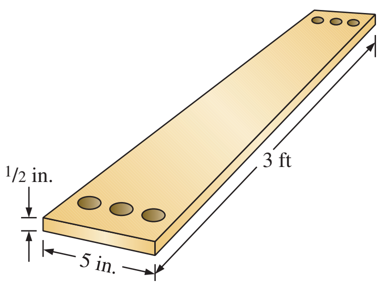


FIG. 3.7
Example 3.3.

EXAMPLE 3.3 What is the resistance of a copper bus-bar, as used in the power distribution panel of a high-rise office building, with the dimensions indicated in Fig. 3.7?

Solution:

$$A_{\text{CM}}\begin{cases} 5.0\text{ in.} = 5000\text{ mils} \\ \frac{1}{2}\text{ in.} = 500\text{ mils} \\ A = (5000\text{ mils})(500\text{ mils}) = 2.5\times10^6\text{ sq mils} \\ \quad = 2.5\times10^6\,\cancel{\text{sq mils}}\left(\dfrac{4/\pi\text{ CM}}{1\,\cancel{\text{sq mil}}}\right) \\ A = 3.183\times10^6\text{ CM} \end{cases}$$

$$R = \rho\frac{l}{A} = \frac{(10.37\text{ CM-}\Omega/\text{ft})(3\text{ ft})}{3.183\times10^6\text{ CM}} = \frac{31.11}{3.183\times10^6}$$

$$R = \mathbf{9.774\times10^{-6}\ \Omega}$$

(quite small, 0.000009774 Ω≅0 Ω)

You will learn in the following chapters that the less the resistance of a conductor, the lower the losses in conduction from the source to the load. Similarly, since resistivity is a major factor in determining the resistance of a conductor, the lower the resistivity, the lower the resistance for the same size conductor. It would appear from Table 3.1 that silver, copper, gold, and aluminum would be the best conductors and the most common. In general, there are other factors, however, such as **malleability** (ability of a material to be shaped), **ductility** (ability of a material to be drawn into long, thin wires), temperature sensitivity, resistance to abuse, and, of course, cost, that must all be weighed when choosing a conductor for a particular application.

In general, copper is the most widely used material because it is quite malleable, ductile, and available; has good thermal characteristics; and is less expensive than silver or gold. It is certainly not cheap, however. Contractors always ensure that the copper wiring has been removed before leveling a building because of its salvage value. Aluminum was once used for general wiring because it is cheaper than copper, but its thermal characteristics created some difficulties. The heating due to current flow and the cooling that occurred when the circuit was turned off resulted in expansion and contraction of the aluminum wire to the point where connections eventually loosened, and resulting in dangerous side effects. Aluminum is still used today, however, in areas such as integrated circuit manufacturing and in situations where the connections can be made secure. Silver and gold are, of course, much more expensive than copper or aluminum, but the cost is justified for certain applications. Silver has excellent plating characteristics for surface preparations, and gold is used quite extensively in integrated circuits. Tungsten has a resistivity three times that of copper, but there are occasions when its physical characteristics (durability, hardness) are the overriding considerations.

3.3 WIRE TABLES

The wire table was designed primarily to standardize the size of wire produced by manufacturers. As a result, the manufacturer has a larger market, and the consumer knows that standard wire sizes will always be available. The table was designed to assist the user in every way possible; it usually includes data such as the cross-sectional area in circular mils, diameter in mils, ohms per 1000 feet at 20°C, and weight per 1000 feet.

The American Wire Gage (AWG) sizes are given in Table 3.2 for solid round copper wire. A column indicating the maximum allowable current in amperes, as determined by the National Fire Protection Association, has also been included.

The chosen sizes have an interesting relationship: For every drop in 3 gage numbers, the area is doubled; and for every drop in 10 gage numbers, the area increases by a factor of 10.

Examining Eq. (3.1), we note also that *doubling the area cuts the resistance in half, and increasing the area by a factor of 10 decreases the resistance of 1/10 the original*, everything else kept constant.

The actual sizes of some of the gage wires listed in Table 3.2 are shown in Fig. 3.8 with a few of their areas of application. A few examples using Table 3.2 follow.

EXAMPLE 3.4 Find the resistance of 650 ft of #8 copper wire (T = 20°C).

Solution: For #8 copper wire (solid), Ω/1000 ft at 20°C = 0.6282 Ω, and

$$650\ \cancel{\text{ft}}\left(\frac{0.6282\ \Omega}{1000\ \cancel{\text{ft}}}\right) = \mathbf{0.408\ \Omega}$$

TABLE 3.2

American Wire Gage (AWG) sizes.

	AWG #	Area (CM)	Ω/1000 ft at 20°C	Maximum Allowable Current for RHW Insulation (A)*
(4/0)	**0000**	211,600	0.0490	**230**
(3/0)	**000**	167,810	0.0618	**200**
(2/0)	**00**	133,080	0.0780	**175**
(1/0)	**0**	105,530	0.0983	**150**
	1	83,694	0.1240	**130**
	2	66,373	0.1563	**115**
	3	52,634	0.1970	**100**
	4	41,742	0.2485	**85**
	5	33,102	0.3133	—
	6	26,250	0.3951	**65**
	7	20,816	0.4982	—
	8	16,509	0.6282	**50**
	9	13,094	0.7921	—
	10	10,381	0.9989	**30**
	11	8,234.0	1.260	—
	12	6,529.9	1.588	**20**
	13	5,178.4	2.003	—
	14	4,106.8	2.525	**15**
	15	3,256.7	3.184	
	16	2,582.9	4.016	
	17	2,048.2	5.064	
	18	1,624.3	6.385	
	19	1,288.1	8.051	
	20	1,021.5	10.15	
	21	810.10	12.80	
	22	642.40	16.14	
	23	509.45	20.36	
	24	404.01	25.67	
	25	320.40	32.37	
	26	254.10	40.81	
	27	201.50	51.47	
	28	159.79	64.90	
	29	126.72	81.83	
	30	100.50	103.2	
	31	79.70	130.1	
	32	63.21	164.1	
	33	50.13	206.9	
	34	39.75	260.9	
	35	31.52	329.0	
	36	25.00	414.8	
	37	19.83	523.1	
	38	15.72	659.6	
	39	12.47	831.8	
	40	9.89	1049.0	

*Not more than three conductors in raceway, cable, or direct burial.

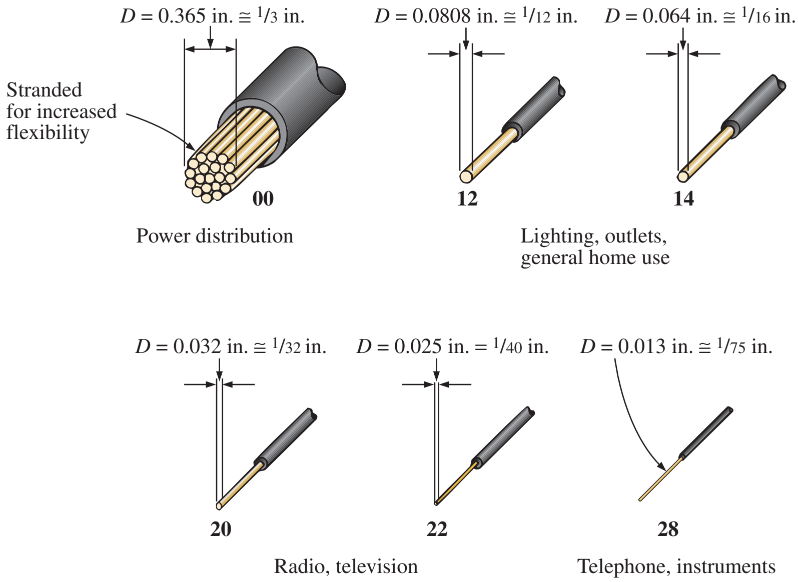


FIG. 3.8
Popular wire sizes and some of their areas of application.

EXAMPLE 3.5 What is the diameter, in inches, of a #12 copper wire?

Solution: For #12 copper wire (solid), $A = 6529.9$ CM, and

$$d_{\text{mils}} = \sqrt{A_{\text{CM}}} = \sqrt{6529.9 \text{ CM}} \cong 80.81 \text{ mils}$$

$$d = \mathbf{0.0808 \text{ in.}} \text{ (or close to } 1/12 \text{ in.)}$$

EXAMPLE 3.6 For the system in Fig. 3.9, the total resistance of *each* power line cannot exceed 0.025 Ω, and the maximum current to be drawn by the load is 95 A. What gage wire should be used?

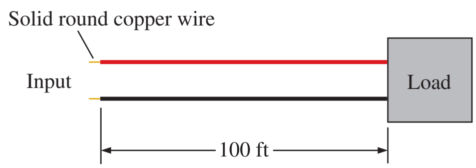


FIG. 3.9
Example 3.6.

Solution:

$$R = \rho\frac{l}{A} \Rightarrow A = \rho\frac{l}{R} = \frac{(10.37 \text{ CM-}\Omega/\text{ft})(100 \text{ ft})}{0.025\ \Omega} = \mathbf{41{,}480 \text{ CM}}$$

Using the wire table, we choose the wire with the next largest area, which is #4, to satisfy the resistance requirement. We note, however, that 95 A must flow through the line. This specification requires that **#3 wire** be used since the #4 wire can carry a maximum current of only 85 A.

3.4 RESISTANCE: METRIC UNITS

The design of resistive elements for various areas of application, including thin-film resistors and integrated circuits, uses metric units for the quantities of Eq. (3.1). In SI units, the resistivity would be measured in ohm-meters, the area in square meters, and the length in meters. However, the meter is generally too large a unit of measure for most applications, and so the centimeter is usually employed. The resulting dimensions for Eq. (3.1) are therefore

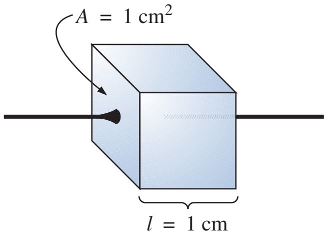


FIG. 3.10
Defining ρ in ohm-centimeters.

ρ:	ohm-centimeters
l:	centimeters
A:	square centimeters

The units for ρ can be derived from

$$\rho = \frac{RA}{l} = \frac{\Omega\text{-cm}^2}{\text{cm}} = \Omega\text{-cm}$$

The resistivity of a material is actually the resistance of a sample such as that appearing in Fig. 3.10. Table 3.3 provides a list of values of ρ in ohm-centimeters. Note that the area now is expressed in square centimeters, which can be determined using the basic equation $A = \pi d^2/4$, eliminating the need to work with circular mils, the special unit of measure associated with circular wires.

TABLE 3.3
Resistivity (ρ) of various materials.

Material	Ω-cm
Silver	1.645×10^{-6}
Copper	$\mathbf{1.723 \times 10^{-6}}$
Gold	2.443×10^{-6}
Aluminum	2.825×10^{-6}
Tungsten	5.485×10^{-6}
Nickel	7.811×10^{-6}
Iron	12.299×10^{-6}
Tantalum	15.54×10^{-6}
Nichrome	99.72×10^{-6}
Tin oxide	250×10^{-6}
Carbon	3500×10^{-6}

EXAMPLE 3.7 Determine the resistance of 100 ft of #28 copper telephone wire if the diameter is 0.0126 in.

Solution: Unit conversions:

$$l = 100\ \text{ft}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right) = 3048\ \text{cm}$$

$$d = 0.0126\ \text{in.}\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right) = 0.032\ \text{cm}$$

Therefore,

$$A = \frac{\pi d^2}{4} = \frac{(3.1416)(0.032\ \text{cm})^2}{4} = 8.04 \times 10^{-4}\ \text{cm}^2$$

$$R = \rho\frac{l}{A} = \frac{(1.723 \times 10^{-6}\Omega\text{-cm})(3048\ \text{cm})}{8.04 \times 10^{-4}\ \text{cm}^2} \cong \mathbf{6.5\ \Omega}$$

Using the units for circular wires and Table 3.2 for the area of a #28 wire, we find

$$R = \rho\frac{l}{A} = \frac{(10.37\ \text{CM-}\Omega/\text{ft})(100\ \text{ft})}{159.79\ \text{CM}} \cong \mathbf{6.5\ \Omega}$$

EXAMPLE 3.8 Determine the resistance of the thin-film resistor in Fig. 3.11 if the **sheet resistance** R_s (defined by $R_s = \rho/d$) is 100 Ω.

Solution: For deposited materials of the same thickness, the sheet resistance factor is usually employed in the design of thin-film resistors.

Eq. (3.1) can be written

$$R = \rho\frac{l}{A} = \rho\frac{l}{dw} = \left(\frac{\rho}{d}\right)\left(\frac{l}{w}\right) = R_s\frac{l}{w}$$

where l is the length of the sample and w is the width. Substituting into the above equation yields

$$R = R_s\frac{l}{w} = \frac{(100\ \Omega)(0.6\ \text{cm})}{0.3\ \text{cm}} = \mathbf{200\ \Omega}$$

as one might expect since $l = 2w$.

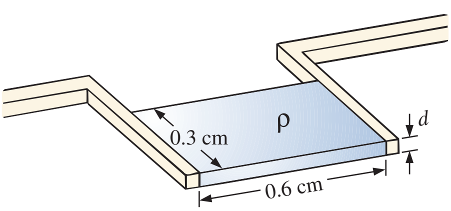


FIG. 3.11
Thin-film resistor (note Fig. 3.25).

The conversion factor between resistivity in circular mil-ohms per foot and ohm-centimeters is the following:

$$\rho\,(\Omega\text{-cm}) = (1.662 \times 10^{-7}) \times (\text{value in CM-}\Omega/\text{ft})$$

For example, for copper, $\rho = 10.37$ CM-Ω/ft:

$$\begin{aligned}\rho\,(\Omega\text{-cm}) &= 1.662 \times 10^{-7}\,(10.37\ \text{CM-}\Omega/\text{ft})\\ &= 1.723 \times 10^{-6}\ \Omega\text{-cm}\end{aligned}$$

as indicated in Table 3.3.

The resistivity in integrated circuit design is typically in ohm-centimeter units, although tables often provide ρ in ohm-meters or microhm-centimeters. Using the conversion technique of Chapter 1, we find that the conversion factor between ohm-centimeters and ohm-meters is the following:

$$1.723 \times 10^{-6}\,\Omega\text{-}\cancel{\text{cm}}\left[\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right] = \frac{1}{100}[1.723 \times 10^{-6}]\,\Omega\text{-m}$$

or the value in ohm-meters is 1/100 the value in ohm-centimeters, and

$$\rho(\Omega\text{-m}) = \left(\frac{1}{100}\right) \times (\text{value in }\Omega\text{-cm}) \tag{3.5}$$

Similarly,

$$\rho\,(\mu\Omega\text{-cm}) = (10^{6}) \times (\text{value in }\Omega\text{-cm}) \tag{3.6}$$

For comparison purposes, typical values of ρ in ohm-centimeters for conductors, semiconductors, and insulators are provided in Table 3.4.

TABLE 3.4
Comparing levels of ρ in Ω-cm.

Conductor (Ω-cm)	Semiconductor (Ω-cm)	Insulator (Ω-cm)
Copper 1.723×10^{-6}	Ge 50 Si 200×10^{3} GaAs 70×10^{6}	In general: 10^{15}

In particular, note the power-of-ten difference between conductors and insulators (10^{21})—a difference of huge proportions. There is a significant difference in levels of ρ for the list of semiconductors, but the power-of-ten difference between the conductor and insulator levels is at least 10^{6} for each of the semiconductors listed.

3.5 TEMPERATURE EFFECTS

Temperature has a significant effect on the resistance of conductors, semiconductors, and insulators.

Conductors

Conductors have a generous number of free electrons, and any introduction of thermal energy will have little impact on the total number of free

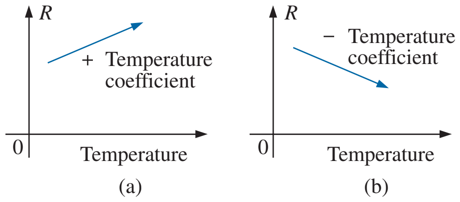


FIG. 3.12
Demonstrating the effect of a positive and a negative temperature coefficient on the resistance of a conductor.

carriers. In fact, the thermal energy only increases the intensity of the random motion of the particles within the material and makes it increasingly difficult for a general drift of electrons in any one direction to be established. The result is that

for good conductors, an increase in temperature results in an increase in the resistance level. Consequently, conductors have a positive temperature coefficient.

The plot in Fig. 3.12(a) has a positive temperature coefficient.

Semiconductors

In semiconductors, an increase in temperature imparts a measure of thermal energy to the system that results in an increase in the number of free carriers in the material for conduction. The result is that

for semiconductor materials, an increase in temperature results in a decrease in the resistance level. Consequently, semiconductors have negative temperature coefficients.

The thermistor and photoconductive cell discussed in Sections 3.11 and 3.12 are excellent examples of semiconductor devices with negative temperature coefficients. The plot in Fig. 3.12(b) has a negative temperature coefficient.

Insulators

As with semiconductors, an increase in temperature results in a decrease in the resistance of an insulator. The result is a negative temperature coefficient.

Inferred Absolute Temperature

Fig. 3.13 reveals that for copper (and most other metallic conductors), the resistance increases almost linearly (in a straight-line relationship) with an increase in temperature. Since temperature can have such a pronounced effect on the resistance of a conductor, it is important that we have some method of determining the resistance at any temperature within operating limits. An equation for this purpose can be obtained by approximating the curve in Fig. 3.13 by the straight dashed line that intersects the temperature scale at −234.5°C. Although the actual curve extends to **absolute zero** (−273.15°C, or 0 K), the straight-line approximation is quite accurate for

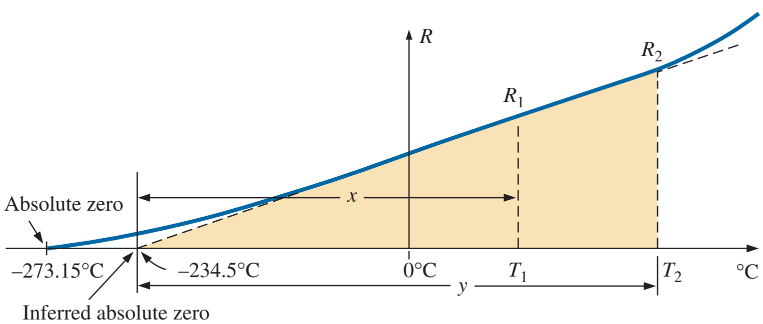


FIG. 3.13
Effect of temperature on the resistance of copper.

the normal operating temperature range. At two different temperatures, T_1 and T_2, the resistance of copper is R_1 and R_2, as indicated on the curve. Using a property of similar triangles, we may develop a mathematical relationship between these values of resistances at different temperatures. Let x equal the distance from −234.5°C to T_1 and y the distance from −234.5°C to T_2, as shown in Fig. 3.13. From similar triangles,

$$\frac{x}{R_1} = \frac{y}{R_2}$$

or

$$\boxed{\frac{234.5 + T_1}{R_1} = \frac{234.5 + T_2}{R_2}} \qquad \textbf{(3.7)}$$

The temperature of −234.5°C is called the **inferred absolute temperature** of copper. For different conducting materials, the intersection of the straight-line approximation occurs at different temperatures. A few typical values are listed in Table 3.5.

The minus sign does not appear with the inferred absolute temperature on either side of Eq. (3.7) because x and y are the *distances* from −234.5°C to T_1 and T_2, respectively, and therefore are simply magnitudes. For T_1 and T_2 less than zero, x and y are less than −234.5°C, and the distances are the differences between the inferred absolute temperature and the temperature of interest.

Eq. (3.7) can easily be adapted to any material by inserting the proper inferred absolute temperature. It may therefore be written as follows:

$$\boxed{\frac{|T_1| + T_1}{R_1} = \frac{|T_1| + T_2}{R_2}} \qquad \textbf{(3.8)}$$

where $|T_1|$ indicates that the inferred absolute temperature of the material involved is inserted as a positive value in the equation. In general, therefore, associate the sign only with T_1 and T_2.

TABLE 3.5

Inferred absolute temperatures (T_i).

Material	°C
Silver	−243
Copper	**−234.5**
Gold	−274
Aluminum	−236
Tungsten	−204
Nickel	−147
Iron	−162
Nichrome	−2,250
Constantan	−125,000

EXAMPLE 3.9 If the resistance of a copper wire is 50 Ω at 20°C, what is its resistance at 100°C (boiling point of water)?

Solution: Eq. (3.7):

$$\frac{234.5°\text{C} + 20°\text{C}}{50\ \Omega} = \frac{234.5°\text{C} + 100°\text{C}}{R_2}$$

$$R_2 = \frac{(50\ \Omega)(334.5°\text{C})}{254.5°\text{C}} = \mathbf{65.72\ \Omega}$$

EXAMPLE 3.10 If the resistance of a copper wire at freezing (0°C) is 30 Ω, what is its resistance at −40°C?

Solution: Eq. (3.7):

$$\frac{234.5°\text{C} + 0}{30\ \Omega} = \frac{234.5°\text{C} - 40°\text{C}}{R_2}$$

$$R_2 = \frac{(30\ \Omega)(194.5°\text{C})}{234.5°\text{C}} = \mathbf{24.88\ \Omega}$$

EXAMPLE 3.11 If the resistance of an aluminum wire at room temperature (20°C) is 100 mΩ (measured by a milliohmmeter), at what temperature will its resistance increase to 120 mΩ?

Solution: Eq. (3.7):

$$\frac{236°\text{C} + 20°\text{C}}{100\ \text{m}\Omega} = \frac{236°\text{C} + T_2}{120\ \text{m}\Omega}$$

and

$$T_2 = 120\ \text{m}\Omega\left(\frac{256°\text{C}}{100\ \text{m}\Omega}\right) - 236°\text{C}$$

$$T_2 = \mathbf{71.2°C}$$

Temperature Coefficient of Resistance

There is a second popular equation for calculating the resistance of a conductor at different temperatures. Defining

$$\boxed{\alpha_{20} = \frac{1}{|T_1| + 20°\text{C}}} \quad (\Omega/°\text{C}/\Omega) \qquad \textbf{(3.9)}$$

as the **temperature coefficient of resistance** at a temperature of 20°C, and R_{20} as the resistance of the sample at 20°C, the resistance R_1 at a temperature T_1 is determined by

$$\boxed{R_1 = R_{20}[1 + \alpha_{20}(T_1 - 20°\text{C})]} \qquad \textbf{(3.10)}$$

The values of α_{20} for different materials have been evaluated, and a few are listed in Table 3.6.

TABLE 3.6

Temperature coefficient of resistance for various conductors at 20°C.

Material	Temperature Coefficient (α_{20})
Silver	0.0038
Copper	**0.00393**
Gold	0.0034
Aluminum	0.00391
Tungsten	0.005
Nickel	0.006
Iron	0.0055
Constantan	0.000008
Nichrome	0.00044

Eq. (3.10) can be written in the following form:

$$\alpha_{20} = \frac{\left(\dfrac{R_1 - R_{20}}{T_1 - 20°\text{C}}\right)}{R_{20}} = \frac{\dfrac{\Delta R}{\Delta T}}{R_{20}}$$

from which the units of Ω/°C/Ω for α_{20} are defined.

Since $\Delta R/\Delta T$ is the slope of the curve in Fig. 3.13, we can conclude that

the higher the temperature coefficient of resistance for a material, the more sensitive the resistance level to changes in temperature.

Referring to Table 3.5, we find that copper is more sensitive to temperature variations than is silver, gold, or aluminum, although the differences are quite small. The slope defined by α_{20} for constantan is so small that the curve is almost horizontal.

Since R_{20} of Eq. (3.10) is the resistance of the conductor at 20°C and $T_1 - 20°\text{C}$ is the change in temperature from 20°C, Eq. (3.10) can be written in the following form:

$$\boxed{R = \rho\frac{l}{A}[1 + \alpha_{20}\,\Delta T]} \qquad \textbf{(3.11)}$$

providing an equation for resistance in terms of all the controlling parameters.

PPM/°C

For resistors, as for conductors, resistance changes with a change in temperature. The specification is normally provided in parts per million per degree Celsius (**PPM/°C**), providing an immediate indication of the sensitivity level of the resistor to temperature. For resistors, a 5000 PPM level is considered high, whereas 20 PPM is quite low. A 1000 PPM/°C characteristic reveals that a 1° change in temperature results in a change in resistance equal to 1000 PPM, or 1000/1,000,000 = 1/1000 of its nameplate value—not a significant change for most applications. However, a 10° change results in a change equal to 1/100 (1%) of its nameplate value, which is becoming significant. The concern, therefore, lies not only with the PPM level but with the range of expected temperature variation.

In equation form, the change in resistance is given by

$$\Delta R = \frac{R_{\text{nominal}}}{10^6}(\text{PPM})(\Delta T) \tag{3.12}$$

where R_{nominal} is the nameplate value of the resistor at room temperature and ΔT is the change in temperature from the reference level of 20°C.

EXAMPLE 3.12 For a 1 kΩ carbon composition resistor with a PPM of 2500, determine the resistance at 60°C.

Solution:

$$\Delta R = \frac{1000\ \Omega}{10^6}(2500)(60°\text{C} - 20°\text{C})$$
$$= 100\ \Omega$$

and

$$R = R_{\text{nominal}} + \Delta R = 1000\ \Omega + 100\ \Omega$$
$$= \mathbf{1100\ \Omega}$$

3.6 SUPERCONDUCTORS

The field of electricity/electronics is one of the most exciting of our time. New developments appear almost weekly from extensive research and development activities. The research drive to develop a room-temperature **superconductor** is generating even more excitement. This advancement rivals the introduction of semiconductor devices such as the transistor (to replace tubes), wireless communication, or the electric light.

What are superconductors? Why is their development so important? In a nutshell,

superconductors are conductors of electric charge that, for all practical purposes, have zero resistance.

In a conventional conductor, electrons travel at average speeds of about 1000 mi/s (they can cross the United States in about 3 seconds), even though Einstein's theory of relativity suggests that the maximum speed of information transmission is the speed of light, or 186,000 mi/s. The relatively slow speed of conventional conduction is due to collisions with atoms in the material, repulsive forces between electrons (like charges repel), thermal agitation that results in indirect paths due to the increased motion of the neighboring atoms, impurities in the conductor, and so on. In the superconductive state, there is a pairing of electrons, denoted by the **Cooper effect,** in which electrons travel in pairs and help each other maintain a significantly higher velocity through the medium. In some ways this is like "drafting" by competitive cyclists or runners. There is an oscillation of energy between partners or even "new" partners (as the need arises) to ensure passage through the conductor at the highest possible velocity with the least total expenditure of energy.

Even though the concept of superconductivity first surfaced in 1911, it was not until 1986 that the possibility of superconductivity at room temperature became a renewed goal of the research community. For over 70 years, superconductivity could be established only at temperatures colder than 23 K. (Kelvin temperature is universally accepted as the unit of measurement for temperature for superconductive effects. Recall that K = 273.15° + °C, so a temperature of 23 K is −250°C, or −418°F.) In 1986, however, physicists Alex Muller and George Bednorz of the IBM Zurich Research Center found a ceramic material—lanthanum barium copper oxide—that exhibited superconductivity at 30 K. This discovery introduced a new direction to the research effort and spurred others to improve on the new standard. (In 1987, both scientists received the Nobel prize for their contribution to an important area of development.)

In just a few short months, Professors Paul Chu of the University of Houston and Man Kven Wu of the University of Alabama raised the temperature to 95 K using a superconductor of yttrium barium copper oxide. The result was a level of excitement in the scientific community that brought research in the area to a new level of effort and investment. The major impact of this discovery was that liquid nitrogen (boiling point of 77 K) rather than liquid helium, (boiling point of 4 K) could now be used to bring the material down to the required temperature. The result is a tremendous saving in the cooling expense since liquid nitrogen is at least ten times less expensive than liquid helium. Pursuing the same direction, some success has been achieved at 125 K and 162 K using a thallium compound (unfortunately, however, thallium is a very poisonous substance).

Fig. 3.14 illustrates that the discovery in 1986 of using a ceramic material in superconductors led to rapid developments in the field. Un-

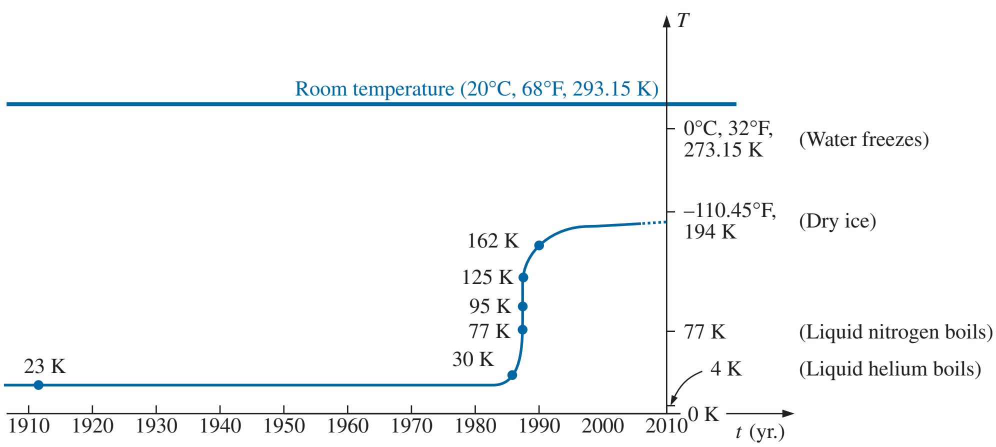


FIG. 3.14
Rising temperatures of superconductors.

fortunately, however, the pace has slowed in recent years. The effort continues and is receiving an increasing level of financing and worldwide attention. Now, increasing numbers of corporations are trying to capitalize on the success already attained, as will be discussed later in this section.

The fact that ceramics provided the recent breakthrough in superconductivity is probably a surprise when you consider that they are also an important class of insulators. However, the ceramics that exhibit the characteristics of superconductivity are compounds that include copper, oxygen, and rare earth elements such as yttrium, lanthanum, and thallium. There are also indicators that the current compounds may be limited to a maximum temperature of 200 K (about 100 K short of room temperature), leaving the door wide open to innovative approaches to compound selection. The temperature at which a superconductor reverts back to the characteristics of a conventional conductor is called the *critical temperature,* denoted by T_c. Note in Fig. 3.15 that the resistivity level changes abruptly at T_c. The sharpness of the transition region is a function of the purity of the sample. Long listings of critical temperatures for a variety of tested compounds can be found in reference materials providing tables of a wide variety to support research in physics, chemistry, geology, and related fields. Two such publications include the CRC (The Chemical Rubber Co.) *Handbook of Tables for Applied Engineering Science* and the CRC *Handbook of Chemistry and Physics.*

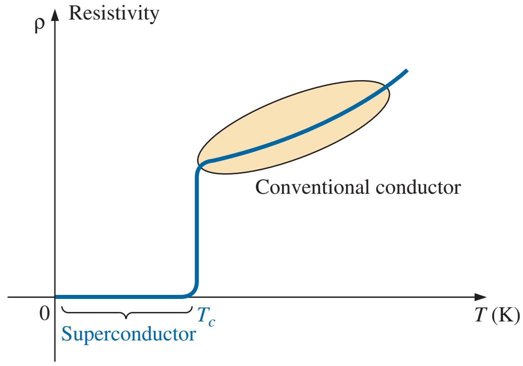


FIG. 3.15
Defining the critical temperature T_c.

Even though ceramic compounds have established higher transition temperatures, there is concern about their brittleness and current density limitations. In the area of integrated circuit manufacturing, current density levels must equal or exceed 1 MA/cm^2, or 1 million amperes through a cross-sectional area about one-half the size of a dime. IBM has attained a level of 4 MA/cm^2 at 77 K, permitting the use of superconductors in the design of some new-generation, high-speed computers.

Although room-temperature success has not been attained, numerous applications for some of the superconductors have been developed. It is simply a matter of balancing the additional cost against the results obtained or deciding whether any results at all can be obtained without the use of this zero-resistance state. Some research efforts require high-energy accelerators or strong magnets attainable only with superconductive materials. Superconductivity is currently applied in the design of Maglev trains (trains that ride on a cushion of air established by opposite magnetic poles) that exceed 300 mi/h, in powerful motors and generators, in nuclear magnetic resonance imaging systems to obtain cross-sectional images of the brain (and other parts of the body), in the design of computers with operating speeds four times that of conventional systems, and in improved power distribution systems.

The range of future uses for superconductors is a function of how much success physicists have in raising the operating temperature and how well they can utilize the successes obtained thus far. However, it may be only a matter of time before magnetically levitated trains increase in number, improved medical diagnostic equipment is available, computers operate at much higher speeds, high-efficiency power and storage systems are available, and transmission systems operate at very high efficiency levels due to this area of developing interest. Only time will reveal the impact that this new direction will have on the quality of life.

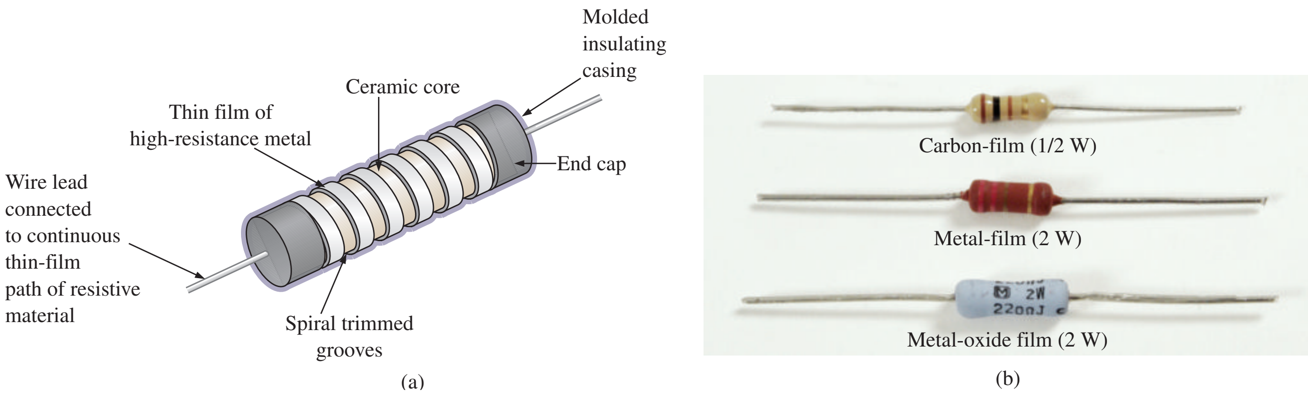


FIG. 3.16
Film resistors: (a) construction; (b) types.

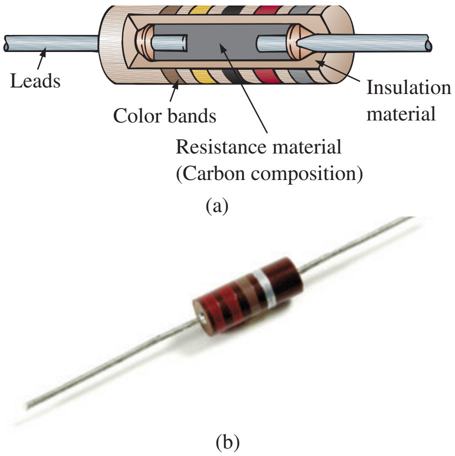


FIG. 3.17
Fixed composition resistors: (a) construction; (b) appearance.

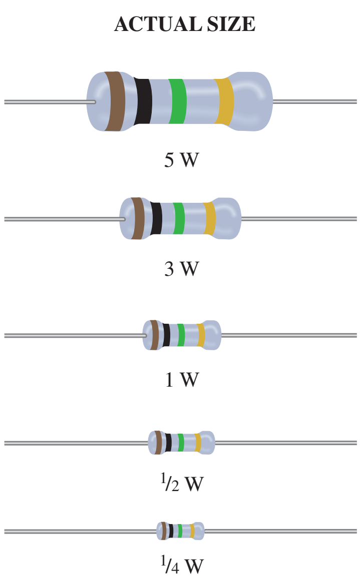


FIG. 3.18
Fixed metal-oxide resistors of different wattage ratings.

3.7 TYPES OF RESISTORS

Fixed Resistors

Resistors are made in many forms, but all belong in either of two groups: fixed or variable. The most common of the low-wattage, fixed-type resistors is the film resistor shown in Fig. 3.16. It is constructed by depositing a thin layer of resistive material (typically carbon, metal, or metal oxide) on a ceramic rod. The desired resistance is then obtained by cutting away some of the resistive material in a helical manner to establish a long, continuous band of high-resistance material from one end of the resistor to the other. In general, carbon-film resistors have a beige body and a lower wattage rating. The metal-film resistor is typically a stronger color, such as brick red or dark green, with higher wattage ratings. The metal-oxide resistor is usually a softer pastel color, such as rating powder blue shown in Fig. 3.16(b), and has the highest wattage rating of the three.

When you search through most electronics catalogs or visit a local electronics dealer such as Radio Shack to purchase resistors, you will find that the most common resistor is the film resistor. In years past, the carbon composition resistor in Fig. 3.17 was the most common, but fewer and fewer companies are manufacturing this variety, with its range of applications reduced to applications in which very high temperatures and inductive effects (Chapter 11) can be a problem. Its resistance is determined by the carbon composition material molded directly to each end of the resistor. The high resistivity characteristics of carbon ($\rho = 21{,}000$ CM-Ω/ft) provide a high-resistance path for the current through the element.

For a particular style and manufacturer, the size of a resistor increases with the power or wattage rating.

The concept of power is covered in detail in Chapter 4, but for the moment recognize that increased power ratings are normally associated with the ability to handle higher current and temperature levels. Fig. 3.18 depicts the actual size of thin-film, metal-oxide resistors in the 1/4 W to 5 W rating range. All the resistors in Fig. 3.18 are 1 MΩ, revealing that

the size of a resistor does not define its resistance level.

A variety of other fixed resistors are depicted in Fig. 3.19. The wire-wound resistors of Fig. 3.19(a) are formed by winding a high-resistance wire around a ceramic core. The entire structure is then baked in a ceramic cement to provide a protective covering. Wire-wound resistors are

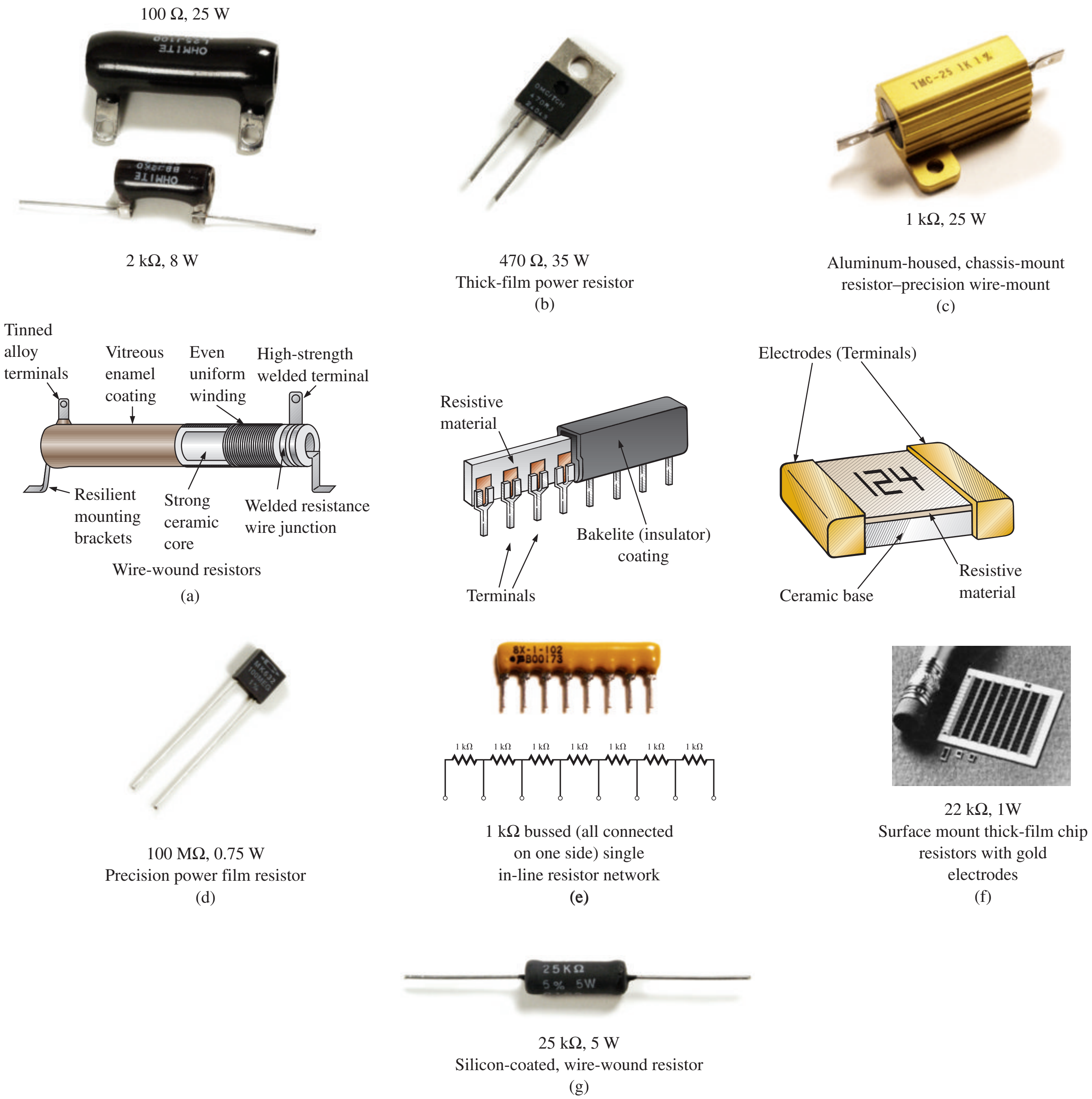


FIG. 3.19
Various types of fixed resistors.

typically used for larger power applications, although they are also available with very small wattage ratings and very high accuracy.

Fig. 3.19(c) and (g) are special types of wire-wound resistors with a low percent tolerance. Note, in particular, the high power ratings for the wire-wound resistors for their relatively small size. Figs. 3.19(b), (d), and (f) are power film resistors that use a thicker layer of film material than used in the variety shown in Fig. 3.16. The chip resistors in Fig. 3.19(f) are used where space is a priority, such as on the surface of circuit board. Units of this type can be less than 1/16 in. in length or width, with thickness as small as 1/30 in., yet they can still handle 0.5 W of power with resistance levels as high as 1000 MΩ—clear evidence that size does not determine the resistance level. The fixed resistor in Fig. 3.19(e) has terminals applied to a layer of resistor material, with the resistance between the terminals a function of the dimensions of the resistive material and the placement of the terminal pads.

Variable Resistors

Variable resistors, as the name implies, have a terminal resistance that can be varied by turning a dial, knob, screw, or whatever seems appropriate for the application. They can have two or three terminals, but most have three terminals. If the two- or three-terminal device is used as a variable resistor, it is usually referred to as a **rheostat.** If the three-terminal device is used for controlling potential levels, it is then commonly called a **potentiometer.** Even though a three-terminal device can be used as a rheostat or a potentiometer (depending on how it is connected), it is typically called a *potentiometer* when listed in trade magazines or requested for a particular application.

The symbol for a three-terminal potentiometer appears in Fig. 3.20(a). When used as a variable resistor (or rheostat), it can be hooked up in one of two ways, as shown in Figs. 3.20(b) and (c). In Fig. 3.20(b), points *a* and *b* are hooked up to the circuit, and the remaining terminal is left hanging. The resistance introduced is determined by that portion of the resistive element between points *a* and *b*. In Fig. 3.20(c), the resistance is again between points *a* and *b*, but now the remaining resistance is "shorted-out" (effect removed) by the connection from *b* to *c*. The universally accepted symbol for a rheostat appears in Fig. 3.20(d).

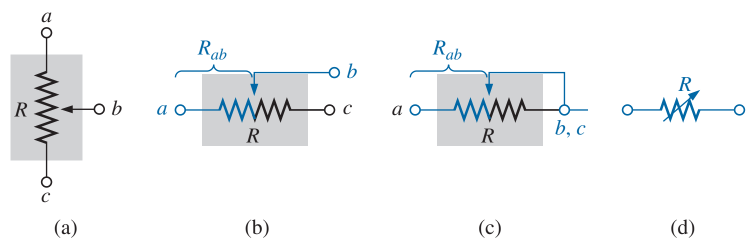


FIG. 3.20
Potentiometer: (a) symbol; (b) and (c) rheostat connections; (d) rheostat symbol.

Most potentiometers have three terminals in the relative positions shown in Fig. 3.21. The knob, dial, or screw in the center of the housing controls the motion of a contact that can move along the resistive element connected between the outer two terminals. The contact is connected to the center terminal, establishing a resistance from movable contact to each outer terminal.

The resistance between the outside terminals a and c in Fig. 3.22(a) (and Fig. 3.21) is always fixed at the full rated value of the potentiometer, regardless of the position of the wiper arm b.

In other words, the resistance between terminals *a* and *c* in Fig. 3.22(a) for a 1 MΩ potentiometer will always be 1 MΩ, no matter how we turn the control element and move the contact. In Fig. 3.22(a), the center contact is not part of the network configuration.

The resistance between the wiper arm and either outside terminal can be varied from a minimum of 0 Ω to a maximum value equal to the full rated value of the potentiometer.

In Fig. 3.22(b), the wiper arm has been placed 1/4 of the way down from point *a* to point *c*. The resulting resistance between points *a* and *b* will

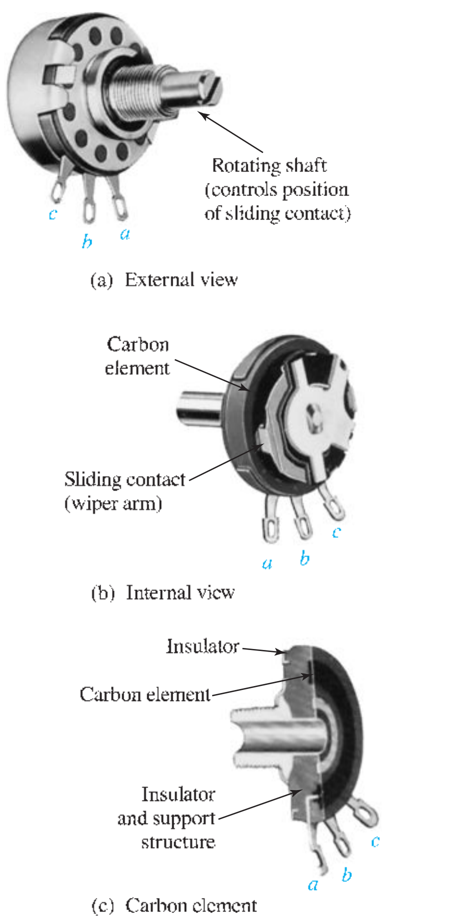
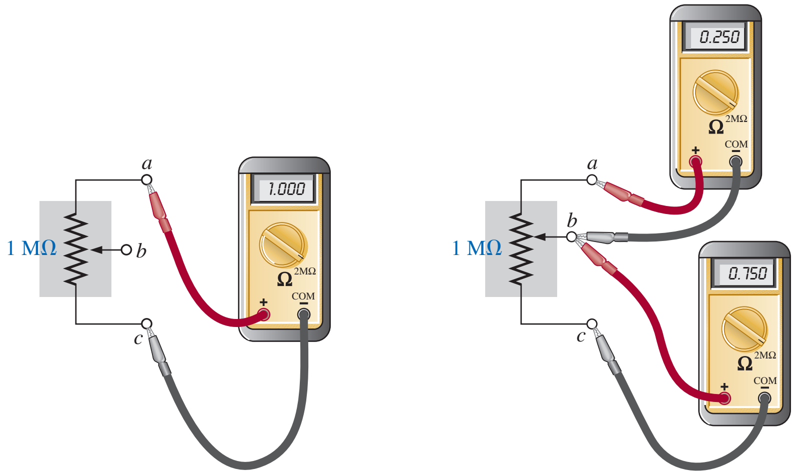


FIG. 3.21
Molded composition-type potentiometer.
(Courtesy of Allen-Bradley Co.)



(a) (b)

FIG. 3.22
Resistance components of a potentiometer: (a) between outside terminals; (b) between wiper arm and each outside terminal.

therefore be 1/4 of the total, or 250 kΩ (for a 1 MΩ potentiometer), and the resistance between *b* and *c* will be 3/4 of the total, or 750 kΩ.

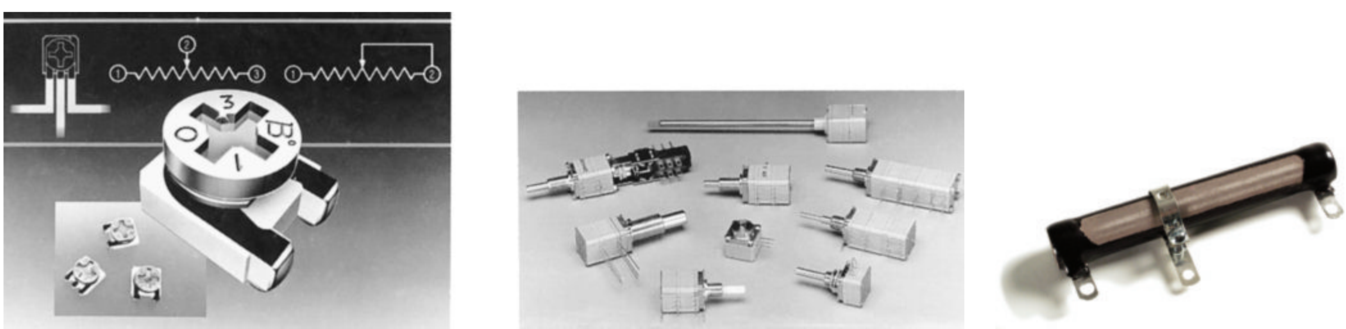
The sum of the resistances between the wiper arm and each outside terminal equals the full rated resistance of the potentiometer.

This was demonstrated in Fig. 3.22(b), where 250 kΩ + 750 kΩ = 1 MΩ. Specifically,

$$R_{ac} = R_{ab} + R_{bc} \tag{3.13}$$

Therefore, as the resistance from the wiper arm to one outside contact increases, the resistance between the wiper arm and the other outside terminal must decrease accordingly. For example, if R_{ab} of a 1 kΩ potentiometer is 200 Ω, then the resistance R_{bc} must be 800 Ω. If R_{ab} is further decreased to 50 Ω, then R_{bc} must increase to 950 Ω, and so on.

The molded carbon composition potentiometer is typically applied in networks with smaller power demands, and it ranges in size from 20 Ω to 22 MΩ (maximum values). A miniature trimmer (less than 1/4 in. in diameter) appears in Fig. 3.23(a), and a variety of potentiometers that use a cermet resistive material appear in Fig. 3.23(b). The contact point of the



(a) (b) (c)

FIG. 3.23
Variable resistors: (a) 4 mm (≈5/32″) trimmer (courtesy of Bourns, Inc.); (b) conductive plastic and cermet elements (courtesy of Honeywell Clarostat); (c) three-point wire-wound resistor.

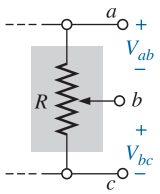


FIG. 3.24
Potentiometer control of voltage levels.

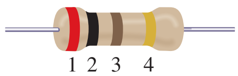


FIG. 3.25
Color coding for fixed resistors.

Number	Color
0	Black
1	Brown
2	Red
3	Orange
4	Yellow
5	Green
6	Blue
7	Violet
8	Gray
9	White
±5% (0.1 multiplier if 3rd band)	Gold
±10% (0.01 multiplier if 3rd band)	Silver

FIG. 3.26
Color coding.

FIG. 3.27
Example 3.13.

three-point wire-wound resistor in Fig. 3.23(c) can be moved to set the resistance between the three terminals.

When the device is used as a potentiometer, the connections are as shown in Fig. 3.24. It can be used to control the level of V_{ab}, V_{bc}, or both, depending on the application. Additional discussion of the potentiometer in a loaded situation can be found in later chapters.

3.8 COLOR CODING AND STANDARD RESISTOR VALUES

A wide variety of resistors, fixed or variable, are large enough to have their resistance in ohms printed on the casing. Some, however, are too small to have numbers printed on them, so a system of **color coding** is used. For the thin-film resistor, four, five, or six bands may be used. The four-band scheme is described. Later in this section the purpose of the fifth and sixth bands will be described.

For the four-band scheme, the bands are *always read from the end that has a band closest to it,* as shown in Fig. 3.25. The bands are numbered as shown for reference in the discussion to follow.

The first two bands represent the first and second digits, respectively.

They are the actual first two numbers that define the numerical value of the resistor.

The third band determines the power-of-ten multiplier for the first two digits (actually the number of zeros that follow the second digit for resistors greater than 10 Ω).

The fourth band is the manufacturer's tolerance, which is an indication of the precision by which the resistor was made.

If the fourth band is omitted, the tolerance is assumed to be ±20%.

The number corresponding to each color is defined in Fig. 3.26. The fourth band will be either ±5% or ±10% as defined by gold and silver, respectively. To remember which color goes with which percent, simply remember that ±5% resistors cost more and gold is more valuable than silver.

Remembering which color goes with each digit takes a bit of practice. In general, the colors start with the very dark shades and move toward the lighter shades. The best way to memorize is to simply repeat over and over that red is 2, yellow is 4, and so on. Simply practice with a friend or a fellow student, and you will learn most of the colors in short order.

EXAMPLE 3.13 Find the value of the resistor in Fig. 3.27.

Solution: Reading from the band closest to the left edge, we find that the first two colors of brown and red represent the numbers 1 and 2, respectively. The third band is orange, representing the number 3 for the power of the multiplier as follows:

$$12 \times 10^3 \ \Omega$$

resulting in a value of 12 kΩ. As indicated above, if 12 kΩ is written as 12,000 Ω, the third band reveals the number of zeros that follow the first two digits.

Now for the fourth band of gold, representing a tolerance of ±5%: To find the range into which the manufacturer has guaranteed the resistor will fall, first convert the 5% to a decimal number by moving the decimal point two places to the left:

$$5\% \Rightarrow 0.05$$

Then multiply the resistor value by this decimal number:

$$0.05(12\text{ k}\Omega) = 600\ \Omega$$

Finally, add the resulting number to the resistor value to determine the maximum value, and subtract the number to find the minimum value. That is,

$$\text{Maximum} = 12{,}000\ \Omega + 600\ \Omega = 12.6\text{ k}\Omega$$
$$\text{Minimum} = 12{,}000\ \Omega - 600\ \Omega = 11.4\text{ k}\Omega$$
$$\text{Range} = \mathbf{11.4\text{ k}\Omega\text{ to }12.6\text{ k}\Omega}$$

The result is that the manufacturer has guaranteed with the 5% gold band that the resistor will fall in the range just determined. In other words, the manufacturer does not guarantee that the resistor will be exactly 12 kΩ but rather that it will fall in a range as defined above.

Using the above procedure, the smallest resistor that can be labeled with the color code is 10 Ω. However,

the range can be extended to include resistors from 0.1 Ω to 10 Ω by simply using gold as a multiplier color (third band) to represent 0.1 and using silver to represent 0.01.

This is demonstrated in the next example.

EXAMPLE 3.14 Find the value of the resistor in Fig. 3.28.

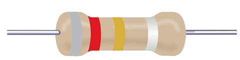


FIG. 3.28
Example 3.14.

Solution: The first two colors are gray and red, representing the numbers 8 and 2. The third color is gold, representing a multiplier of 0.1. Using the multiplier, we obtain a resistance of

$$(0.1)(82\ \Omega) = 8.2\ \Omega$$

The fourth band is silver, representing a tolerance of ±10%. Converting to a decimal number and multiplying through yields

$$10\% = 0.10 \quad\text{and}\quad (0.1)(8.2\ \Omega) = 0.82\ \Omega$$

$$\text{Maximum} = 8.2\ \Omega + 0.82\ \Omega = 9.02\ \Omega$$
$$\text{Minimum} = 8.2\ \Omega - 0.82\ \Omega = 7.38\ \Omega$$

so that $$\text{Range} = \mathbf{7.38\ \Omega\text{ to }9.02\ \Omega}$$

Although it will take some time to learn the numbers associated with each color, it is certainly encouraging to become aware that

the same color scheme to represent numbers is used for all the important elements of electrical circuits.

Later on, you will find that the numerical value associated with each color is the same for capacitors and inductors. Therefore, once learned, the scheme has repeated areas of application.

Some manufacturers prefer to use a **five-band color code.** In such cases, as shown in the top portion of Fig. 3.29, three digits are provided before the multiplier. The fifth band remains the tolerance indicator. If the manufacturer decides to include the temperature coefficient, a sixth band will appear as shown in the lower portion of Fig. 3.29, with the color indicating the PPM level.

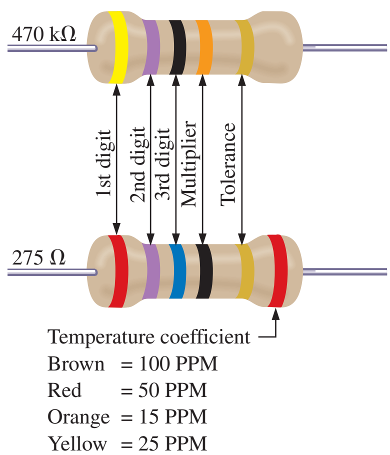


FIG. 3.29
Five-band color coding for fixed resistors.

For four, five, or six bands, if the tolerance is less than 5%, the following colors are used to reflect the % tolerances:

brown* = ±*1%, red* = ±*2%, green* = ±*0.5%, blue* = ±*0.25%, and violet* = ±*0.1%.

You might expect that resistors would be available for a full range of values such as 10 Ω, 20 Ω, 30 Ω, 40 Ω, 50 Ω, and so on. However, this is not the case with some typical commercial values, such as 27 Ω, 56 Ω, and 68 Ω. There is a reason for the chosen values, which is best demonstrated by examining the list of standard values of commercially available resistors in Table 3.7. The values in boldface blue are available with 5%, 10%, and 20% tolerances, making them the most common of the commercial variety. The values in boldface black are typically available with 5% and 10% tolerances, and those in normal print are available only in the 5% variety.

TABLE 3.7
Standard values of commercially available resistors.

Ohms (Ω)					Kilohms (kΩ)		Megohms (MΩ)	
0.10	**1.0**	**10**	**100**	**1000**	**10**	**100**	**1.0**	**10.0**
0.11	1.1	11	110	1100	11	110	1.1	11.0
0.12	**1.2**	**12**	**120**	**1200**	**12**	**120**	**1.2**	**12.0**
0.13	1.3	13	130	1300	13	130	1.3	13.0
0.15	**1.5**	**15**	**150**	**1500**	**15**	**150**	**1.5**	**15.0**
0.16	1.6	16	160	1600	16	160	1.6	16.0
0.18	**1.8**	**18**	**180**	**1800**	**18**	**180**	**1.8**	**18.0**
0.20	2.0	20	200	2000	20	200	2.0	20.0
0.22	**2.2**	**22**	**220**	**2200**	**22**	**220**	**2.2**	**22.0**
0.24	2.4	24	240	2400	24	240	2.4	
0.27	**2.7**	**27**	**270**	**2700**	**27**	**270**	**2.7**	
0.30	3.0	30	300	3000	30	300	3.0	
0.33	**3.3**	**33**	**330**	**3300**	**33**	**330**	**3.3**	
0.36	3.6	36	360	3600	36	360	3.6	
0.39	**3.9**	**39**	**390**	**3900**	**39**	**390**	**3.9**	
0.43	4.3	43	430	4300	43	430	4.3	
0.47	**4.7**	**47**	**470**	**4700**	**47**	**470**	**4.7**	
0.51	5.1	51	510	5100	51	510	5.1	
0.56	**5.6**	**56**	**560**	**5600**	**56**	**560**	**5.6**	
0.62	6.2	62	620	6200	62	620	6.2	
0.68	**6.8**	**68**	**680**	**6800**	**68**	**680**	**6.8**	
0.75	7.5	75	750	7500	75	750	7.5	
0.82	**8.2**	**82**	**820**	**8200**	**82**	**820**	**8.2**	
0.91	9.1	91	910	9100	91	910	9.1	

Examining the impact of the tolerance level will help explain the choice of numbers for the commercial values. Take the sequence 47 Ω–68 Ω–100 Ω, which are all available with 20% tolerances. In Fig. 3.30(a), the tolerance band for each has been determined and plotted on a single axis. Note that with this tolerance (which is all that the manufacturer will guarantee), the full range of resistor values is available from 37.6 Ω to 120 Ω. In other words, the manufacturer is guaranteeing

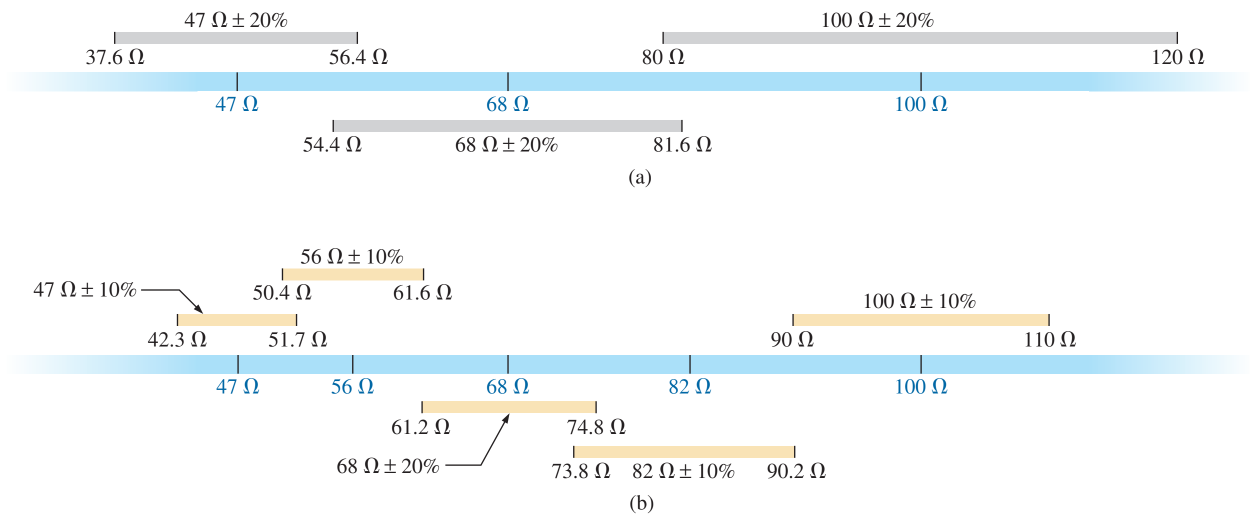


FIG. 3.30

Guaranteeing the full range of resistor values for the given tolerance: (a) 20%; (b) 10%.

the full range, using the tolerances to fill in the gaps. Dropping to the 10% level introduces the 56 Ω and 82 Ω resistors to fill in the gaps, as shown in Fig. 3.30(b). Dropping to the 5% level would require additional resistor values to fill in the gaps. In total, therefore, the resistor values were chosen to ensure that the full range was covered, as determined by the tolerances employed. Of course, if a specific value is desired but is not one of the standard values, combinations of standard values often result in a total resistance very close to the desired level. If this approach is still not satisfactory, a potentiometer can be set to the exact value and then inserted in the network.

Throughout the text, you will find that many of the resistor values are not standard values. This was done to reduce the mathematical complexity, which might make it more difficult to understand the procedure or analysis technique being introduced. In the problem sections, however, standard values are frequently used to ensure that you start to become familiar with the commercial values available.

Surface Mount Resistors

In general, surface mount resistors are marked in three ways: color coding, three symbols, and two symbols.

The **color coding** is the same as just described earlier in this section for through-hole resistors.

The **three-symbol** approach uses three digits. The first two define the first two digits of the value; the last digit, the power of the power-of-ten multiplier.

For instance:

$$820 \text{ is } 82 \times 10^0\ \Omega = \mathbf{82\ \Omega}$$
$$222 \text{ is } 22 \times 10^2\ \Omega = 2200\ \Omega = \mathbf{2.2\ k\Omega}$$
$$010 \text{ is } 1 \times 10^0\ \Omega = \mathbf{1\ \Omega}$$

The **two-symbol** marking uses a letter followed by a number. The letter defines the value as listed below. Note that all the numbers of the commercially available list of Table 3.7 are included.

A = 1.0	**B** = 1.1	**C** = 1.2	**D** = 1.3
E = 1.5	**F** = 1.6	**G** = 1.8	**H** = 2
J = 2.2	**K** = 2.4	**L** = 2.7	**M** = 3
N = 3.3	**P** = 3.6	**Q** = 3.9	**R** = 4.3
S = 4.7	**T** = 5.1	**U** = 5.6	**V** = 6.2
W = 6.8	**X** = 7.5	**Y** = 8.2	**Z** = 9.1

The second symbol is the power of the power-of-ten multiplier. For example:

$$\text{C3} = 1.2 \times 10^3\,\Omega = \mathbf{1.2\ k\Omega}$$
$$\text{T0} = 5.1 \times 10^0\,\Omega = \mathbf{5.1\ \Omega}$$
$$\text{Z1} = 9.1 \times 10^1\,\Omega = \mathbf{91\ \Omega}$$

Additional symbols may precede or follow the codes above and may differ depending on the manufacturer. These may provide information on the internal resistance structure, power rating, surface material, tapping, and tolerance.

3.9 CONDUCTANCE

By finding the reciprocal of the resistance of a material, we have a measure of how well the material conducts electricity. The quantity is called **conductance,** has the symbol *G,* and is measured in *siemens* (S) (note Fig. 3.31). In equation form, conductance is

$$\boxed{G = \frac{1}{R}} \qquad \text{(siemens, S)} \qquad \textbf{(3.14)}$$

A resistance of 1 MΩ is equivalent to a conductance of 10^{-6} S, and a resistance of 10 Ω is equivalent to a conductance of 10^{-1} S. The larger the conductance, therefore, the less the resistance and the greater the conductivity.

In equation form, the conductance is determined by

$$\boxed{G = \frac{A}{\rho l}} \qquad \text{(S)} \qquad \textbf{(3.15)}$$

indicating that increasing the area or decreasing either the length or the resistivity increases the conductance.

EXAMPLE 3.15

a. Determine the conductance of a 1 Ω, 50 kΩ, and 10 MΩ resistor.
b. How does the conductance level change with increase in resistance?

Solution: Eq. (3.14):

a. $1\Omega\text{: } G = \frac{1}{R} = \frac{1}{1\ \Omega} = \mathbf{1\ S}$

$$50\ \text{k}\Omega\text{: } G = \frac{1}{R} = \frac{1}{50\ \text{k}\Omega} = \frac{1}{50 \times 10^3\ \Omega} = 0.02 \times 10^{-3}\ \text{S} = \mathbf{0.02\ mS}$$

$$10\ \text{M}\Omega\text{: } G = \frac{1}{R} = \frac{1}{10\ \text{M}\Omega} = \frac{1}{10 \times 10^6\ \Omega} = 0.1 \times 10^{-6}\ \text{S} = \mathbf{0.1\ \mu S}$$

FIG. 3.31
Werner von Siemens.

German (Lenthe, Berlin)
(1816–92)
Electrical Engineer
Telegraph Manufacturer,
Siemens & Halske AG

Developed an *electroplating process* during a brief stay in prison for acting as a second in a duel between fellow officers of the Prussian army. Inspired by the electronic telegraph invented by Sir Charles Wheatstone in 1817, he improved on the design and proceeded to lay cable with the help of his brother Carl across the Mediterranean and from Europe to India. His inventions included the first *self-excited generator,* which depended on the *residual* magnetism of its electromagnet rather than an inefficient permanent magnet. In 1888 he was raised to the rank of nobility with the addition of *von* to his name. The current firm of Siemens AG has manufacturing outlets in some 35 countries with sales offices in some 125 countries.

b. The conductance level decreases rapidly with significant increase in resistance levels.

EXAMPLE 3.16 What is the relative increase or decrease in conductivity of a conductor if the area is reduced by 30% and the length is increased by 40%? The resistivity is fixed.

Solution: Eq. (3.15):

$$G_i = \frac{1}{R_i} = \frac{1}{\dfrac{\rho_i l_i}{A_i}} = \frac{A_i}{\rho_i l_i}$$

with the subscript i for the initial value. Using the subscript n for new value:

$$G_n = \frac{A_n}{\rho_n l_n} = \frac{0.70A_i}{\rho_i(1.4l_i)} = \frac{0.70}{1.4}\,\frac{A_i}{\rho_i l_i} = \frac{0.70G_i}{1.4}$$

and $G_n = \mathbf{0.5G_i}$

3.10 OHMMETERS

The **ohmmeter** is an instrument used to perform the following tasks and several other useful functions:

1. ***Measure the resistance of individual or combined elements.***
2. ***Detect open-circuit (high-resistance) and short-circuit (low-resistance) situations.***
3. ***Check the continuity of network connections and identify wires of a multilead cable.***
4. ***Test some semiconductor (electronic) devices.***

For most applications, the ohmmeters used most frequently are the ohmmeter section of a VOM or DMM. The details of the internal circuitry and the method of using the meter will be left primarily for a laboratory exercise. In general, however, the resistance of a resistor can be measured by simply connecting the two leads of the meter across the resistor, as shown in Fig. 3.32. There is no need to be concerned about which lead goes on which end; the result is the same in either case since resistors offer the same resistance to the flow of charge (current) in either direction. If the VOM is used, a switch must be set to the proper resistance range, and a nonlinear scale (usually the top scale of the meter) must be properly read to obtain the resistance value. The DMM also requires choosing the best scale setting for the resistance to be measured, but the result appears as a numerical display, with the proper placement of the decimal point determined by the chosen scale. When measuring the resistance of a single resistor, it is usually best to remove the resistor from the network before making the measurement. If this is difficult or impossible, at least one end of the resistor must not be connected to the network, or the reading may include the effects of the other elements of the system.

If the two leads of the meter are touching in the ohmmeter mode, the resulting resistance is zero. A connection can be checked as shown in Fig. 3.33 by simply hooking up the meter to either side of the connection. If the resistance is zero, the connection is secure. If it is other than zero, the connection could be weak; if it is infinite, there is no connection at all.

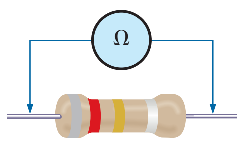


FIG. 3.32
Measuring the resistance of a single element.

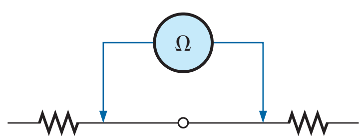


FIG. 3.33
Checking the continuity of a connection.

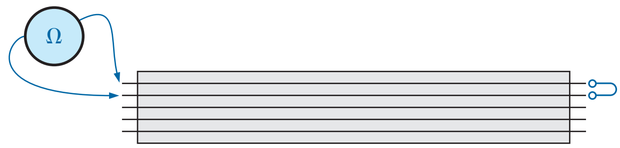


FIG. 3.34
Identifying the leads of a multilead cable.

If one wire of a harness is known, a second can be found as shown in Fig. 3.34. Simply connect the end of the known lead to the end of any other lead. When the ohmmeter indicates zero ohms (or very low resistance), the second lead has been identified. The above procedure can also be used to determine the first known lead by simply connecting the meter to any wire at one end and then touching all the leads at the other end until a zero ohm indication is obtained.

Preliminary measurements of the condition of some electronic devices such as the diode and the transistor can be made using the ohmmeter. The meter can also be used to identify the terminals of such devices.

One important note about the use of any ohmmeter:

Never hook up an ohmmeter to a live circuit!

The reading will be meaningless, and you may damage the instrument. The ohmmeter section of any meter is designed to pass a small sensing current through the resistance to be measured. A large external current could damage the movement and would certainly throw off the calibration of the instrument. In addition:

Never store a VOM or a DMM in the resistance mode.

If the two leads of the meter touch, the small sensing current could drain the internal battery. VOMs should be stored with the selector switch on the highest voltage range, and the selector switch of DMMs should be in the off position.

3.11 THERMISTORS

The **thermistor** is a two-terminal semiconductor device whose resistance, as the name suggests, is temperature sensitive. A representative characteristic appears in Fig. 3.35 with the graphic symbol for the device. Note the nonlinearity of the curve and the drop in resistance from about 5000 Ω to 100 Ω for an increase in temperature from 20°C to 100°C. The decrease in resistance with an increase in temperature indicates a negative temperature coefficient.

The temperature of the device can be changed internally or externally. An increase in current through the device raises its temperature, causing a drop in its terminal resistance. Any externally applied heat source results in an increase in its body temperature and a drop in resistance. This type of action (internal or external) lends itself well to control mechanisms. Many different types of thermistors are shown in Fig. 3.36. Materials used in the manufacture of thermistors include oxides of cobalt, nickel, strontium, and manganese.

Note the use of a log scale (to be discussed in Chapter 23) in Fig. 3.35 for the vertical axis. The log scale permits the display of a wider range of

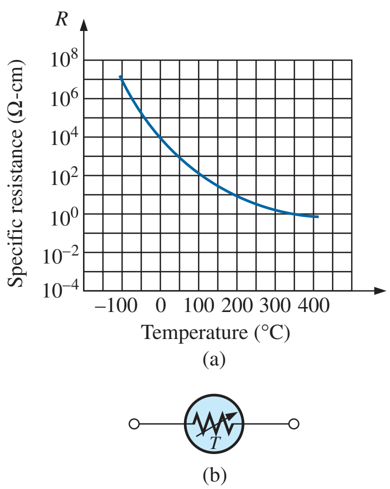


FIG. 3.35
Thermistor: (a) characteristics; (b) symbol.

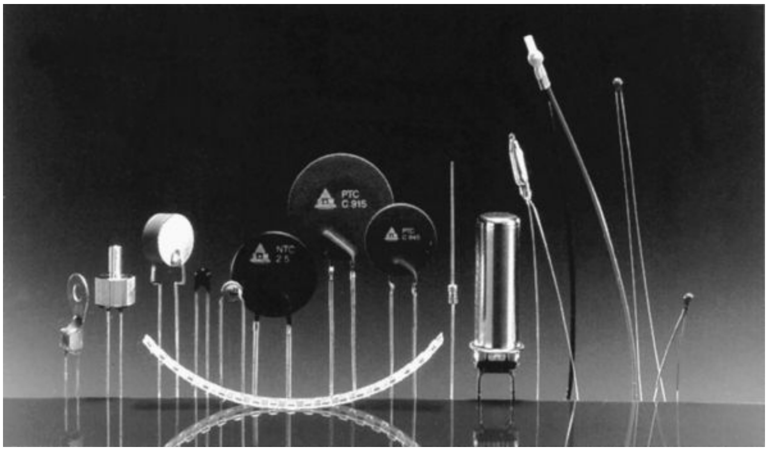


FIG. 3.36
NTC (negative temperature coefficient) and PTC (positive temperature coefficient) thermistors.
(Courtesy of Siemens Components, Inc.)

specific resistance levels than a linear scale such as the horizontal axis. Note that it extends from 0.0001 Ω-cm to 100,000,000 Ω-cm over a very short interval. The log scale is used for both the vertical and the horizontal axis in Fig. 3.37.

3.12 PHOTOCONDUCTIVE CELL

The **photoconductive cell** is a two-terminal semiconductor device whose terminal resistance is determined by the intensity of the incident light on its exposed surface. As the applied illumination increases in intensity, the energy state of the surface electrons and atoms increases, with a resultant increase in the number of "free carriers" and a corresponding drop in resistance. A typical set of characteristics and the photoconductive cell's graphic symbol appear in Fig. 3.37. Note the negative illumination coefficient. Several cadmium sulfide photoconductive cells appear in Fig. 3.38.

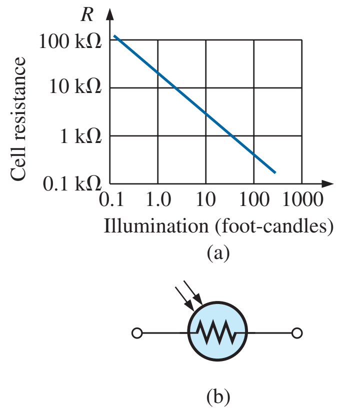


FIG. 3.37
Photoconductive cell: (a) characteristics. (b) symbol.

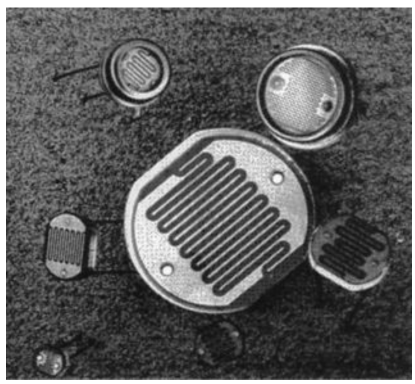
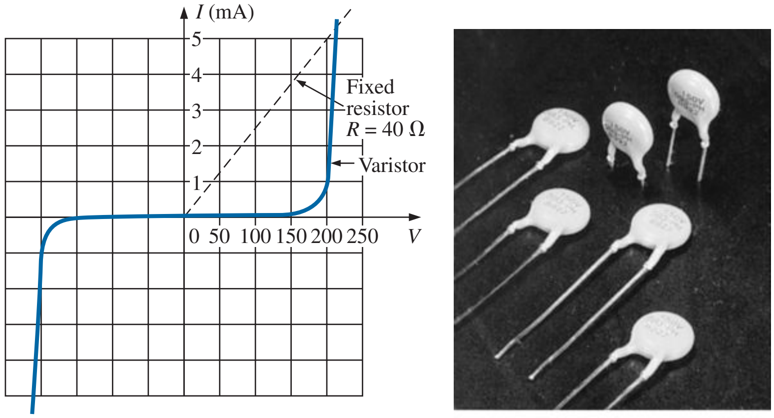


FIG. 3.38
Photoconductive cells.
(Courtesy of EG&G VACTEC, Inc.)

3.13 VARISTORS

Varistors are voltage-dependent, nonlinear resistors used to suppress high-voltage transients; that is, their characteristics enable them to limit the voltage that can appear across the terminals of a sensitive device or system. A typical set of characteristics appears in Fig. 3.39(a), along with a linear resistance characteristic for comparison purposes. Note that at a particular "firing voltage," the current rises rapidly, but the voltage is limited to a level just above this firing potential. In other words, the magnitude of the voltage that can appear across this device cannot exceed that level defined by its characteristics. Through proper design techniques, this device can therefore limit the voltage appearing across sensitive regions of a network. The current is simply limited by the network to which it is connected. A photograph of a number of commercial units appears in Fig. 3.39(b).



(a) (b)

FIG. 3.39

Varistors available with maximum dc voltage ratings between 18 V and 615 V.
(Courtesy of Philips Components, Inc.)

3.14 APPLICATIONS

The following are examples of how resistance can be used to perform a variety of tasks, from heating to measuring the stress or strain on a supporting member of a structure. In general, resistance is a component of every electrical or electronic application.

Electric Baseboard Heating Element

One of the most common applications of resistance is in household fixtures such as toasters and baseboard heating where the heat generated by current passing through a resistive element is employed to perform a useful function.

Recently, as we remodeled our house, the local electrician informed us that we were limited to 16 ft of electric baseboard on a single circuit. That naturally had me wondering about the wattage per foot, the resulting current level, and whether the 16-ft limitation was a national standard. Reading the label on the 2-ft section appearing in Fig. 3.40(a). I found VOLTS AC 240/208, WATTS 750/575 (the power rating is described in Chapter 4), AMPS 3.2/2.8. Since my panel is rated 240 V (as are those in most residential homes), the wattage rating per foot is 575 W/2 or 287.5 W at a current of 2.8 A. The total wattage for the 16 ft is therefore 16×287.5 W or 4600 W.

In Chapter 4, you will find that the power to a resistive load is related to the current and applied voltage by the equation $P = VI$. The total resulting current can then be determined using this equation in the following manner: $I = P/V = 4600\text{ W}/240\text{ V} = 19.17\text{ A}$. The result was that we needed a circuit breaker larger than 19.17 A; otherwise, the circuit breaker would trip every time we turned the heat on. In my case, the electrician used a 30 A breaker to meet the National Fire Code requirement that does not permit exceeding 80% of the rated current for a conductor or breaker. In most panels, a 30 A breaker takes two slots of the panel, whereas the more common 20 A breaker takes only one slot. If you have

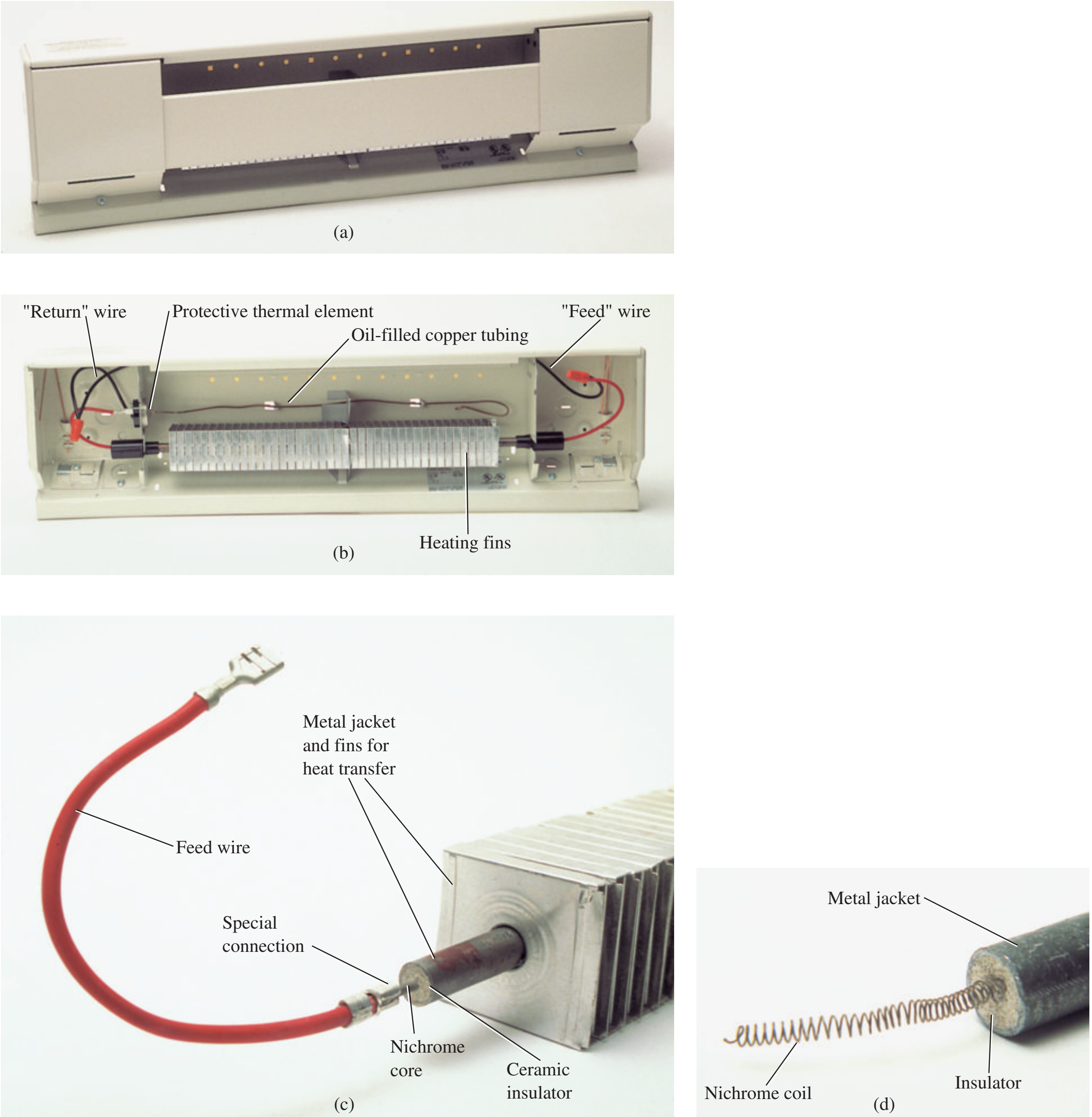


FIG. 3.40
Electric baseboard: (a) 2-ft section; (b) interior; (c) heating element; (d) nichrome coil.

a moment, take a look in your own panel and note the rating of the breakers used for various circuits of your home.

Going back to Table 3.2, we find that the #12 wire commonly used for most circuits in the home has a maximum rating of 20 A and would not be suitable for the electric baseboard. Since #11 is usually not commercially available, a #10 wire with a maximum rating of 30 A was used. You might wonder why the current drawn from the supply is 19.17 A while that required for one unit was only 2.8 A. This difference is due to the parallel combination of sections of the heating elements, a configuration that will be described in Chapter 6. It is now clear why they specify a 16-ft limitation on a single circuit. Additional elements would raise the current to a level that would exceed the code level for #10 wire and would approach the maximum rating of the circuit breaker.

Fig. 3.40(b) shows a photo of the interior construction of the heating element. The red feed wire on the right is connected to the core of the heating element, and the black wire at the other end passes through a protective heater element and back to the terminal box of the unit (the place where the exterior wires are brought in and connected). If you look carefully at the end of the heating unit as shown in Fig. 3.40(c), you will find that the heating wire that runs through the core of the heater is not connected directly to the round jacket holding the fins in place. A ceramic material (insulator) separates the heating wire from the fins to remove any possibility of conduction between the current passing through the bare heating element and the outer fin structure. Ceramic materials are used because they are excellent conductors of heat. They also have a high retentivity for heat so the surrounding area remains heated for a period of time even after the current has been turned off. As shown in Fig. 3.40(d), the heating wire that runs through the metal jacket is normally a nichrome composite (because pure nichrome is quite brittle) wound in the shape of a coil to compensate for expansion and contraction with heating and also to permit a longer heating element in standard-length baseboard. On opening the core, we found that the nichrome wire in the core of a 2-ft baseboard was actually 7 ft long, or a 3.5 : 1 ratio. The thinness of the wire was particularly noteworthy, measuring out at about 8 mils in diameter, not much thicker than a hair. Recall from this chapter that the longer the conductor and the thinner the wire, the greater the resistance. We took a section of the nichrome wire and tried to heat it with a reasonable level of current and the application of a hair dryer. The change in resistance was almost unnoticeable. In other words, all our effort to increase the resistance with the basic elements available to us in the lab was fruitless. This was an excellent demonstration of the meaning of the temperature coefficient of resistance in Table 3.6. Since the coefficient is so small for nichrome, the resistance does not measurably change unless the change in temperature is truly significant. The curve in Fig. 3.13 would therefore be close to horizontal for nichrome. For baseboard heaters, this is an excellent characteristic because the heat developed, and the power dissipated, will not vary with time as the conductor heats up with time. The flow of heat from the unit will remain fairly constant.

The feed and return cannot be soldered to the nichrome heater wire for two reasons. First, you cannot solder nichrome wires to each other or to other types of wire. Second, if you could, there might be a problem because the heat of the unit could rise above 880°F at the point where the wires are connected, the solder could melt, and the connection could be broken. Nichrome must be spot welded or crimped onto the copper wires of the unit. Using Eq. (3.1) and the 8-mil measured diameter, and assuming pure nichrome for the moment, the resistance of the 7-ft length is

$$
\begin{aligned}
R &= \frac{\rho l}{A} \\
&= \frac{(600)(7')}{(8\text{ mils})^2} = \frac{4200}{64} \\
R &= \mathbf{65.6\ \Omega}
\end{aligned}
$$

In Chapter 4, a power equation will be introduced in detail relating power, current, and resistance in the following manner: $P = I^2R$. Using the above data and solving for the resistance, we obtain

$$R = \frac{P}{I^2} = \frac{575\text{ W}}{(2.8\text{ A})^2}$$
$$R = \mathbf{73.34\ \Omega}$$

which is very close to the value calculated above from the geometric shape since we cannot be absolutely sure about the resistivity value for the composite.

During normal operation, the wire heats up and passes that heat on to the fins, which in turn heat the room via the air flowing through them. The flow of air through the unit is enhanced by the fact that hot air rises, so when the heated air leaves the top of the unit, it draws cold air from the bottom to contribute to the convection effect. Closing off the top or bottom of the unit would effectively eliminate the convection effect, and the room would not heat up. A condition could occur in which the inside of the heater became too hot, causing the metal casing also to get too hot. This concern is the primary reason for the thermal protective element introduced above and appearing in Fig. 3.40(b). The long, thin copper tubing in Fig. 3.40 is actually filled with an oil-type fluid that expands when heated. If too hot, it expands, depresses a switch in the housing, and turns off the heater by cutting off the current to the heater wire.

Dimmer Control in an Automobile

A two-point rheostat is the primary element in the control of the light intensity on the dashboard and accessories of a car. The basic network appears in Fig. 3.41 with typical voltage and current levels. When the light switch is closed (usually by pulling the light control knob out from the dashboard), current is established through the 50 Ω rheostat and then to the various lights on the dashboard (including the panel lights, ashtray light, radio display, and glove compartment light). As the knob of the control switch is turned, it controls the amount of resistance between points *a* and *b* of the rheostat. The more resistance between points *a* and

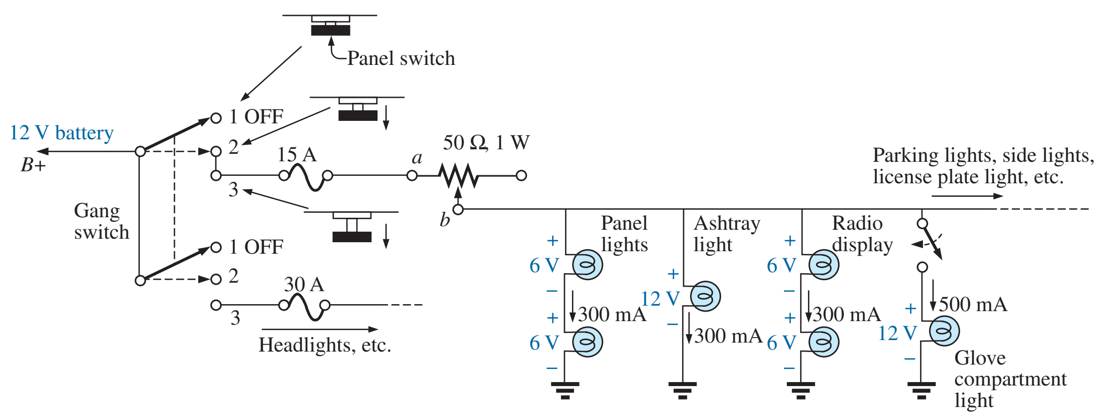


FIG. 3.41
Dashboard dimmer control in an automobile.

b, the less the current and the less the brightness of the various lights. Note the additional switch in the glove compartment light which is activated by the opening of the door of the compartment. Aside from the glove compartment light, all the lights in Fig. 3.41 will be on at the same time when the light switch is activated. The first branch after the rheostat contains two bulbs of 6 V rating rather than the 12 V bulbs appearing in the other branches. The smaller bulbs of this branch produce a softer, more even light for specific areas of the panel. Note that the sum of the two bulbs (in series) is 12 V to match that across the other branches. The division of voltage in any network is covered in detail in Chapters 5 and 6.

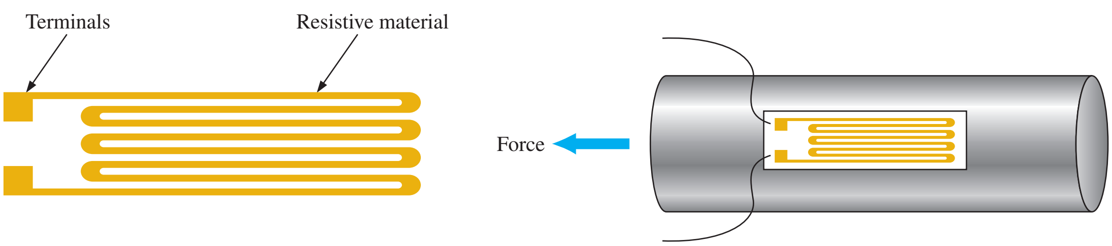
Typical current levels for the various branches have also been provided in Fig. 3.41. You will learn in Chapter 6 that the current drain from the battery and through the fuse and rheostat approximately equals the sum of the currents in the branches of the network. The result is that the fuse must be able to handle current in amperes, so a 15 A fuse was used (even though the bulbs appear in Fig. 3.41 as 12 V bulbs to match the battery).

Whenever the operating voltage and current levels of a component are known, the internal "hot" resistance of the unit can be determined using Ohm's law, introduced in detail in Chapter 4. Basically this law relates voltage, current, and resistance by $I = V/R$. For the 12 V bulb at a rated current of 300 mA, the resistance is $R = V/I = 12\text{ V}/300\text{ mA} = 40\ \Omega$. For the 6 V bulbs, it is $6\text{ V}/300\text{ mA} = 20\ \Omega$. Additional information regarding the power levels and resistance levels is discussed in later chapters.

The preceding description assumed an ideal level of 12 V for the battery. In actuality, 6.3 V and 14 V bulbs are used to match the charging level of most automobiles.

Strain Gauges

Any change in the shape of a structure can be detected using strain gauges whose resistance changes with applied stress or flex. An example of a strain gauge is shown in Fig. 3.42. Metallic strain gauges are constructed of a fine wire or thin metallic foil in a grid pattern. The terminal resistance of the strain gauge will change when exposed to compression or extension. One simple example of the use of resistive strain gauges is to monitor earthquake activity. When the gauge is placed across an area of suspected earthquake activity, the slightest separation in the earth changes the terminal resistance, and the processor displays a result sen-



(a) Typical strain gauge configuration.

(b) The strain gauge is bonded to the surface to be measured along the line of force. When the surface lengthens, the strain gauge stretches.

FIG. 3.42
Resistive strain gauge.

sitive to the amount of separation. Another example is in alarm systems where the slightest change in the shape of a supporting beam when someone walks overhead results in a change in terminal resistance, and an alarm sounds. Other examples include placing strain gauges on bridges to maintain an awareness of their rigidity and on very large generators to check whether various moving components are beginning to separate because of a wearing of the bearings or spacers. The small mouse control within a computer keyboard can be a series of strain gauges that reveal the direction of compression or extension applied to the controlling element on the keyboard. Movement in one direction can extend or compress a resistance gauge which can monitor and control the motion of the mouse on the screen.

3.15 MATHCAD

Throughout the text the mathematical software package Mathcad 12 is used to introduce a variety of operations that a math software package can perform. There is no need to obtain a copy of the software package to continue with the material covered in this text. The coverage is at a very introductory level simply to introduce the scope and power of the package. All the exercises appearing at the end of each chapter can be done without Mathcad.

Once the package is installed, all operations begin with the basic screen in Fig. 3.43. The operations must be performed in the sequence appearing in Fig. 3.44; that is, from left to right and then from top to bottom. For example, if an equation on the second line is to operate on a specific variable, the variable must be defined to the left of or above the equation.

To perform any mathematical calculation, simply click on the screen at any convenient point to establish a crosshair on the display (the location of the first entry). Then type in the mathematical operation such as

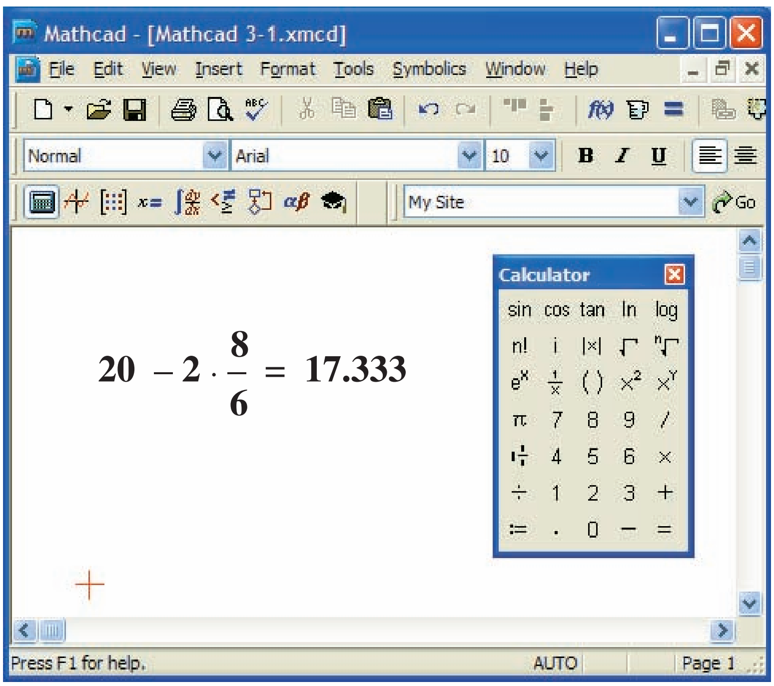


FIG. 3.43
Using Mathcad to perform a basic mathematical operation.

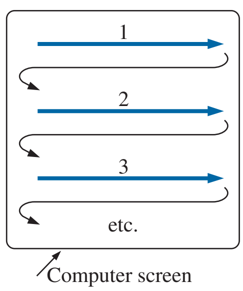


FIG. 3.44
Defining the order of mathematical operations for Mathcad.

20 − 2 · 8/6 as shown in Fig. 3.43; the instant the equal sign is selected, the result, **17.333,** will appear as shown in Fig. 3.43. The multiplication is obtained using the asterisk (*) appearing at the top of the number 8 key (under the SHIFT CONTROL key). The division is set by the / key at the bottom right of the keyboard. The equal sign can be selected from the top right corner of the keyboard. Another option is to apply the sequence **View-Toolbars-Calculator** to obtain the calculator in Fig. 3.43. Then use the calculator to enter the entire expression and the result will appear as soon as the equal sign is selected.

As an example in which variables must be defined, the resistance of a 200-ft length of copper wire with a diameter of 0.01 in. will be determined. First, as shown in Fig. 3.45, the variables for resistivity, length, and diameter must be defined. This is accomplished by first calling for the **Greek** palette through **View-Toolbars-Greek** and selecting the Greek letter *rho* (ρ) followed by a combined **Shift-colon** operation. A colon and an equal sign will appear, after which **10.37** is entered. For all the calculations to follow, the value of ρ has been defined. A left click on the screen removes the rectangular enclosure and places the variable and its value in memory. Proceed in the same way to define the length l and the diameter d. Next, the diameter in mils is defined by multiplying the diameter in inches by 1000, and the area is defined by the diameter in mils squared. Note that m had to be defined to the left of the expression for the area, and the variable d was defined in the line above. The power of 2 was obtained by first selecting the superscript symbol (^) at the top of the number 6 on the keyboard and then entering the number 2 in the Mathcad bracket. Or you can simply type the letter m and choose x^2 from the **Calculator** palette. In fact, all the operations of multiplication,

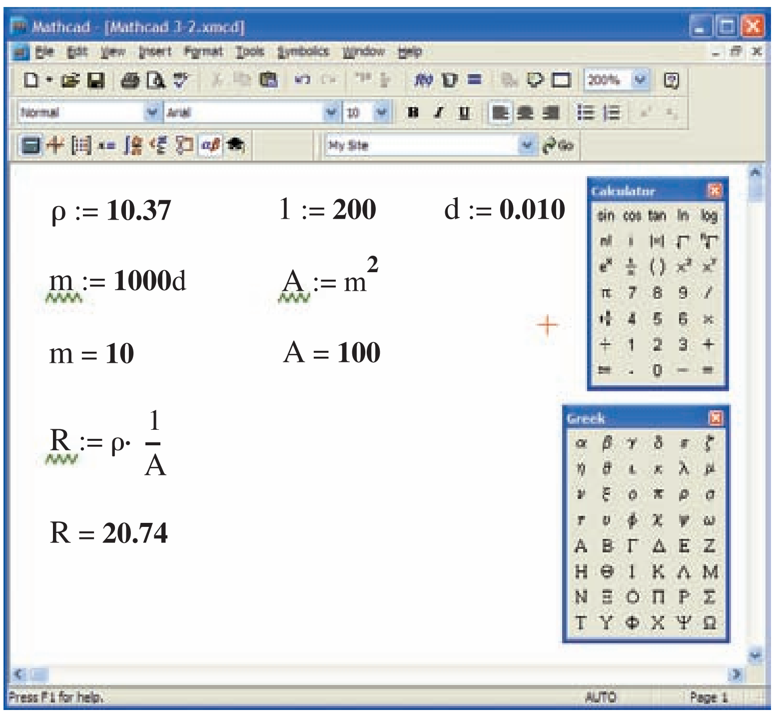


FIG. 3.45

Using Mathcad to calculate the resistance of a copper conductor.

division, etc., required to determine the resistance R can be lifted from the **Calculator** palette.

On the next line in Fig. 3.45, the values of m and A were calculated by simply typing in m followed by the keyboard equal sign. Finally, the equation for the resistance R is defined in terms of the variables, and the result is obtained. The true value of developing mathematical equations in the above sequence is that you can place the program in memory and, when the need arises, call it up and change a variable or two—the result will appear immediately. There is no need to reenter all the definitions—just change the numerical value.

The following chapters have additional examples of how using Mathcad to perform calculation can save time and ensure accuracy.

PROBLEMS

SECTION 3.2 Resistance: Circular Wires

1. Convert the following to mils:
a. 0.5 in. **b.** 0.02 in. **c.** 1/4 in.
d. 1 in. **e.** 0.02 ft **f.** 0.1 cm

2. Calculate the area in circular mils (CM) of wires having the following diameters:
a. 30 mils **b.** 0.016 in. **c.** 1/8 in.
d. 1 cm **e.** 0.02 ft **f.** 0.0042 m

3. The area in circular mils is
a. 1600 CM **b.** 820 CM **c.** 40,000 CM
d. 625 CM **e.** 6.25 CM **f.** 100 CM
What is the diameter of each wire in inches?

4. What is the resistance of a copper wire 200 ft long and 0.01 in. in diameter ($T = 20°C$)?

5. Find the resistance of a silver wire 50 yd long and 4 mils in diameter ($T = 20°C$).

6. **a.** What is the area in circular mils of an aluminum conductor that is 80 ft long with a resistance of 2.5 Ω?
b. What is its diameter in inches?

7. A 2.2 Ω resistor is to be made of nichrome wire. If the available wire is 1/32 in. in diameter, how much wire is required?

8. **a.** What is the diameter in inches of a copper wire that has a resistance of 3.3 Ω and is as long as a football field (100 yd) ($T = 20°C$)?
b. Without working out the numerical solution, determine whether the area of an aluminum wire will be smaller or larger than that of the copper wire. Explain.
c. Repeat (b) for a silver wire.

9. In Fig. 3.46, three conductors of different materials are presented.
a. Without working out the numerical solution, which do you think has the most resistance between its ends? Explain.
b. Find the resistance of each section and compare your answer with the result of (a) ($T = 20°C$).

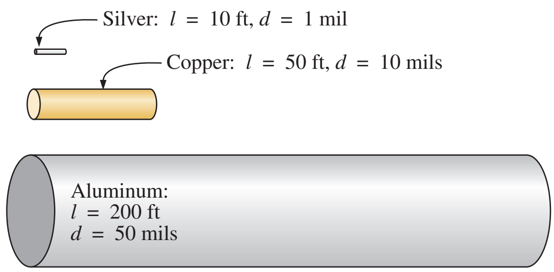


FIG. 3.46
Problem 9.

10. A wire 1000 ft long has a resistance of 0.5 kΩ and an area of 94 CM. Of what material is the wire made ($T = 20°C$)?

***11.** **a.** What is the resistance of a copper bus-bar for a high-rise building with the dimensions shown ($T = 20°C$) in Fig. 3.47?
b. Repeat (a) for aluminum and compare the results.

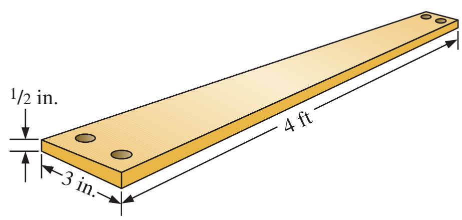


FIG. 3.47
Problem 11.

12. Determine the increase in resistance of a copper conductor if the area is reduced by a factor of 4 and the length is doubled. The original resistance was 0.2 Ω. The temperature remains fixed.

***13.** What is the new resistance level of a copper wire if the length is changed from 200 ft to 100 yd, the area is changed from 40,000 CM to 0.04 in.^{2}, and the original resistance was 800 mΩ?

SECTION 3.3 Wire Tables

14. **a.** Using Table 3.2, find the resistance of 450 ft of #11 and #14 AWG wires.
b. Compare the resistances of the two wires.
c. Compare the areas of the two wires.

15. **a.** Using Table 3.2, find the resistance of 1800 ft of #8 and #18 AWG wires.
b. Compare the resistances of the two wires.
c. Compare the areas of the two wires.

16. **a.** For the system in Fig. 3.48, the resistance of each line cannot exceed 0.006 Ω, and the maximum current drawn by the load is 110 A. What minimum size gage wire should be used?
b. Repeat (a) for a maximum resistance of 0.003 Ω, $d = 30$ ft, and a maximum current of 110 A.

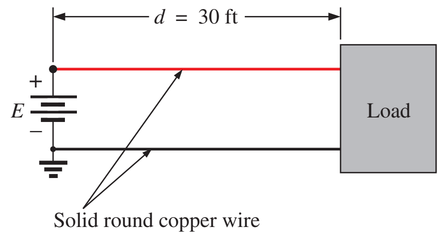


FIG. 3.48
Problem 16.

***17.** **a.** From Table 3.2, determine the maximum permissible current density (A/CM) for an AWG #0000 wire.
b. Convert the result of (a) to A/in.^{2}
c. Using the result of (b), determine the cross-sectional area required to carry a current of 5000 A.

SECTION 3.4 Resistance: Metric Units

18. Using metric units, determine the length of a copper wire that has a resistance of 0.2 Ω and a diameter of 1/12 in.

19. Repeat Problem 11 using metric units; that is, convert the given dimensions to metric units before determining the resistance.

20. If the sheet resistance of a tin oxide sample is 100 Ω, what is the thickness of the oxide layer?

21. Determine the width of a carbon resistor having a sheet resistance of 15 0 Ω if the length is 1/2 in. and the resistance is 500 Ω.

***22.** Derive the conversion factor between ρ (CM-Ω/ft) and ρ (Ω-cm) by
a. Solving for ρ for the wire in Fig. 3.49 in CM-Ω/ft.
b. Solving for ρ for the same wire in Fig. 3.49 in Ω-cm by making the necessary conversions.
c. Use the equation $\rho_2 = k\rho_1$ to determine the conversion factor k if ρ_1 is the solution of part (a) and ρ_2 the solution of part (b).

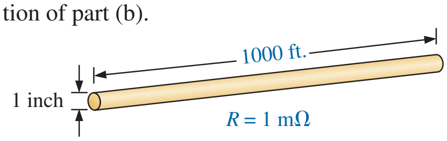


FIG. 3.49
Problem 22.

SECTION 3.5 Temperature Effects

23. The resistance of a copper wire is 2 Ω at 10°C. What is its resistance at 80°C?

24. The resistance of an aluminum bus-bar is 0.02 Ω at 0°C. What is its resistance at 100°C?

25. The resistance of a copper wire is 4 Ω at 70°F. What is its resistance at 32°F?

26. The resistance of a copper wire is 0.76 Ω at 30°C. What is its resistance at −40°C?

27. If the resistance of a silver wire is 0.04 Ω at −30°C, what is its resistance at 32°F?

***28.** **a.** The resistance of a copper wire is 0.002 Ω at room temperature (68°F). What is its resistance at 32°F (freezing) and 212°F (boiling)?
b. For part (a), determine the change in resistance for each 10° change in temperature between room temperature and 212°F.

29. **a.** The resistance of a copper wire is 1 Ω at 4°C. At what temperature (°C) will it be 1.1 Ω?
b. At what temperature will it be 0.1 Ω?

***30.** **a.** If the resistance of a 1000-ft length of copper wire is 10 Ω at room temperature (20°C), what will its resistance be at 50 K (Kelvin units) using Eq. (3.8)?
b. Repeat part (a) for a temperature of 38.65 K. Comment on the results obtained by reviewing the curve of Fig. 3.13.
c. What is the temperature of absolute zero in Fahrenheit units?

31. **a.** Verify the value of α_{20} for copper in Table 3.6 by substituting the inferred absolute temperature into Eq. (3.9).
b. Using Eq. (3.10) find the temperature at which the resistance of a copper conductor will increase to 1 Ω from a level of 0.8 Ω at 20°C.

32. Using Eq. (3.10), find the resistance of a copper wire at 16°C if its resistance at 20°C is 0.4 Ω.

***33.** Determine the resistance of a 1000-ft coil of #12 copper wire sitting in the desert at a temperature of 115°F.

34. A 22 Ω wire-wound resistor is rated at +200 PPM for a temperature range of −10°C to +75°C. Determine its resistance at 65°C.

35. A 100 Ω wire-wound resistor is rated at +100 PPM for a temperature range of 0°C to +100°C. Determine its resistance at 50°C.

SECTION 3.6 Superconductors

36. Visit your local library and find a table listing the critical temperatures for a variety of materials. List at least five materials with critical temperatures that are not mentioned in this text. Choose a few materials that have relatively high critical temperatures.

37. Find at least one article on the application of superconductivity in the commercial sector, and write a short summary, including all interesting facts and figures.

***38.** Using the required 1 MA/cm^2 density level for integrated circuit manufacturing, determine what the resulting current would be through a #12 house wire. Compare the result obtained with the allowable limit of Table 3.2.

*39. Research the SQUID magnetic field detector and review its basic mode of operation and an application or two.

SECTION 3.7 Types of Resistors

40. **a.** What is the approximate increase in size from a 1 W to a 2 W carbon resistor?
b. What is the approximate increase in size from a 1/2 W to a 2 W carbon resistor?
c. In general, can we conclude that for the same type of resistor, an increase in wattage rating requires an increase in size (volume)? Is it almost a linear relationship? That is, does twice the wattage require an increase in size of 2:1?

41. If the resistance between the outside terminals of a linear potentiometer is 10 kΩ, what is its resistance between the wiper (movable) arm and an outside terminal if the resistance between the wiper arm and the other outside terminal is 3.5 kΩ?

42. If the wiper arm of a linear potentiometer is one-quarter the way around the contact surface, what is the resistance between the wiper arm and each terminal if the total resistance is 2.5 kΩ?

*43. Show the connections required to establish 4 kΩ between the wiper arm and one outside terminal of a 10 kΩ potentiometer while having only zero ohms between the other outside terminal and the wiper arm.

SECTION 3.8 Color Coding and Standard Resistor Values

44. Find the range in which a resistor having the following color bands must exist to satisfy the manufacturer's tolerance:

	1st band	2nd band	3rd band	4th band
a.	green	blue	yellow	gold
b.	red	red	brown	silver
c.	brown	black	brown	—

45. Find the color code for the following 10% resistors:
a. 120 Ω **b.** 0.2 Ω
c. 68 kΩ **d.** 3.3 MΩ

46. Is there an overlap in coverage between 20% resistors? That is, determine the tolerance range for a 10 Ω 20% resistor and a 15 Ω 20% resistor, and note whether their tolerance ranges overlap.

47. Repeat Problem 46 for 10% resistors of the same value.

48. Find the value of the following surface mount resistors:
a. 621 **b.** 333
c. Q2 **d.** C6

SECTION 3.9 Conductance

49. Find the conductance of each of the following resistances:
a. 120 Ω
b. 4 kΩ
c. 2.2 MΩ
d. Compare the three results.

50. Find the conductance of 1000 ft of #12 AWG wire made of
a. copper
b. aluminum
c. iron

*51. The conductance of a wire is 100 S. If the area of the wire is increased by 2/3 and the length is reduced by the same amount, find the new conductance of the wire if the temperature remains fixed.

SECTION 3.10 Ohmmeters

52. How would you check the status of a fuse with an ohmmeter?

53. How would you determine the on and off states of a switch using an ohmmeter?

54. How would you use an ohmmeter to check the status of a light bulb?

SECTION 3.11 Thermistors

*55. **a.** Find the resistance of the thermistor having the characteristics of Fig. 3.35 at −50°C, 50°C, and 200°C. Note that it is a log scale. If necessary, consult a reference with an expanded log scale.
b. Does the thermistor have a positive or a negative temperature coefficient?
c. Is the coefficient a fixed value for the range −100°C to 400°C? Why?
d. What is the approximate rate of change of ρ with temperature at 100°C?

SECTION 3.12 Photoconductive Cell

56. **a.** Using the characteristics of Fig. 3.37, determine the resistance of the photoconductive cell at 10 and 100 foot-candles of illumination. As in Problem 55, note that it is a log scale.
b. Does the cell have a positive or a negative illumination coefficient?
c. Is the coefficient a fixed value for the range 0.1 to 1000 foot-candles? Why?
d. What is the approximate rate of change of R with illumination at 10 foot-candles?

SECTION 3.13 Varistors

57. **a.** Referring to Fig. 3.39(a), find the terminal voltage of the device at 0.5 mA, 1 mA, 3 mA, and 5 mA.
b. What is the total change in voltage for the indicated range of current levels?
c. Compare the ratio of maximum to minimum current levels above to the corresponding ratio of voltage levels.

SECTION 3.15 Mathcad

58. Verify the results of Example 3.2 using Mathcad.

59. Verify the results of Example 3.10 using Mathcad.

GLOSSARY

Absolute zero The temperature at which all molecular motion ceases; −273.15°C.

Circular mil (CM) The cross-sectional area of a wire having a diameter of one mil.

Color coding A technique using bands of color to indicate the resistance levels and tolerance of resistors.

Conductance (G) An indication of the relative ease with which current can be established in a material. It is measured in siemens (S).

Cooper effect The "pairing" of electrons as they travel through a medium.

Ductility The property of a material that allows it to be drawn into long, thin wires.

Inferred absolute temperature The temperature through which a straight-line approximation for the actual resistance-versus-temperature curve intersects the temperature axis.

Malleability The property of a material that allows it to be worked into many different shapes.

Negative temperature coefficient of resistance The value revealing that the resistance of a material will decrease with an increase in temperature.

Ohm (Ω) The unit of measurement applied to resistance.

Ohmmeter An instrument for measuring resistance levels.

Photoconductive cell A two-terminal semiconductor device whose terminal resistance is determined by the intensity of the incident light on its exposed surface.

Positive temperature coefficient of resistance The value revealing that the resistance of a material will increase with an increase in temperature.

Potentiometer A three-terminal device through which potential levels can be varied in a linear or nonlinear manner.

PPM/°C Temperature sensitivity of a resistor in parts per million per degree Celsius.

Resistance A measure of the opposition to the flow of charge through a material.

Resistivity (ρ) A constant of proportionality between the resistance of a material and its physical dimensions.

Rheostat An element whose terminal resistance can be varied in a linear or nonlinear manner.

Sheet resistance Defined by ρ/d for thin-film and integrated circuit design.

Superconductor Conductors of electric charge that have for all practical purposes zero ohms.

Thermistor A two-terminal semiconductor device whose resistance is temperature sensitive.

Varistor A voltage-dependent, nonlinear resistor used to suppress high-voltage transients.