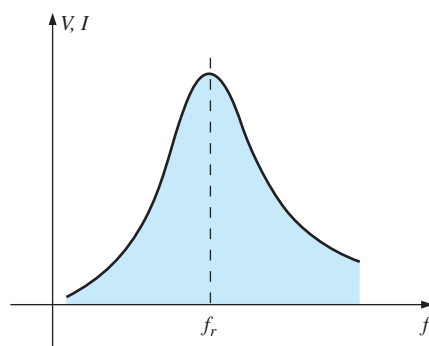


## OBJECTIVES

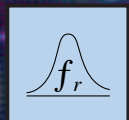
- *Become familiar with the frequency response of a series resonant circuit and how to calculate the resonant and cutoff frequencies.*
- *Be able to calculate a tuned network's quality factor, bandwidth, and power levels at important frequency levels.*
- *Become familiar with the frequency response of a parallel resonant circuit and how to calculate the resonant and cutoff frequencies.*
- *Understand the impact of the quality factor on the frequency response of a series or parallel resonant network.*
- *Begin to appreciate the difference between defining parallel resonance at either the frequency where the input impedance is a maximum or where the network has a unity power factor.*

## 20.1 INTRODUCTION

This chapter introduces the very important *resonant* (or *tuned*) *circuit*, which is fundamental to the operation of a wide variety of electrical and electronic systems in use today. The resonant circuit is a combination of  $R$ ,  $L$ , and  $C$  elements having a frequency response characteristic similar to the one appearing in Fig. 20.1. Note in the figure that the response is a maximum for the frequency  $f_r$ , decreasing to the right and left of this frequency. In other words, for a particular range of frequencies, the response will be near or equal to the maximum. The frequencies to the far left or right have very low voltage or current levels and, for all practical purposes, have little effect on the system's response. The radio or television receiver has a response curve for each broadcast station of the type indicated in Fig. 20.1. When the receiver is set (or tuned) to a particular station, it is set on or near the frequency  $f_r$  in Fig. 20.1. Stations transmitting at



**FIG. 20.1**  
*Resonance curve.*





frequencies to the far right or left of this resonant frequency are not carried through with significant power to affect the program of interest. The tuning process (setting the dial to  $f_r$ ) as described above is the reason for the terminology *tuned circuit*. When the response is at or near the maximum, the circuit is said to be in a state of **resonance**.

The concept of resonance is not limited to electrical or electronic systems. If mechanical impulses are applied to a mechanical system at the proper frequency, the system will enter a state of resonance in which sustained vibrations of very large amplitude will develop. The frequency at which this occurs is called the *natural frequency* of the system. The classic example of this effect was the Tacoma Narrows Bridge built in 1940 over Puget Sound in Washington State. Four months after the bridge, with its suspended span of 2800 ft, was completed, a 42 mi/h pulsating gale set the bridge into oscillations at its natural frequency. The amplitude of the oscillations increased to the point where the main span broke up and fell into the water below. It has since been replaced by the new Tacoma Narrows Bridge, completed in 1950.

The resonant electrical circuit *must* have both inductance and capacitance. In addition, resistance will always be present due either to the lack of ideal elements or to the control offered on the shape of the resonance curve. When resonance occurs due to the application of the proper frequency ( $f_r$ ), the energy absorbed by one reactive element is the same as that released by another reactive element within the system. In other words, energy pulsates from one reactive element to the other. Therefore, once an ideal (pure  $C, L$ ) system has reached a state of resonance, it requires no further reactive power since it is self-sustaining. In a practical circuit, there is some resistance associated with the reactive elements that will result in the eventual “damping” of the oscillations between reactive elements.

There are two types of resonant circuits: *series* and *parallel*. Each will be considered in some detail in this chapter.

## SERIES RESONANCE

### 20.2 SERIES RESONANT CIRCUIT

A resonant circuit (series or parallel) must have an inductive and a capacitive element. A resistive element is always present due to the internal resistance of the source ( $R_s$ ), the internal resistance of the inductor ( $R_l$ ), and any added resistance to control the shape of the response curve ( $R_{\text{design}}$ ). The basic configuration for the series resonant circuit appears in Fig. 20.2(a) with the resistive elements listed above. The “cleaner” ap-

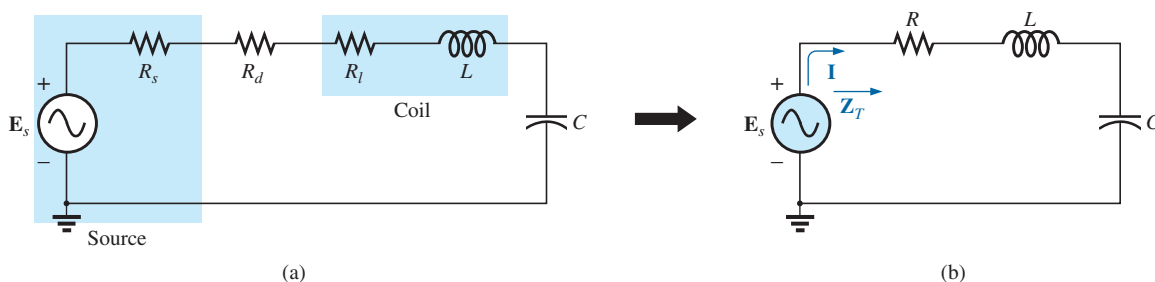


FIG. 20.2

Series resonant circuit.



pearance in Fig. 20.2(b) is a result of combining the series resistive elements into one total value. That is,

$$R = R_s + R_l + R_d \quad (20.1)$$

The total impedance of this network at any frequency is determined by

$$\mathbf{Z}_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

The resonant conditions described in the introduction occurs when

$$X_L = X_C \quad (20.2)$$

removing the reactive component from the total impedance equation. The total impedance at resonance is then

$$\mathbf{Z}_{T_s} = R \quad (20.3)$$

representing the minimum value of  $\mathbf{Z}_T$  at any frequency. The subscript  $s$  is employed to indicate series resonant conditions.

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance [Eq. (20.2)]:

$$X_L = X_C$$

Substituting yields

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

and

$$\omega_s = \frac{1}{\sqrt{LC}} \quad (20.4)$$

or

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad \begin{array}{l} f = \text{hertz (Hz)} \\ L = \text{henries (H)} \\ C = \text{farads (F)} \end{array} \quad (20.5)$$

The current through the circuit at resonance is

$$\mathbf{I} = \frac{E \angle 0^\circ}{R \angle 0^\circ} = \frac{E}{R} \angle 0^\circ$$

which is the maximum current for the circuit in Fig. 20.2 for an applied voltage  $\mathbf{E}$  since  $\mathbf{Z}_T$  is a minimum value. Consider also that *the input voltage and current are in phase at resonance*.

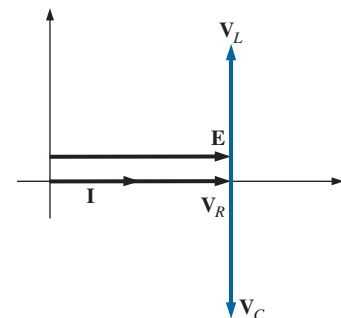
Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but  $180^\circ$  out of phase at resonance:

$$\left. \begin{array}{l} \mathbf{V}_L = (I \angle 0^\circ)(X_L \angle 90^\circ) = IX_L \angle 90^\circ \\ \mathbf{V}_C = (I \angle 0^\circ)(X_C \angle -90^\circ) = IX_C \angle -90^\circ \end{array} \right\} \begin{array}{l} 180^\circ \\ \text{out of} \\ \text{phase} \end{array}$$

and, since  $X_L = X_C$ , the magnitude of  $V_L$  equals  $V_C$  at resonance; that is,

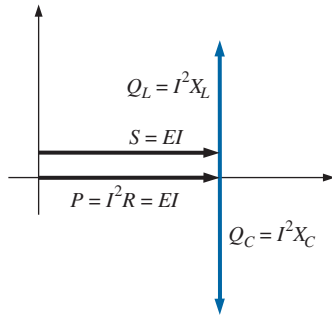
$$V_{L_s} = V_{C_s} \quad (20.6)$$

Fig. 20.3, a phasor diagram of the voltages and current, clearly indicates that the voltage across the resistor at resonance is the input voltage, and  $\mathbf{E}$ ,  $\mathbf{I}$ , and  $\mathbf{V}_R$  are in phase at resonance.



**FIG. 20.3**

Phasor diagram for the series resonant circuit at resonance.



**FIG. 20.4**  
Power triangle for the series resonant circuit at resonance.

The average power to the resistor at resonance is equal to  $I^2 R$ , and the reactive power to the capacitor and inductor are  $I^2 X_C$  and  $I^2 X_L$ , respectively.

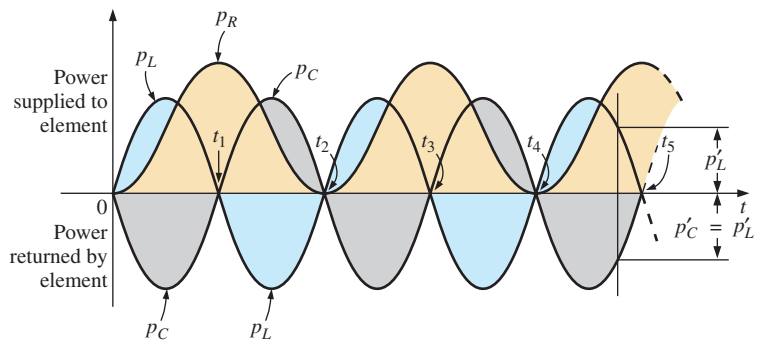
The power triangle at resonance (Fig. 20.4) shows that the total apparent power is equal to the average power dissipated by the resistor since  $Q_L = Q_C$ . The power factor of the circuit at resonance is

$$F_p = \cos \theta = \frac{P}{S}$$

and

$$F_{ps} = 1 \tag{20.7}$$

Plotting the power curves of each element on the same set of axes (Fig. 20.5), we note that, even though the total reactive power at any instant is equal to zero (note that  $t = t'$ ), energy is still being absorbed and released by the inductor and capacitor at resonance.



**FIG. 20.5**  
Power curves at resonance for the series resonant circuit.

A closer examination reveals that the energy absorbed by the inductor from time 0 to  $t_1$  is the same as the energy released by the capacitor from 0 to  $t_1$ . The reverse occurs from  $t_1$  to  $t_2$ , and so on. Therefore, the total apparent power continues to be equal to the average power, even though the inductor and capacitor are absorbing and releasing energy. This condition occurs only at resonance. The slightest change in frequency introduces a reactive component into the power triangle, which increases the apparent power of the system above the average power dissipation, and resonance no longer exists.

### 20.3 THE QUALITY FACTOR ( $Q$ )

The **quality factor**  $Q$  of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

$$Q_s = \frac{\text{reactive power}}{\text{average power}} \tag{20.8}$$

The quality factor is also an indication of how much energy is placed in storage (continual transfer from one reactive element to the other) compared to that dissipated. The lower the level of dissipation for the same reactive power, the larger the  $Q_s$  factor and the more concentrated and intense the region of resonance.



Substituting for an inductive reactance in Eq. (20.8) at resonance gives us

$$Q_s = \frac{I^2 X_L}{I^2 R}$$

and

$$Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R} \tag{20.9}$$

If the resistance  $R$  is just the resistance of the coil ( $R_l$ ), we can speak of the  $Q$  of the coil, where

$$Q_{\text{coil}} = Q_l = \frac{X_L}{R_l} \tag{20.10}$$

Since the quality factor of a coil is typically the information provided by manufacturers of inductors, it is often given the symbol  $Q$  without an associated subscript. It appears from Eq. (20.10) that  $Q_l$  increases linearly with frequency since  $X_L = 2\pi fL$ . That is, if the frequency doubles, then  $Q_l$  also increases by a factor of 2. This is approximately true for the low range to the midrange of frequencies such as shown for the coils in Fig. 20.6. Unfortunately, however, as the frequency increases, the effective resistance of the coil also increases, due primarily to skin effect phenomena, and the resulting  $Q_l$  decreases. In addition, the capacitive effects between the windings increases, further reducing the  $Q_l$  of the coil. For this reason,  $Q_l$  must be specified for a particular frequency or frequency range. For wide frequency applications, a plot of  $Q_l$  versus frequency is often provided. The maximum  $Q_l$  for most commercially available coils is less than 200, with most having a maximum near 100. Note in Fig. 20.6 that for coils of the same type,  $Q_l$  drops off more quickly for higher levels of inductance.

If we substitute

$$\omega_s = 2\pi f_s$$

and then

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

into Eq. (20.9), we have

$$\begin{aligned} Q_s &= \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R} \left( \frac{1}{2\pi\sqrt{LC}} \right) L \\ &= \frac{L}{R} \left( \frac{1}{\sqrt{LC}} \right) = \left( \frac{\sqrt{L}}{\sqrt{L}} \right) \frac{L}{R\sqrt{LC}} \end{aligned}$$

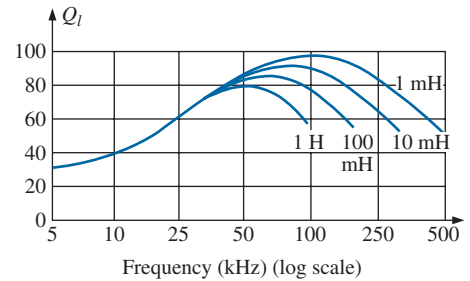
and

$$Q_s = \frac{1}{R} \sqrt{\frac{L}{C}} \tag{20.11}$$

providing  $Q_s$  in terms of the circuit parameters.

For series resonant circuits used in communication systems,  $Q_s$  is usually greater than 1. By applying the voltage divider rule to the circuit in Fig. 20.2, we obtain

$$V_L = \frac{X_L E}{Z_T} = \frac{X_L E}{R} \quad (\text{at resonance})$$



**FIG. 20.6**

$Q_l$  versus frequency for a series of inductors of similar construction.



and 
$$V_{L_s} = Q_s E \tag{20.12}$$

or 
$$V_C = \frac{X_C E}{Z_T} = \frac{X_C E}{R}$$

and 
$$V_{C_s} = Q_s E \tag{20.13}$$

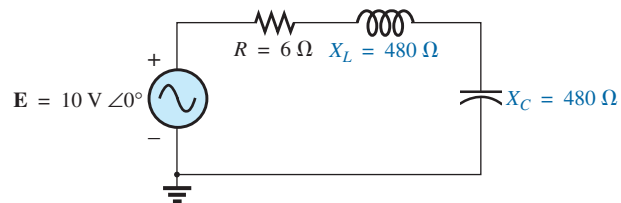
Since  $Q_s$  is usually greater than 1, the voltage across the capacitor or inductor of a series resonant circuit can be significantly greater than the input voltage. In fact, in many cases the  $Q_s$  is so high that careful design and handling (including adequate insulation) are mandatory with respect to the voltage across the capacitor and inductor.

In the circuit in Fig. 20.7, for example, which is in the state of resonance,

$$Q_s = \frac{X_L}{R} = \frac{480 \Omega}{6 \Omega} = 80$$

and 
$$V_L = V_C = Q_s E = (80)(10 \text{ V}) = \mathbf{800 \text{ V}}$$

which is certainly a potential of significant magnitude.



**FIG. 20.7**  
High- $Q$  series resonant circuit.

### 20.4 $Z_T$ VERSUS FREQUENCY

The total impedance of the series  $R$ - $L$ - $C$  circuit in Fig. 20.2 at any frequency is determined by

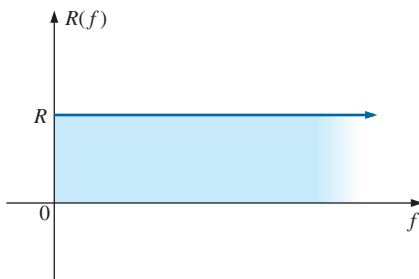
$$Z_T = R + j X_L - j X_C \quad \text{or} \quad Z_T = R + j (X_L - X_C)$$

The magnitude of the impedance  $Z_T$  versus frequency is determined by

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

The total-impedance-versus-frequency curve for the series resonant circuit in Fig. 20.2 can be found by applying the impedance-versus-frequency curve for each element of the equation just derived, written in the following form:

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2} \tag{20.14}$$



**FIG. 20.8**  
Resistance versus frequency.

where  $Z_T(f)$  “means” the total impedance as a *function* of frequency. For the frequency range of interest, we assume that the resistance  $R$  does not change with frequency, resulting in the plot in Fig. 20.8. The curve for the inductance, as determined by the reactance equation, is a straight line intersecting the origin with a slope equal to the inductance of the coil. The mathematical expression for any straight line in a two-dimensional plane is given by

$$y = mx + b$$

Thus, for the coil,

$$X_L = 2\pi fL + 0 = (2\pi L)(f) + 0$$

$$\begin{matrix} \downarrow & & \downarrow & \downarrow & \downarrow \\ y = & & m \cdot x & + & b \end{matrix}$$

(where  $2\pi L$  is the slope), producing the results shown in Fig. 20.9.

For the capacitor,

$$X_C = \frac{1}{2\pi fC} \quad \text{or} \quad X_C f = \frac{1}{2\pi C}$$

which becomes  $yx = k$ , the equation for a hyperbola, where

$$\begin{aligned} y \text{ (variable)} &= X_C \\ x \text{ (variable)} &= f \\ k \text{ (constant)} &= \frac{1}{2\pi C} \end{aligned}$$

The hyperbolic curve for  $X_C(f)$  is plotted in Fig. 20.10. In particular, note its very large magnitude at low frequencies and its rapid dropoff as the frequency increases.

If we place Figs. 20.9 and 20.10 on the same set of axes, we obtain the curves in Fig. 20.11. The condition of resonance is now clearly defined by the point of intersection, where  $X_L = X_C$ . For frequencies less than  $f_s$ , it is also quite clear that the network is primarily capacitive ( $X_C > X_L$ ). For frequencies above the resonant condition,  $X_L > X_C$ , and the network is inductive.

Applying

$$\begin{aligned} Z_T(f) &= \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2} \\ &= \sqrt{[R(f)]^2 + [X(f)]^2} \end{aligned}$$

to the curves in Fig. 20.11, where  $X(f) = X_L(f) - X_C(f)$ , we obtain the curve for  $Z_T(f)$  as shown in Fig. 20.12. The minimum impedance occurs at the resonant frequency and is equal to the resistance  $R$ . Note that the curve is not symmetrical about the resonant frequency (especially at higher values of  $Z_T$ ).

The phase angle associated with the total impedance is

$$\theta = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) \quad (20.15)$$

For the  $\tan^{-1}x$  function (resulting when  $X_L > X_C$ ), the larger  $x$  is, the larger the angle  $\theta$  (closer to  $90^\circ$ ). However, for regions where  $X_C > X_L$ , one must also be aware that

$$\tan^{-1}(-x) = -\tan^{-1}x \quad (20.16)$$

At low frequencies,  $X_C > X_L$ , and  $\theta$  approaches  $-90^\circ$  (capacitive), as shown in Fig. 20.13, whereas at high frequencies,  $X_L > X_C$ , and  $\theta$  approaches  $90^\circ$ . In general, therefore, for a series resonant circuit:

$f < f_s$ :	network capacitive; <b>I</b> leads <b>E</b>
$f > f_s$ :	network inductive; <b>E</b> leads <b>I</b>
$f = f_s$ :	network resistive; <b>E</b> and <b>I</b> are in phase

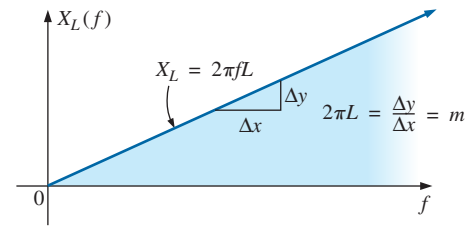


FIG. 20.9

Inductive reactance versus frequency.

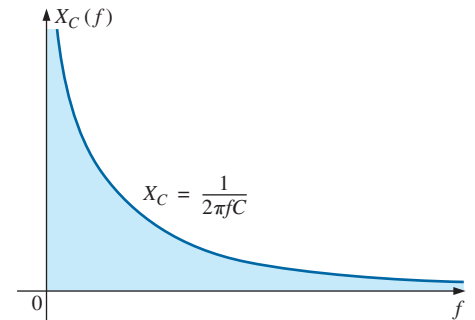


FIG. 20.10

Capacitive reactance versus frequency.

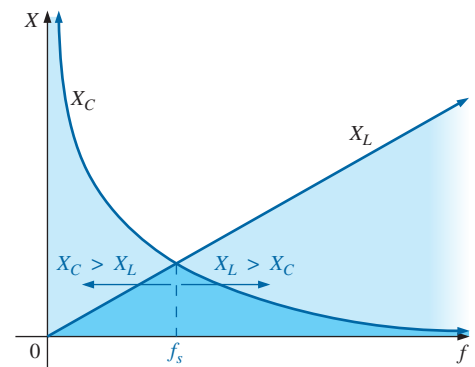


FIG. 20.11

Placing the frequency response of the inductive and capacitive reactance of a series R-L-C circuit on the same set of axes.

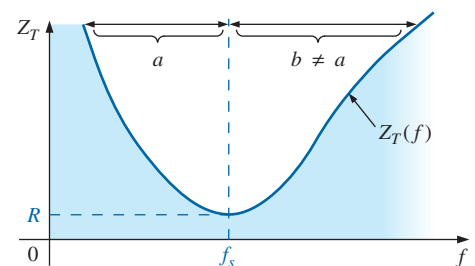
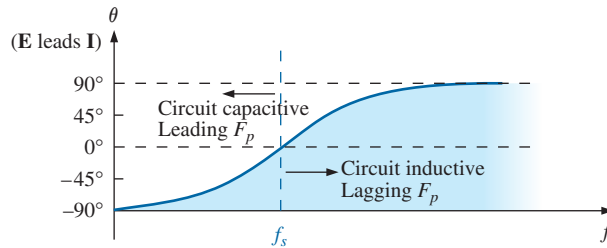


FIG. 20.12

$Z_T$  versus frequency for the series resonant circuit.

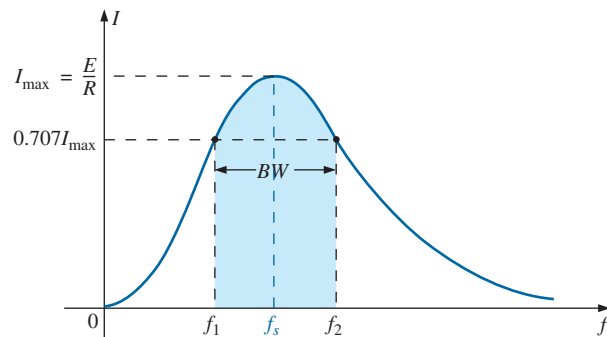


**FIG. 20.13**

Phase plot for the series resonant circuit.

### 20.5 SELECTIVITY

If we now plot the magnitude of the current  $I = E/Z_T$  versus frequency for a *fixed* applied voltage  $E$ , we obtain the curve shown in Fig. 20.14, which rises from zero to a maximum value of  $E/R$  (where  $Z_T$  is a minimum) and then drops toward zero (as  $Z_T$  increases) at a slower rate than it rose to its peak value. The curve is actually the inverse of the impedance-versus-frequency curve. Since the  $Z_T$  curve is not absolutely symmetrical about the resonant frequency, the curve of the current versus frequency has the same property.



**FIG. 20.14**

$I$  versus frequency for the series resonant circuit.

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of the maximum current are called the **band frequencies, cutoff frequencies, half-power frequencies, or corner frequencies**. They are indicated by  $f_1$  and  $f_2$  in Fig. 20.14. The range of frequencies between the two is referred to as the **bandwidth** (abbreviated  $BW$ ) of the resonant circuit.

Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency; that is,

$$P_{\text{HPF}} = \frac{1}{2} P_{\text{max}} \tag{20.17}$$

The above condition is derived using the fact that

$$P_{\text{max}} = I_{\text{max}}^2 R$$

and 
$$P_{\text{HPF}} = I^2 R = (0.707 I_{\text{max}})^2 R = (0.5)(I_{\text{max}}^2 R) = \frac{1}{2} P_{\text{max}}$$



Since the resonant circuit is adjusted to select a band of frequencies, the curve in Fig. 20.14 is called the **selectivity curve**. The term is derived from the fact that one must be *selective* in choosing the frequency to ensure that it is in the bandwidth. The smaller the bandwidth, the higher the selectivity. The shape of the curve, as shown in Fig. 20.15, depends on each element of the series  $R$ - $L$ - $C$  circuit. If the resistance is made smaller with a fixed inductance and capacitance, the bandwidth decreases and the selectivity increases. Similarly, if the ratio  $L/C$  increases with fixed resistance, the bandwidth again decreases with an increase in selectivity.

In terms of  $Q_s$ , if  $R$  is larger for the same  $X_L$ , then  $Q_s$  is less, as determined by the equation  $Q_s = \omega_s L/R$ .

**A small  $Q_s$ , therefore, is associated with a resonant curve having a large bandwidth and a small selectivity, while a large  $Q_s$  indicates the opposite.**

**For circuits where  $Q_s \geq 10$ , a widely accepted approximation is that the resonant frequency bisects the bandwidth and that the resonant curve is symmetrical about the resonant frequency.**

These conditions are shown in Fig. 20.16, indicating that the cutoff frequencies are then equidistant from the resonant frequency.

For any  $Q_s$ , the preceding is not true. The cutoff frequencies  $f_1$  and  $f_2$  can be found for the general case (any  $Q_s$ ) by first using the fact that a drop in current to 0.707 of its resonant value corresponds to an increase in impedance equal to  $1/0.707 = \sqrt{2}$  times the resonant value, which is  $R$ .

Substituting  $\sqrt{2}R$  into the equation for the magnitude of  $Z_T$ , we find that

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

becomes

$$\sqrt{2}R = \sqrt{R^2 + (X_L - X_C)^2}$$

or, squaring both sides, that

$$2R^2 = R^2 + (X_L - X_C)^2$$

and,

$$R^2 = (X_L - X_C)^2$$

Taking the square root of both sides gives us

$$R = X_L - X_C \quad \text{or} \quad R - X_L + X_C = 0$$

Let us first consider the case where  $X_L > X_C$ , which relates to  $f_2$  or  $\omega_2$ . Substituting  $\omega_2 L$  for  $X_L$  and  $1/\omega_2 C$  for  $X_C$  and bringing both quantities to the left of the equal sign, we have

$$R - \omega_2 L + \frac{1}{\omega_2 C} = 0 \quad \text{or} \quad R\omega_2 - \omega_2^2 L + \frac{1}{C} = 0$$

which can be written

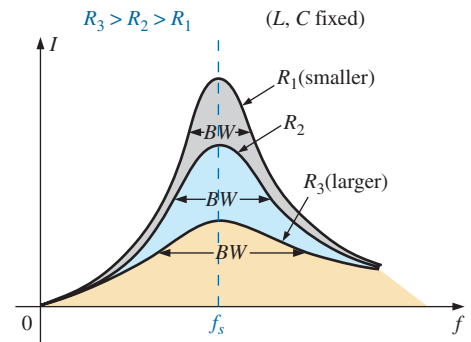
$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

Solving the quadratic, we have

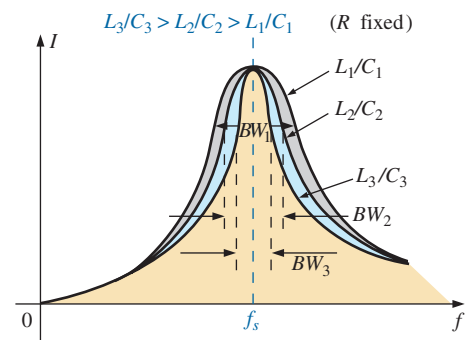
$$\omega_2 = \frac{-(-R/L) \pm \sqrt{[-(R/L)]^2 - [-(4/LC)]}}{2}$$

and

$$\omega_2 = +\frac{R}{2L} \pm \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$



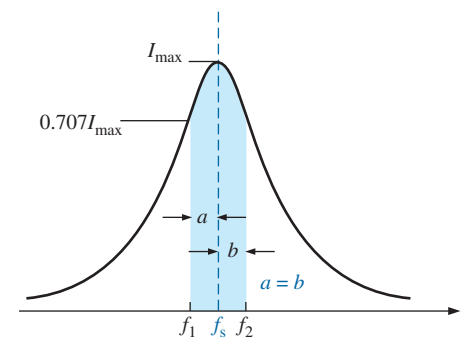
(a)



(b)

**FIG. 20.15**

Effect of  $R$ ,  $L$ , and  $C$  on the selectivity curve for the series resonant circuit.


**FIG. 20.16**

Approximate series resonance curve for  $Q_s \geq 10$ .



$$\text{with } f_2 = \frac{1}{2\pi} \left[ \frac{R}{2L} + \frac{1}{2} \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right] \quad (\text{Hz}) \quad (20.18)$$

The negative sign in front of the second factor was dropped because  $(1/2)\sqrt{(R/L)^2 + 4/LC}$  is always greater than  $R/(2L)$ . If it were not dropped, there would be a negative solution for the radian frequency  $\omega$ .

If we repeat the same procedure for  $X_C > X_L$ , which relates to  $\omega_1$  or  $f_1$  such that  $Z_T = \sqrt{R^2 + (X_C - X_L)^2}$ , the solution  $f_1$  becomes

$$f_1 = \frac{1}{2\pi} \left[ -\frac{R}{2L} + \frac{1}{2} \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right] \quad (\text{Hz}) \quad (20.19)$$

The bandwidth ( $BW$ ) is

$$BW = f_2 - f_1 = \text{Eq. (20.18)} - \text{Eq. (20.19)}$$

and

$$BW = f_2 - f_1 = \frac{R}{2\pi L} \quad (20.20)$$

Substituting  $R/L = \omega_s/Q_s$  from  $Q_s = \omega_s L/R$  and  $1/2\pi = f_s/\omega_s$  from  $\omega_s = 2\pi f_s$  gives us

$$BW = \frac{R}{2\pi L} = \left(\frac{1}{2\pi}\right)\left(\frac{R}{L}\right) = \left(\frac{f_s}{\omega_s}\right)\left(\frac{\omega_s}{Q_s}\right)$$

or

$$BW = \frac{f_s}{Q_s} \quad (20.21)$$

which is a very convenient form since it relates the bandwidth to the  $Q_s$  of the circuit. As mentioned earlier, Eq. (20.21) verifies that the larger the  $Q_s$ , the smaller the bandwidth, and vice versa.

Written in a slightly different form, Eq. (20.21) becomes

$$\frac{f_2 - f_1}{f_s} = \frac{1}{Q_s} \quad (20.22)$$

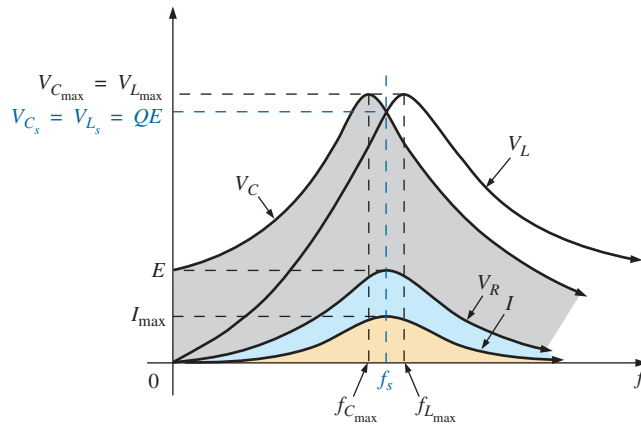
The ratio  $(f_2 - f_1)/f_s$  is sometimes called the *fractional bandwidth*, providing an indication of the width of the bandwidth compared to the resonant frequency.

It can also be shown through mathematical manipulations of the pertinent equations that the resonant frequency is related to the geometric mean of the band frequencies; that is,

$$f_s = \sqrt{f_1 f_2} \quad (20.23)$$

## 20.6 $V_R$ , $V_L$ , AND $V_C$

Plotting the magnitude (effective value) of the voltages  $V_R$ ,  $V_L$ , and  $V_C$  and the current  $I$  versus frequency for the series resonant circuit on the same set of axes, we obtain the curves shown in Fig. 20.17. Note that the  $V_R$  curve has the same shape as the  $I$  curve and a peak value equal to the magnitude of the input voltage  $E$ . The  $V_C$  curve builds up slowly at first from



**FIG. 20.17**

$V_R, V_L, V_C,$  and  $I$  versus frequency for a series resonant circuit.

a value equal to the input voltage since the reactance of the capacitor is infinite (open circuit) at zero frequency and the reactance of the inductor is zero (short circuit) at this frequency. As the frequency increases,  $1/\omega C$  of the equation

$$V_C = IX_C = (I) \left( \frac{1}{\omega C} \right)$$

becomes smaller, but  $I$  increases at a rate faster than that at which  $1/\omega C$  drops. Therefore,  $V_C$  rises and will continue to rise due to the quickly rising current, until the frequency nears resonance. As it approaches the resonant condition, the rate of change of  $I$  decreases. When this occurs, the factor  $1/\omega C$ , which decreased as the frequency rose, overcomes the rate of change of  $I$ , and  $V_C$  starts to drop. The peak value occurs at a frequency just before resonance. After resonance, both  $V_C$  and  $I$  drop in magnitude, and  $V_C$  approaches zero.

The higher the  $Q_s$  of the circuit, the closer  $f_{C_{max}}$  will be to  $f_s$ , and the closer  $V_{C_{max}}$  will be to  $Q_s E$ . For circuits with  $Q_s \geq 10$ ,  $f_{C_{max}} \cong f_s$ , and  $V_{C_{max}} \cong Q_s E$ .

The curve for  $V_L$  increases steadily from zero to the resonant frequency since both quantities  $\omega L$  and  $I$  of the equation  $V_L = IX_L = (I)(\omega L)$  increase over this frequency range. At resonance,  $I$  has reached its maximum value, but  $\omega L$  is still rising. Therefore,  $V_L$  reaches its maximum value after resonance. After reaching its peak value, the voltage  $V_L$  drops toward  $E$  since the drop in  $I$  overcomes the rise in  $\omega L$ . It approaches  $E$  because  $X_L$  will eventually be infinite, and  $X_C$  will be zero.

As  $Q_s$  of the circuit increases, the frequency  $f_{L_{max}}$  drops toward  $f_s$ , and  $V_{L_{max}}$  approaches  $Q_s E$ . For circuits with  $Q_s \geq 10$ ,  $f_{L_{max}} \cong f_s$ , and  $V_{L_{max}} \cong Q_s E$ .

The  $V_L$  curve has a greater magnitude than the  $V_C$  curve for any frequency above resonance, and the  $V_C$  curve has a greater magnitude than the  $V_L$  curve for any frequency below resonance. This again verifies that the series  $R$ - $L$ - $C$  circuit is predominantly capacitive from zero to the resonant frequency and predominantly inductive for any frequency above resonance.

For the condition  $Q_s \geq 10$ , the curves in Fig. 20.17 appear as shown in Fig. 20.18. Note that they each peak (on an approximate basis) at the resonant frequency and have a similar shape.

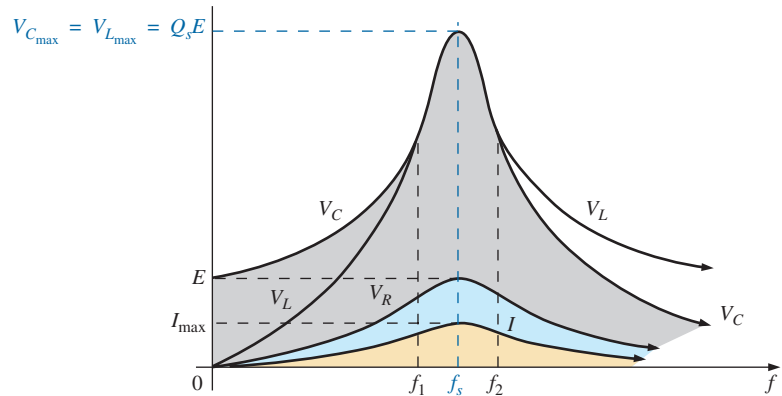


FIG. 20.18

$V_R$ ,  $V_L$ ,  $V_C$ , and  $I$  for a series resonant circuit where  $Q_s \geq 10$ .

In review,

1.  $V_C$  and  $V_L$  are at their maximum values at or near resonance (depending on  $Q_s$ ).
2. At very low frequencies,  $V_C$  is very close to the source voltage and  $V_L$  is very close to zero volts, whereas at very high frequencies,  $V_L$  approaches the source voltage and  $V_C$  approaches zero volts.
3. Both  $V_R$  and  $I$  peak at the resonant frequency and have the same shape.

## 20.7 EXAMPLES (SERIES RESONANCE)

### EXAMPLE 20.1

- a. For the series resonant circuit in Fig. 20.19, find  $\mathbf{I}$ ,  $\mathbf{V}_R$ ,  $\mathbf{V}_L$ , and  $\mathbf{V}_C$  at resonance.
- b. What is the  $Q_s$  of the circuit?
- c. If the resonant frequency is 5000 Hz, find the bandwidth.
- d. What is the power dissipated in the circuit at the half-power frequencies?

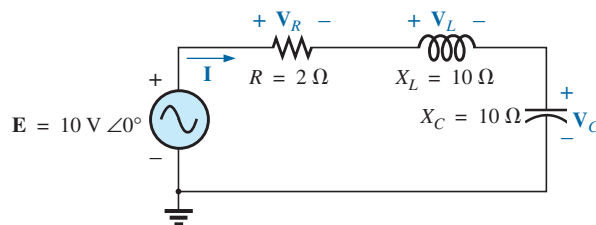


FIG. 20.19

Example 20.1.

### Solutions:

a.  $\mathbf{Z}_{T_s} = R = 2 \Omega$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_{T_s}} = \frac{10 \text{ V } \angle 0^\circ}{2 \Omega \angle 0^\circ} = 5 \text{ A } \angle 0^\circ$$

$$\mathbf{V}_R = \mathbf{E} = 10 \text{ V } \angle 0^\circ$$



$$\mathbf{V}_L = (I \angle 0^\circ)(X_L \angle 90^\circ) = (5 \text{ A } \angle 0^\circ)(10 \Omega \angle 90^\circ) \\ = \mathbf{50 \text{ V } \angle 90^\circ}$$

$$\mathbf{V}_C = (I \angle 0^\circ)(X_C \angle -90^\circ) = (5 \text{ A } \angle 0^\circ)(10 \Omega \angle -90^\circ) \\ = \mathbf{50 \text{ V } \angle -90^\circ}$$

$$\text{b. } Q_s = \frac{X_L}{R} = \frac{10 \Omega}{2 \Omega} = \mathbf{5}$$

$$\text{c. } BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{5000 \text{ Hz}}{5} = \mathbf{1000 \text{ Hz}}$$

$$\text{d. } P_{\text{HPF}} = \frac{1}{2} P_{\text{max}} = \frac{1}{2} I_{\text{max}}^2 R = \left(\frac{1}{2}\right)(5 \text{ A})^2(2 \Omega) = \mathbf{25 \text{ W}}$$

**EXAMPLE 20.2** The bandwidth of a series resonant circuit is 400 Hz.

- If the resonant frequency is 4000 Hz, what is the value of  $Q_s$ ?
- If  $R = 10 \Omega$ , what is the value of  $X_L$  at resonance?
- Find the inductance  $L$  and capacitance  $C$  of the circuit.

**Solutions:**

$$\text{a. } BW = \frac{f_s}{Q_s} \quad \text{or} \quad Q_s = \frac{f_s}{BW} = \frac{4000 \text{ Hz}}{400 \text{ Hz}} = \mathbf{10}$$

$$\text{b. } Q_s = \frac{X_L}{R} \quad \text{or} \quad X_L = Q_s R = (10)(10 \Omega) = \mathbf{100 \Omega}$$

$$\text{c. } X_L = 2\pi f_s L \quad \text{or} \quad L = \frac{X_L}{2\pi f_s} = \frac{100 \Omega}{2\pi(4000 \text{ Hz})} = \mathbf{3.98 \text{ mH}}$$

$$X_C = \frac{1}{2\pi f_s C} \quad \text{or} \quad C = \frac{1}{2\pi f_s X_C} = \frac{1}{2\pi(4000 \text{ Hz})(100 \Omega)} \\ = \mathbf{397.89 \text{ nF}}$$

**EXAMPLE 20.3** A series  $R$ - $L$ - $C$  circuit has a series resonant frequency of 12,000 Hz.

- If  $R = 5 \Omega$ , and if  $X_L$  at resonance is 300  $\Omega$ , find the bandwidth.
- Find the cutoff frequencies.

**Solutions:**

$$\text{a. } Q_s = \frac{X_L}{R} = \frac{300 \Omega}{5 \Omega} = 60$$

$$BW = \frac{f_s}{Q_s} = \frac{12,000 \text{ Hz}}{60} = \mathbf{200 \text{ Hz}}$$

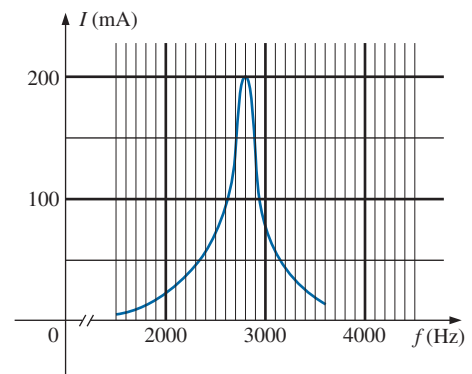
- b. Since  $Q_s \geq 10$ , the bandwidth is bisected by  $f_s$ . Therefore,

$$f_2 = f_s + \frac{BW}{2} = 12,000 \text{ Hz} + 100 \text{ Hz} = \mathbf{12,100 \text{ Hz}}$$

$$\text{and } f_1 = 12,000 \text{ Hz} - 100 \text{ Hz} = \mathbf{11,900 \text{ Hz}}$$

**EXAMPLE 20.4**

- Determine the  $Q_s$  and bandwidth for the response curve in Fig. 20.20.
- For  $C = 101.5 \text{ nF}$ , determine  $L$  and  $R$  for the series resonant circuit.
- Determine the applied voltage.



**FIG. 20.20**  
Example 20.4

**Solutions:**

- a. The resonant frequency is 2800 Hz. At 0.707 times the peak value,

$$BW = \mathbf{200 \text{ Hz}}$$

$$\text{and } Q_s = \frac{f_s}{BW} = \frac{2800 \text{ Hz}}{200 \text{ Hz}} = \mathbf{14}$$

$$\begin{aligned} \text{b. } f_s &= \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad L = \frac{1}{4\pi^2 f_s^2 C} \\ &= \frac{1}{4\pi^2 (2.8 \text{ kHz})^2 (101.5 \text{ nF})} \\ &= \mathbf{31.83 \text{ mH}} \end{aligned}$$

$$\begin{aligned} Q_s = \frac{X_L}{R} \quad \text{or} \quad R &= \frac{X_L}{Q_s} = \frac{2\pi(2800 \text{ Hz})(31.832 \text{ mH})}{14} \\ &= \mathbf{40 \Omega} \end{aligned}$$

$$\begin{aligned} \text{c. } I_{\max} &= \frac{E}{R} \quad \text{or} \quad E = I_{\max} R \\ &= (200 \text{ mA})(40 \Omega) = \mathbf{8 \text{ V}} \end{aligned}$$

**EXAMPLE 20.5** A series  $R$ - $L$ - $C$  circuit is designed to resonate at  $\omega_s = 10^5$  rad/s, have a bandwidth of  $0.15\omega_s$ , and draw 16 W from a 120 V source at resonance.

- Determine the value of  $R$ .
- Find the bandwidth in hertz.
- Find the nameplate values of  $L$  and  $C$ .
- Determine the  $Q_s$  of the circuit.
- Determine the fractional bandwidth.

**Solutions:**

$$\text{a. } P = \frac{E^2}{R} \quad \text{and} \quad R = \frac{E^2}{P} = \frac{(120 \text{ V})^2}{16 \text{ W}} = \mathbf{900 \Omega}$$

$$\begin{aligned} \text{b. } f_s &= \frac{\omega_s}{2\pi} = \frac{10^5 \text{ rad/s}}{2\pi} = 15,915.49 \text{ Hz} \\ BW &= 0.15f_s = 0.15(15,915.49 \text{ Hz}) = \mathbf{2387.32 \text{ Hz}} \end{aligned}$$

- c. Eq. (20.20):

$$BW = \frac{R}{2\pi L} \quad \text{and} \quad L = \frac{R}{2\pi BW} = \frac{900 \Omega}{2\pi(2387.32 \text{ Hz})} = \mathbf{60 \text{ mH}}$$

$$\begin{aligned} f_s &= \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad C = \frac{1}{4\pi^2 f_s^2 L} \\ &= \frac{1}{4\pi^2 (15,915.49 \text{ Hz})^2 (60 \times 10^{-3})} \\ &= \mathbf{1.67 \text{ nF}} \end{aligned}$$

$$\text{d. } Q_s = \frac{X_L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi(15,915.49 \text{ Hz})(60 \text{ mH})}{900 \Omega} = \mathbf{6.67}$$

$$\text{e. } \frac{f_2 - f_1}{f_s} = \frac{BW}{f_s} = \frac{1}{Q_s} = \frac{1}{6.67} = \mathbf{0.15}$$

## PARALLEL RESONANCE

### 20.8 PARALLEL RESONANT CIRCUIT

The basic format of the series resonant circuit is a series  $R$ - $L$ - $C$  combination in series with an applied voltage source. The parallel resonant circuit has the basic configuration in Fig. 20.21, a parallel  $R$ - $L$ - $C$  combination in parallel with an applied current source.

For the series circuit, the impedance was a minimum at resonance, producing a significant current that resulted in a high output voltage for  $V_C$  and  $V_L$ . For the parallel resonant circuit, the impedance is relatively high at resonance, producing a significant voltage for  $V_C$  and  $V_L$  through the Ohm's law relationship ( $V_C = IZ_T$ ). For the network in Fig. 20.21, resonance occurs when  $X_L = X_C$  and the resonant frequency has the same format obtained for series resonance.

If the practical equivalent in Fig. 20.22 had the format in Fig. 20.21, the analysis would be as direct and lucid as that experienced for series resonance. However, in the practical world, the internal resistance of the coil must be placed in series with the inductor, as shown in Fig. 20.22. The resistance  $R_l$  can no longer be included in a simple series or parallel combination with the source resistance and any other resistance added for design purposes. Even though  $R_l$  is usually relatively small in magnitude compared with other resistance and reactance levels of the network, it does have an important impact on the parallel resonant condition, as demonstrated in the sections to follow. In other words, the network in Fig. 20.21 is an ideal situation that can be assumed only for specific network conditions.

Our first effort is to find a parallel network equivalent (at the terminals) for the series  $R$ - $L$  branch in Fig. 20.22 using the technique introduced in Section 15.10. That is,

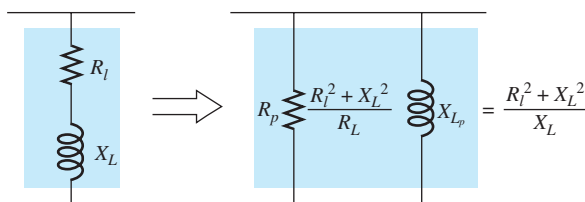
$$Z_{R-L} = R_l + jX_L$$

$$\begin{aligned} \text{and } Y_{R-L} &= \frac{1}{Z_{R-L}} = \frac{1}{R_l + jX_L} = \frac{R_l}{R_l^2 + X_L^2} - j \frac{X_L}{R_l^2 + X_L^2} \\ &= \frac{1}{\frac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j \left( \frac{R_l^2 + X_L^2}{X_L} \right)} = \frac{1}{R_p} + \frac{1}{jX_{Lp}} \end{aligned}$$

$$\text{with } R_p = \frac{R_l^2 + X_L^2}{R_l} \quad (20.24)$$

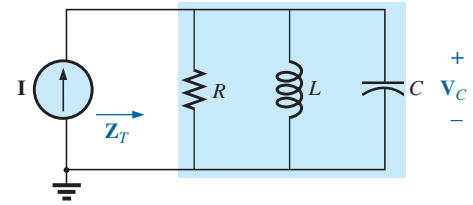
$$\text{and } X_{Lp} = \frac{R_l^2 + X_L^2}{X_L} \quad (20.25)$$

as shown in Fig. 20.23.



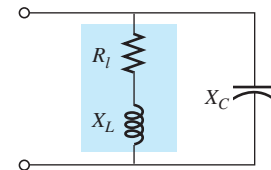
**FIG. 20.23**

Equivalent parallel network for a series  $R$ - $L$  combination.



**FIG. 20.21**

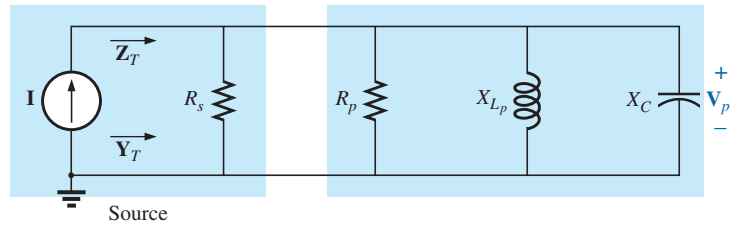
Ideal parallel resonant network.



**FIG. 20.22**

Practical parallel  $L$ - $C$  network.

Redrawing the network in Fig. 20.22 with the equivalent in Fig. 20.23 and a practical current source having an internal resistance  $R_s$ , results in the network in Fig. 20.24.

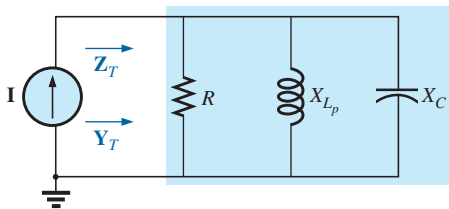


**FIG. 20.24**

Substituting the equivalent parallel network for the series R-L combination in Fig. 20.22.

If we define the parallel combination of  $R_s$  and  $R_p$  by the notation

$$R = R_s \parallel R_p \tag{20.26}$$



**FIG. 20.25**

Substituting  $R = R_s \parallel R_p$  for the network in Fig. 20.24.

the network in Fig. 20.25 results. It has the same format as the ideal configuration in Fig. 20.21.

We are now at a point where we can define the resonance conditions for the practical parallel resonant configuration. Recall that for series resonance, the resonant frequency was the frequency at which the impedance was a minimum, the current a maximum, and the input impedance purely resistive, and the network had a unity power factor. For parallel networks, since the resistance  $R_p$  in our equivalent model is frequency dependent, the frequency at which maximum  $V_C$  is obtained is not the same as required for the unity-power-factor characteristic. Since both conditions are often used to define the resonant state, the frequency at which each occurs is designated by different subscripts.

### Unity Power Factor, $f_p$

For the network in Fig. 20.25,

$$\begin{aligned} Y_T &= \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{R} + \frac{1}{jX_{L_p}} + \frac{1}{-jX_C} \\ &= \frac{1}{R} - j\left(\frac{1}{X_{L_p}}\right) + j\left(\frac{1}{X_C}\right) \end{aligned}$$

and 
$$Y_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right) \tag{20.27}$$

For unity power factor, the reactive component must be zero as defined by

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

Therefore, 
$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

and 
$$X_{L_p} = X_C \tag{20.28}$$



Substituting for  $X_{L_p}$  yields

$$\boxed{\frac{R_i^2 + X_L^2}{X_L} = X_C} \quad (20.29)$$

The resonant frequency,  $f_p$ , can now be determined from Eq. (20.29) as follows:

$$R_i^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C}\right)\omega L = \frac{L}{C}$$

or 
$$X_L^2 = \frac{L}{C} - R_i^2$$

with 
$$2\pi f_p L = \sqrt{\frac{L}{C} - R_i^2}$$

and 
$$f_p = \frac{1}{2\pi L} \sqrt{\frac{L}{C} - R_i^2}$$

Multiplying the top and bottom of the factor within the square root sign by  $C/L$  produces

$$f_p = \frac{1}{2\pi L} \sqrt{\frac{1 - R_i^2(C/L)}{C/L}} = \frac{1}{2\pi L \sqrt{C/L}} \sqrt{1 - \frac{R_i^2 C}{L}}$$

and 
$$\boxed{f_p = \frac{1}{2\pi \sqrt{LC}} \sqrt{1 - \frac{R_i^2 C}{L}}} \quad (20.30)$$

or 
$$\boxed{f_p = f_s \sqrt{1 - \frac{R_i^2 C}{L}}} \quad (20.31)$$

where  $f_p$  is the resonant frequency of a parallel resonant circuit (for  $F_p = 1$ ) and  $f_s$  is the resonant frequency as determined by  $X_L = X_C$  for series resonance. Note that unlike a series resonant circuit, the resonant frequency  $f_p$  is a function of resistance (in this case  $R_i$ ). Note also, however, the absence of the source resistance  $R_s$  in Eqs. (20.30) and (20.31). Since the factor  $\sqrt{1 - (R_i^2 C/L)}$  is less than 1,  $f_p$  is less than  $f_s$ . Recognize also that as the magnitude of  $R_i$  approaches zero,  $f_p$  rapidly approaches  $f_s$ .

### Maximum Impedance, $f_m$

At  $f = f_p$  the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of  $R_p$ . The frequency at which maximum impedance occurs is defined by  $f_m$  and is slightly more than  $f_p$ , as demonstrated in Fig. 20.26. The frequency  $f_m$  is determined by differentiating (calculus) the general equation for  $Z_T$  with respect to frequency and then determining the frequency at which the resulting equation is equal to zero. The algebra is quite extensive and cumbersome and is not included here. The resulting equation, however, is the following:

$$\boxed{f_m = f_s \sqrt{1 - \frac{1}{4} \left(\frac{R_i^2 C}{L}\right)}} \quad (20.32)$$

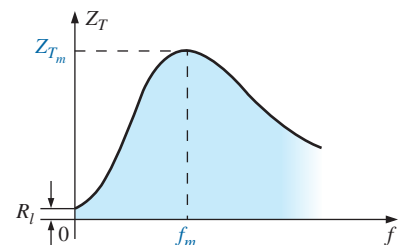


FIG. 20.26

$Z_T$  versus frequency for the parallel resonant circuit.



Note the similarities with Eq. (20.31). Since the square root factor of Eq. (20.32) is always more than the similar factor of Eq. (20.31),  $f_m$  is always closer to  $f_s$  and more than  $f_p$ . In general,

$$f_s > f_m > f_p \tag{20.33}$$

Once  $f_m$  is determined, the network in Fig. 20.25 can be used to determine the magnitude and phase angle of the total impedance at the resonance condition simply by substituting  $f = f_m$  and performing the required calculations. That is,

$$Z_{T_m} = R \parallel X_{L_p} \parallel X_C \quad f = f_m \tag{20.34}$$

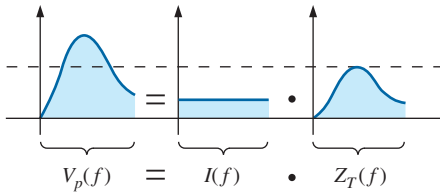
### 20.9 SELECTIVITY CURVE FOR PARALLEL RESONANT CIRCUITS

The  $Z_T$ -versus-frequency curve in Fig. 20.26 clearly reveals that a parallel resonant circuit exhibits maximum impedance at resonance ( $f_m$ ), unlike the series resonant circuit, which experiences minimum resistance levels at resonance. Note also that  $Z_T$  is approximately  $R_l$  at  $f = 0$  Hz since  $Z_T = R_s \parallel R_l \cong R_l$ .

Since the current  $I$  of the current source is constant for any value of  $Z_T$  or frequency, the voltage across the parallel circuit will have the same shape as the total impedance  $Z_T$ , as shown in Fig. 20.27.

For the parallel circuit, the resonance curve of interest is that of the voltage  $V_C$  across the capacitor. The reason for this interest in  $V_C$  derives from electronic considerations that often place the capacitor at the input to another stage of a network.

Since the voltage across parallel elements is the same,



**FIG. 20.27**  
Defining the shape of the  $V_p(f)$  curve.

$$V_C = V_p = IZ_T \tag{20.35}$$

The resonant value of  $V_C$  is therefore determined by the value of  $Z_{T_m}$  and the magnitude of the current source  $I$ .

The quality factor of the parallel resonant circuit continues to be determined by the ratio of the reactive power to the real power. That is,

$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

where  $R = R_s \parallel R_p$ , and  $V_p$  is the voltage across the parallel branches. The result is

$$Q_p = \frac{R}{X_{L_p}} = \frac{R_s \parallel R_p}{X_{L_p}} \tag{20.36a}$$

or since  $X_{L_p} = X_C$  at resonance,

$$Q_p = \frac{R_s \parallel R_p}{X_C} \tag{20.36b}$$

For the ideal current source ( $R_s = \infty \Omega$ ) or when  $R_s$  is sufficiently large compared to  $R_p$ , we can make the following approximation:

$$R = R_s \parallel R_p \cong R_p$$

and

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = \frac{(R_l^2 + X_L^2)/R_l}{(R_l^2 + X_L^2)/X_L}$$

so that

$$\boxed{Q_p = \frac{X_L}{R_l} = Q_l} \quad R_s \gg R_p \quad (20.37)$$

which is simply the quality factor  $Q_l$  of the coil.

In general, the bandwidth is still related to the resonant frequency and the quality factor by

$$\boxed{BW = f_2 - f_1 = \frac{f_r}{Q_p}} \quad (20.38)$$

The cutoff frequencies  $f_1$  and  $f_2$  can be determined using the equivalent network in Fig. 20.25 and the unity power condition for resonance. The half-power frequencies are defined by the condition that the output voltage is 0.707 times the maximum value. However, for parallel resonance with a current source driving the network, the frequency response for the driving point impedance is the same as that for the output voltage. This similarity permits defining each cutoff frequency as the frequency at which the input impedance is 0.707 times its maximum value. Since the maximum value is the equivalent resistance  $R$  in Fig. 20.25, the cutoff frequencies are associated with an impedance equal to  $0.707R$  or  $(1/\sqrt{2})R$ .

Setting the input impedance for the network in Fig. 20.25 equal to this value results in the following relationship:

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = 0.707R$$

which can be written as

$$\mathbf{Z} = \frac{1}{\frac{1}{R} \left[ 1 + jR\left(\omega C - \frac{1}{\omega L}\right) \right]} = \frac{R}{\sqrt{2}}$$

or

$$\frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{R}{\sqrt{2}}$$

and finally

$$\frac{1}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{1}{\sqrt{2}}$$

The only way the equality can be satisfied is if the magnitude of the imaginary term on the bottom left is equal to 1 because the magnitude of  $1 + j1$  must be equal to  $\sqrt{2}$ .

The following relationship, therefore, defines the cutoff frequencies for the system:

$$R\left(\omega C - \frac{1}{\omega L}\right) = 1$$

Substituting  $\omega = 2\pi f$  and rearranging results in the following quadratic equation:

$$f^2 - \frac{f}{2\pi RC} - \frac{1}{4\pi^2 LC} = 0$$

having the form  $af^2 + bf + c = 0$

with  $a = 1$   $b = -\frac{1}{2\pi RC}$  and  $c = -\frac{1}{4\pi^2 LC}$

Substituting into the equation:

$$f = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

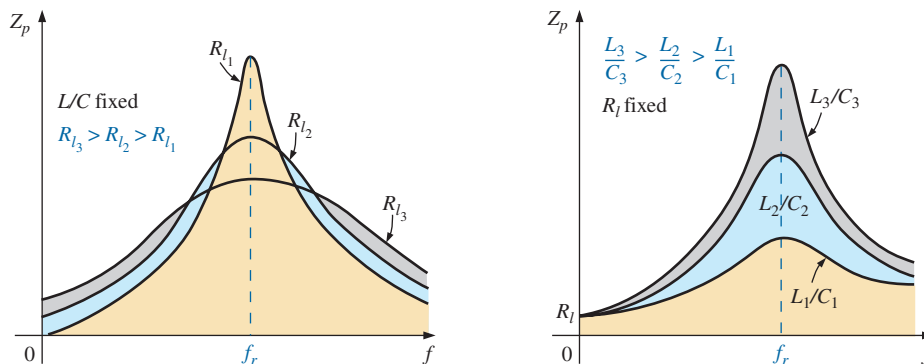
results in the following after a series of careful mathematical manipulations:

$$f_1 = \frac{1}{4\pi C} \left[ \frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right] \tag{20.39a}$$

$$f_2 = \frac{1}{4\pi C} \left[ \frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right] \tag{20.39b}$$

Since the term in the brackets of Eq. (20.39a) is always negative, simply associate  $f_1$  with the magnitude of the result.

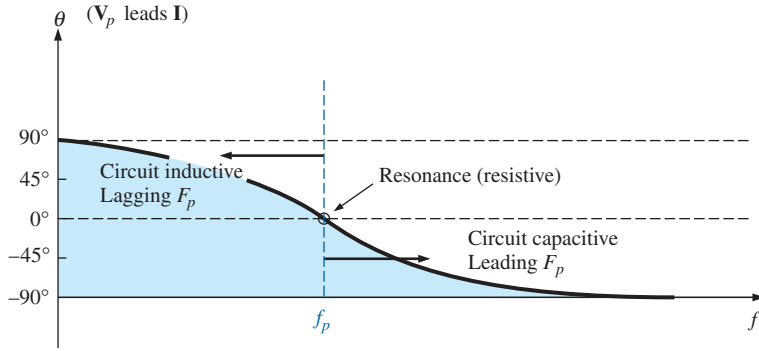
The effect of  $R_l$ ,  $L$ , and  $C$  on the shape of the parallel resonance curve, as shown in Fig. 20.28 for the input impedance, is quite similar to their effect on the series resonance curve. Whether or not  $R_l$  is zero, the parallel resonant circuit frequently appears in a network schematic as shown in Fig. 20.28.



**FIG. 20.28**  
Effect of  $R_l$ ,  $L$ , and  $C$  on the parallel resonance curve.

At resonance, an increase in  $R_l$  or a decrease in the ratio  $L/C$  results in a decrease in the resonant impedance, with a corresponding increase in the current. The bandwidth of the resonance curves is given by Eq. (20.38). For increasing  $R_l$  or decreasing  $L$  (or  $L/C$  for constant  $C$ ), the bandwidth increases as shown in Fig. 20.28.

At low frequencies, the capacitive reactance is quite high, and the inductive reactance is low. Since the elements are in parallel, the total im-


**FIG. 20.29**

Phase plot for the parallel resonant circuit.

pedance at low frequencies is therefore inductive. At high frequencies, the reverse is true, and the network is capacitive. At resonance ( $f_p$ ), the network appears resistive. These facts lead to the phase plot in Fig. 20.29. Note that it is the inverse of that appearing for the series resonant circuit because at low frequencies the series resonant circuit was capacitive and at high frequencies it was inductive.

## 20.10 EFFECT OF $Q_L \geq 10$

The content of the previous section may suggest that the analysis of parallel resonant circuits is significantly more complex than encountered for series resonant circuits. Fortunately, however, this is not the case since, for the majority of parallel resonant circuits, the quality factor of the coil  $Q_l$  is sufficiently large to permit a number of approximations that simplify the required analysis.

### Inductive Reactance, $X_{L_p}$

If we expand  $X_{L_p}$  as

$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L} = \frac{R_l^2(X_L)}{X_L(X_L)} + X_L = \frac{X_L}{Q_l^2} + X_L$$

then, for  $Q_l \geq 10$ ,  $X_L/Q_l^2 \cong 0$  compared to  $X_L$ , and

$$\boxed{X_{L_p} \cong X_L} \quad Q_l \geq 10 \quad (20.40)$$

and since resonance is defined by  $X_{L_p} = X_C$ , the resulting condition for resonance is reduced to:

$$\boxed{X_L \cong X_C} \quad Q_l \geq 10 \quad (20.41)$$

### Resonant Frequency, $f_p$ (Unity Power Factor)

We can rewrite the factor  $R_l^2 C/L$  of Eq. (20.31) as

$$\frac{R_l^2 C}{L} = \frac{1}{\frac{L}{R_l^2 C}} = \frac{1}{\frac{(\omega)}{(\omega)} \frac{L}{R_l^2 C}} = \frac{1}{\frac{\omega L}{R_l^2 \omega C}} = \frac{1}{\frac{X_L X_C}{R_l^2}}$$



and substitute Eq. (20.41) ( $X_L \cong X_C$ ):

$$\frac{1}{X_L X_C} = \frac{1}{X_L^2} = \frac{1}{Q_l^2} \frac{1}{R_l^2}$$

Eq. (20.31) then becomes

$$f_p = f_s \sqrt{1 - \frac{1}{Q_l^2}} \quad Q_l \geq 10 \quad (20.42)$$

clearly revealing that as  $Q_l$  increases,  $f_p$  becomes closer and closer to  $f_s$ .

For  $Q_l \geq 10$ ,

$$1 - \frac{1}{Q_l^2} \cong 1$$

and

$$f_p \cong f_s = \frac{1}{2\pi\sqrt{LC}} \quad Q_l \geq 10 \quad (20.43)$$

### Resonant Frequency, $f_m$ (Maximum $V_C$ )

Using the equivalency  $R_l^2 C/L = 1/Q_l^2$  derived for Eq. (20.42), Eq. (20.32) takes on the following form:

$$f_m \cong f_s \sqrt{1 - \frac{1}{4} \left( \frac{1}{Q_l^2} \right)} \quad Q_l \geq 10 \quad (20.44)$$

The fact that the negative term under the square root will always be less than that appearing in the equation for  $f_p$  reveals that  $f_m$  will always be closer to  $f_s$  than  $f_p$ .

For  $Q_l \geq 10$ , the negative term becomes very small and can be dropped from consideration, leaving:

$$f_m \cong f_s = \frac{1}{2\pi\sqrt{LC}} \quad Q_l \geq 10 \quad (20.45)$$

In total, therefore, for  $Q_l \geq 10$ ,

$$f_p \cong f_m \cong f_s \quad Q_l \geq 10 \quad (20.46)$$

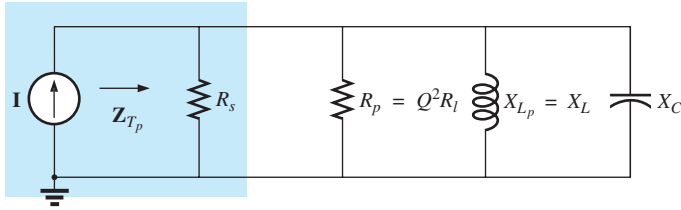
### $R_p$

$$\begin{aligned} R_p &= \frac{R_l^2 + X_L^2}{R_l} = R_l + \frac{X_L^2}{R_l} \left( \frac{R_l}{R_l} \right) = R_l + \frac{X_L^2}{R_l^2} R_l \\ &= R_l + Q_l^2 R_l = (1 + Q_l^2) R_l \end{aligned}$$

For  $Q_l \geq 10$ ,  $1 + Q_l^2 \cong Q_l^2$ , and

$$R_p \cong Q_l^2 R_l \quad Q_l \geq 10 \quad (20.47)$$

Applying the approximations just derived to the network in Fig. 20.24 results in the approximate equivalent network for  $Q_L \geq 10$  in Fig. 20.30, which is certainly a lot “cleaner” in general appearance.



**FIG. 20.30**

Approximate equivalent circuit for  $Q_L \geq 10$ .

Substituting  $Q_L = \frac{X_L}{R_l}$  into Eq. (20.47),

$$R_p \cong Q_L^2 R_l = \left( \frac{X_L}{R_l} \right)^2 R_l = \frac{X_L^2}{R_l} = \frac{X_L X_C}{R_l} = \frac{2\pi f L}{R_l (2\pi f C)}$$

and

$$R_p \cong \frac{L}{R_l C} \quad Q_L \geq 10 \quad (20.48)$$

### $Z_{T_p}$

The total impedance at resonance is now defined by

$$Z_{T_p} \cong R_s \parallel R_p = R_s \parallel Q_L^2 R_l \quad Q_L \geq 10 \quad (20.49)$$

For an ideal current source ( $R_s = \infty \Omega$ ), or if  $R_s \gg R_p$ , the equation reduces to

$$Z_{T_p} \cong Q_L^2 R_l \quad Q_L \geq 10, R_s \gg R_p \quad (20.50)$$

### $Q_p$

The quality factor is now defined by

$$Q_p = \frac{R}{X_{L_p}} \cong \frac{R_s \parallel Q_L^2 R_l}{X_L} \quad (20.51)$$

Quite obviously, therefore,  $R_s$  does have an impact on the quality factor of the network and the shape of the resonant curves.

If an ideal current source ( $R_s = \infty \Omega$ ) is used, or if  $R_s \gg R_p$ ,

$$Q_p \cong \frac{R_s \parallel Q_L^2 R_l}{X_L} = \frac{Q_L^2 R_l}{X_L} = \frac{Q_L^2}{X_L/R_l} = \frac{Q_L^2}{Q_L}$$

and

$$Q_p \cong Q_L \quad Q_L \geq 10, R_s \gg R_p \quad (20.52)$$



### BW

The bandwidth defined by  $f_p$  is

$$BW = f_2 - f_1 = \frac{f_p}{Q_p} \tag{20.53}$$

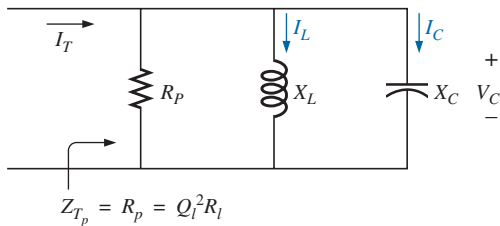
By substituting  $Q_p$  from above and performing a few algebraic manipulations, we can show that

$$BW = f_2 - f_1 \cong \frac{1}{2\pi} \left[ \frac{R_l}{L} + \frac{1}{R_s C} \right] \tag{20.54}$$

clearly revealing the impact of  $R_s$  on the resulting bandwidth. Of course, if  $R_s = \infty \Omega$  (ideal current source):

$$BW = f_2 - f_1 \cong \frac{R_l}{2\pi L} \quad R_s = \infty \Omega \tag{20.55}$$

### $I_L$ and $I_C$



**FIG. 20.31**

Establishing the relationship between  $I_C$  and  $I_L$  and the current  $I_T$ .

A portion of Fig. 20.30 is reproduced in Fig. 20.31, with  $I_T$  defined as shown.

As indicated,  $Z_{T_p}$  at resonance is  $Q_l^2 R_l$ . The voltage across the parallel network is, therefore,

$$V_C = V_L = V_R = I_T Z_{T_p} = I_T Q_l^2 R_l$$

The magnitude of the current  $I_C$  can then be determined using Ohm's law, as follows:

$$I_C = \frac{V_C}{X_C} = \frac{I_T Q_l^2 R_l}{X_C}$$

Substituting  $X_C = X_L$  when  $Q_l \geq 10$ ,

$$I_C = \frac{I_T Q_l^2 R_l}{X_L} = I_T \frac{Q_l^2}{X_L} = I_T \frac{Q_l^2}{R_l}$$

and

$$I_C \cong Q_l I_T \quad Q_l \geq 10 \tag{20.56}$$

revealing that the capacitive current is  $Q_l$  times the magnitude of the current entering the parallel resonant circuit. For large  $Q_l$ , the current  $I_C$  can be significant.

A similar derivation results in

$$I_L \cong Q_l I_T \quad Q_l \geq 10 \tag{20.57}$$

### Conclusions

The equations resulting from the application of the condition  $Q_l \geq 10$  are obviously a great deal easier to apply than those obtained earlier. It is, therefore, a condition that should be checked early in an analy-





sis to determine which approach must be applied. Although the condition  $Q_l \geq 10$  was applied throughout, many of the equations are still good approximations for  $Q_l < 10$ . For instance, if  $Q_l = 5$ ,  $X_{L_p} = (X_L/Q_l^2) + X_L = (X_L/25) + X_L = 1.04X_L$ , which is very close to  $X_L$ . In fact, for  $Q_l = 2$ ,  $X_{L_p} = (X_L/4) + X_L = 1.25X_L$ , which is not  $X_L$ , but it is only 25% off. In general, be aware that the approximate equations can be applied with good accuracy with  $Q_l < 10$ . The smaller the level of  $Q_l$ , however, the less valid the approximation. The approximate equations are certainly valid for a range of values of  $Q_l < 10$  if a rough approximation to the actual response is desired rather than one accurate to the hundredths place.

### 20.11 SUMMARY TABLE

In an effort to limit any confusion resulting from the introduction of  $f_p$  and  $f_m$  and an approximate approach dependent on  $Q_l$ , the summary in Table 20.1 was developed. You can always use the equations for any  $Q_l$ , but a proficiency in applying the approximate equations defined by  $Q_l$  will pay dividends in the long run.

For the future, the analysis of a parallel resonant network may proceed as follows:

1. Determine  $f_s$  to obtain some idea of the resonant frequency. Recall that for most situations,  $f_s$ ,  $f_m$ , and  $f_p$  will be relatively close to each other.
2. Calculate an approximate  $Q_l$  using  $f_s$  from below, and compare it to the condition  $Q_l \geq 10$ . If the condition is satisfied, the approximate approach should be the chosen path unless a high degree of accuracy is required.
3. If  $Q_l$  is less than 10, the approximate approach can be applied, but it must be understood that the smaller the level of  $Q_l$ , the less accurate the solution. However, considering the typical variations

**TABLE 20.1**  
Parallel resonant circuit ( $f_s = 1/(2\pi\sqrt{LC})$ ).

	Any $Q_l$	$Q_l \geq 10$	$Q_l \geq 10, R_s \gg Q_l^2 R_l$
$f_p$	$f_s \sqrt{1 - \frac{R_l^2 C}{L}}$	$f_s$	$f_s$
$f_m$	$f_s \sqrt{1 - \frac{1}{4} \left[ \frac{R_l^2 C}{L} \right]}$	$f_s$	$f_s$
$Z_{T_p}$	$R_s \parallel R_p = R_s \parallel \left( \frac{R_l^2 + X_L^2}{R_l} \right)$	$R_s \parallel Q_l^2 R_l$	$Q_l^2 R_l$
$Z_{T_m}$	$R_s \parallel \mathbf{Z}_{R-L} \parallel \mathbf{Z}_C$	$R_s \parallel Q_l^2 R_l$	$Q_l^2 R_l$
$Q_p$	$\frac{Z_{T_p}}{X_{L_p}} = \frac{Z_{T_p}}{X_C}$	$\frac{Z_{T_p}}{X_L} = \frac{Z_{T_p}}{X_C}$	$Q_l$
$BW$	$\frac{f_p}{Q_p}$ or $\frac{f_m}{Q_p}$	$\frac{f_p}{Q_p} = \frac{f_s}{Q_p}$	$\frac{f_p}{Q_l} = \frac{f_s}{Q_l}$
$I_L, I_C$	Network analysis	$I_L = I_C = Q_l I_T$	$I_L = I_C = Q_l I_T$

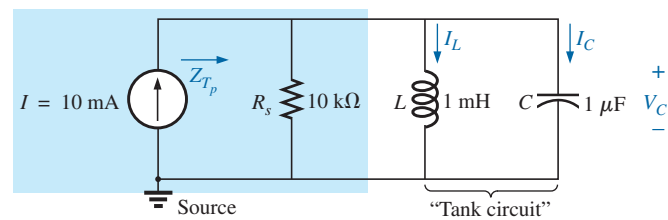


from nameplate values for many of our components and that a resonant frequency to the tenths place is seldom required, the use of the approximate approach for many practical situations is usually quite valid.

## 20.12 EXAMPLES (PARALLEL RESONANCE)

**EXAMPLE 20.6** Given the parallel network in Fig. 20.32 composed of “ideal” elements:

- Determine the resonant frequency  $f_p$ .
- Find the total impedance at resonance.
- Calculate the quality factor, bandwidth, and cutoff frequencies  $f_1$  and  $f_2$  of the system.
- Find the voltage  $V_C$  at resonance.
- Determine the currents  $I_L$  and  $I_C$  at resonance.



**FIG. 20.32**

Example 20.6.

### Solutions:

- The fact that  $R_l$  is zero ohms results in a very high  $Q_l (= X_L/R_l)$ , permitting the use of the following equation for  $f_p$ :

$$\begin{aligned} f_p = f_s &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1 \text{ mH})(1 \mu\text{F})}} \\ &= \mathbf{5.03 \text{ kHz}} \end{aligned}$$

- For the parallel reactive elements:

$$\mathbf{Z_L} \parallel \mathbf{Z_C} = \frac{(X_L \angle 90^\circ)(X_C \angle -90^\circ)}{+j(X_L - X_C)}$$

but  $X_L = X_C$  at resonance, resulting in a zero in the denominator of the equation and a very high impedance that can be approximated by an open circuit. Therefore,

$$\mathbf{Z_{Tp}} = R_s \parallel \mathbf{Z_L} \parallel \mathbf{Z_C} = R_s = \mathbf{10 \text{ k}\Omega}$$

- $$Q_p = \frac{R_s}{X_{Lp}} = \frac{R_s}{2\pi f_p L} = \frac{10 \text{ k}\Omega}{2\pi(5.03 \text{ kHz})(1 \text{ mH})} = \mathbf{316.41}$$

$$BW = \frac{f_p}{Q_p} = \frac{5.03 \text{ kHz}}{316.41} = \mathbf{15.90 \text{ Hz}}$$

Eq. (20.39a):

$$f_1 = \frac{1}{4\pi C} \left[ \frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$\begin{aligned}
 &= \frac{1}{4\pi(1\ \mu\text{F})} \left[ \frac{1}{10\ \text{k}\Omega} - \sqrt{\frac{1}{(10\ \text{k}\Omega)^2} + \frac{4(1\ \mu\text{F})}{1\ \text{mH}}} \right] \\
 &= \mathbf{5.03\ \text{kHz}}
 \end{aligned}$$

Eq. (20.39b):

$$\begin{aligned}
 f_2 &= \frac{1}{4\pi C} \left[ \frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right] \\
 &= \mathbf{5.04\ \text{kHz}}
 \end{aligned}$$

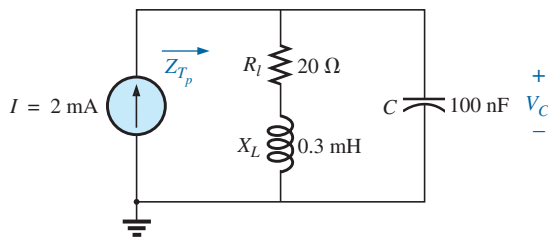
d.  $V_C = IZ_{T_p} = (10\ \text{mA})(10\ \text{k}\Omega) = \mathbf{100\ \text{V}}$

e.  $I_L = \frac{V_L}{X_L} = \frac{V_C}{2\pi f_p L} = \frac{100\ \text{V}}{2\pi(5.03\ \text{kHz})(1\ \text{mH})} = \frac{100\ \text{V}}{31.6\ \Omega} = \mathbf{3.16\ \text{A}}$

$$I_C = \frac{V_C}{X_C} = \frac{100\ \text{V}}{31.6\ \Omega} = \mathbf{3.16\ \text{A}} \quad (= Q_p I)$$

Example 20.6 demonstrates the impact of  $R_s$  on the calculations associated with parallel resonance. The source impedance is the only factor to limit the input impedance and the level of  $V_C$ .

**EXAMPLE 20.7** For the parallel resonant circuit in Fig. 20.33 with  $R_s = \infty\ \Omega$ :



**FIG. 20.33**  
Example 20.7.

- Determine  $f_s$ ,  $f_m$ , and  $f_p$ , and compare their levels.
- Calculate the maximum impedance and the magnitude of the voltage  $V_C$  at  $f_m$ .
- Determine the quality factor  $Q_p$ .
- Calculate the bandwidth.
- Compare the above results with those obtained using the equations associated with  $Q_l \geq 10$ .

**Solutions:**

a.  $f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3\ \text{mH})(100\ \text{nF})}} = \mathbf{29.06\ \text{kHz}}$

$$\begin{aligned}
 f_m &= f_s \sqrt{1 - \frac{1}{4} \left[ \frac{R_l^2 C}{L} \right]} \\
 &= (29.06\ \text{kHz}) \sqrt{1 - \frac{1}{4} \left[ \frac{(20\ \Omega)^2 (100\ \text{nF})}{0.3\ \text{mH}} \right]} \\
 &= \mathbf{25.58\ \text{kHz}}
 \end{aligned}$$



$$f_p = f_s \sqrt{1 - \frac{R_l^2 C}{L}} = (29.06 \text{ kHz}) \sqrt{1 - \left[ \frac{(20 \Omega)^2 (100 \text{ nF})}{0.3 \text{ mH}} \right]}$$

$$= \mathbf{27.06 \text{ kHz}}$$

Both  $f_m$  and  $f_p$  are less than  $f_s$ , as predicted. In addition,  $f_m$  is closer to  $f_s$  than  $f_p$ , as forecast.  $f_m$  is about 0.5 kHz less than  $f_s$ , whereas  $f_p$  is about 2 kHz less. The differences among  $f_s$ ,  $f_m$ , and  $f_p$  suggest a low  $Q$  network.

b.  $\mathbf{Z}_{T_m} = (R_l + jX_L) \parallel -jX_C$  at  $f = f_m$

$$X_L = 2\pi f_m L = 2\pi(28.58 \text{ kHz})(0.3 \text{ mH}) = 53.87 \Omega$$

$$X_C = \frac{1}{2\pi f_m C} = \frac{1}{2\pi(28.58 \text{ kHz})(100 \text{ nF})} = 55.69 \Omega$$

$$R_l + jX_L = 20 \Omega + j53.87 \Omega = 57.46 \Omega \angle 69.63^\circ$$

$$\mathbf{Z}_{T_m} = \frac{(57.46 \Omega \angle 69.63^\circ)(55.69 \Omega \angle -90^\circ)}{20 \Omega + j53.87 \Omega - j55.69 \Omega}$$

$$= \mathbf{159.34 \Omega \angle -15.17^\circ}$$

$$V_{C_{\max}} = IZ_{T_m} = (2 \text{ mA})(159.34 \Omega) = \mathbf{318.68 \text{ mV}}$$

c.  $R_s = \infty \Omega$ ; therefore,

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l}$$

$$= \frac{2\pi(27.06 \text{ kHz})(0.3 \text{ mH})}{20 \Omega} = \frac{51 \Omega}{20 \Omega} = \mathbf{2.55}$$

The low  $Q$  confirms our conclusion of part (a). The differences among  $f_s$ ,  $f_m$ , and  $f_p$  are significantly less for higher  $Q$  networks.

d.  $BW = \frac{f_p}{Q_p} = \frac{27.06 \text{ kHz}}{2.55} = \mathbf{10.61 \text{ kHz}}$

e. For  $Q_l \geq 10$ ,  $f_m = f_p = f_s = \mathbf{29.06 \text{ kHz}}$

$$Q_p = Q_l = \frac{2\pi f_s L}{R_l} = \frac{2\pi(29.06 \text{ kHz})(0.3 \text{ mH})}{20 \Omega} = \mathbf{2.74}$$

(versus 2.55 above)

$$Z_{T_p} = Q_l^2 R_l = (2.74)^2 \cdot 20 \Omega = \mathbf{150.15 \Omega \angle 0^\circ}$$

(versus  $159.34 \Omega \angle -15.17^\circ$  above)

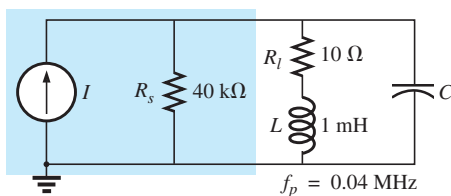
$$V_{C_{\max}} = IZ_{T_p} = (2 \text{ mA})(150.15 \Omega) = \mathbf{300.3 \text{ mV}}$$

(versus 318.68 mV above)

$$BW = \frac{f_p}{Q_p} = \frac{29.06 \text{ kHz}}{2.74} = \mathbf{10.61 \text{ kHz}}$$

(versus 10.61 kHz above)

The results reveal that, even for a relatively low  $Q$  system, the approximate solutions are still close compared to those obtained using the full equations. The primary difference is between  $f_s$  and  $f_p$  (about 7%), with the difference between  $f_s$  and  $f_m$  at less than 2%. For the future, using  $f_s$  to determine  $Q_l$  will certainly provide a measure of  $Q_l$  that can be used to determine whether the approximate approach is appropriate.



**FIG. 20.34**  
Example 20.8.

**EXAMPLE 20.8** For the network in Fig. 20.34 with  $f_p$  provided:

- Determine  $Q_l$ .
- Determine  $R_p$ .



- Calculate  $Z_{T_p}$ .
- Find  $C$  at resonance.
- Find  $Q_p$ .
- Calculate the  $BW$  and cutoff frequencies.

**Solutions:**

$$a. Q_l = \frac{X_L}{R_l} = \frac{2\pi f_p L}{R_l} = \frac{2\pi(0.04 \text{ MHz})(1 \text{ mH})}{10 \Omega} = 25.12$$

- $Q_l \geq 10$ . Therefore,

$$R_p \cong Q_l^2 R_l = (25.12)^2 (10 \Omega) = 6.31 \text{ k}\Omega$$

$$c. Z_{T_p} = R_s \parallel R_p = 40 \text{ k}\Omega \parallel 6.31 \text{ k}\Omega = 5.45 \text{ k}\Omega$$

- $Q_l \geq 10$ . Therefore,

$$f_p \cong \frac{1}{2\pi\sqrt{LC}}$$

$$\text{and } C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (0.04 \text{ MHz})^2 (1 \text{ mH})} = 15.83 \text{ nF}$$

- $Q_l \geq 10$ . Therefore,

$$Q_p = \frac{Z_{T_p}}{X_L} = \frac{R_s \parallel Q_l^2 R_l}{2\pi f_p L} = \frac{5.45 \text{ k}\Omega}{2\pi(0.04 \text{ MHz})(1 \text{ mH})} = 21.68$$

$$f. BW = \frac{f_p}{Q_p} = \frac{0.04 \text{ MHz}}{21.68} = 1.85 \text{ kHz}$$

$$f_1 = \frac{1}{4\pi C} \left[ \frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$= \frac{1}{4\pi(15.9 \text{ nF})} \left[ \frac{1}{5.45 \text{ k}\Omega} - \sqrt{\frac{1}{(5.45 \text{ k}\Omega)^2} + \frac{4(15.9 \text{ nF})}{1 \text{ mH}}} \right]$$

$$= 5.005 \times 10^6 [183.486 \times 10^{-6} - 7.977 \times 10^{-3}]$$

$$= 5.005 \times 10^6 [-7.794 \times 10^{-3}]$$

$$= 39 \text{ kHz} \quad (\text{ignoring the negative sign})$$

$$f_2 = \frac{1}{4\pi C} \left[ \frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$= 5.005 \times 10^6 [183.486 \times 10^{-6} + 7.977 \times 10^{-3}]$$

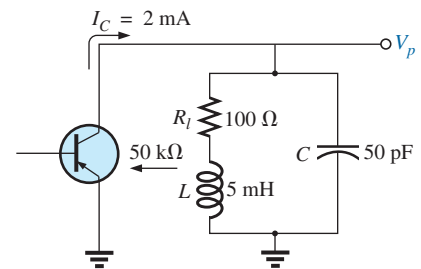
$$= 5.005 \times 10^6 [8.160 \times 10^{-3}]$$

$$= 40.84 \text{ kHz}$$

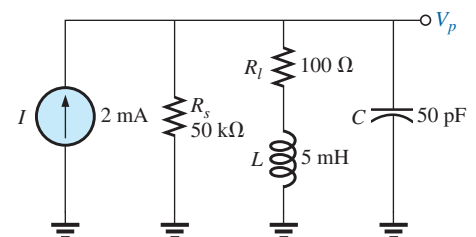
Note that  $f_2 - f_1 = 40.84 \text{ kHz} - 39 \text{ kHz} = 1.84 \text{ kHz}$ , confirming our solution for the bandwidth above. Note also that the bandwidth is not symmetrical about the resonant frequency, with 1 kHz below and 840 Hz above.

**EXAMPLE 20.9** The equivalent network for the transistor configuration in Fig. 20.35 is provided in Fig. 20.36.

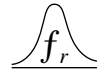
- Find  $f_p$ .
- Determine  $Q_p$ .
- Calculate the  $BW$ .
- Determine  $V_p$  at resonance.
- Sketch the curve of  $V_C$  versus frequency.



**FIG. 20.35**  
Example 20.9.



**FIG. 20.36**  
Equivalent network for the transistor configuration in Fig. 20.35.



**Solutions:**

a.  $f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5\text{ mH})(50\text{ pF})}} = 318.31\text{ kHz}$

$X_L = 2\pi f_s L = 2\pi(318.31\text{ kHz})(5\text{ mH}) = 10\text{ k}\Omega$

$Q_l = \frac{X_L}{R_l} = \frac{10\text{ k}\Omega}{100\text{ k}\Omega} = 100 > 10$

Therefore,  $f_p = f_s = \mathbf{318.31\text{ kHz}}$ . Using Eq. (20.31) results in  $\cong 318.5\text{ kHz}$ .

b.  $Q_p = \frac{R_s \parallel R_p}{X_L}$

$R_p = Q_l^2 R_l = (100)^2 100\ \Omega = 1\text{ M}\Omega$

$Q_p = \frac{50\text{ k}\Omega \parallel 1\text{ M}\Omega}{10\text{ k}\Omega} = \frac{47.62\text{ k}\Omega}{10\text{ k}\Omega} = \mathbf{4.76}$

Note the drop in  $Q$  from  $Q_l = 100$  to  $Q_p = 4.76$  due to  $R_s$ .

c.  $BW = \frac{f_p}{Q_p} = \frac{318.31\text{ kHz}}{4.76} = \mathbf{66.87\text{ kHz}}$

On the other hand,

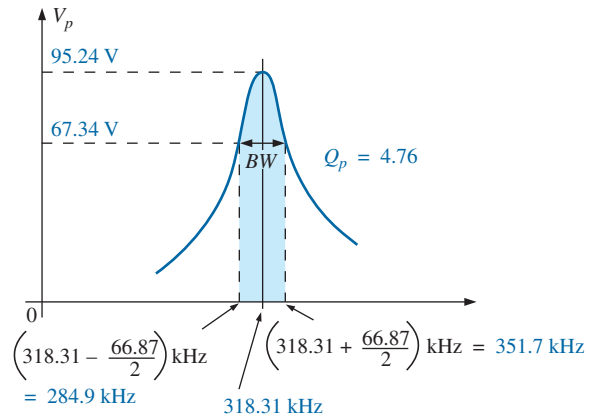
$$BW = \frac{1}{2\pi} \left( \frac{R_l}{L} + \frac{1}{R_s C} \right) = \frac{1}{2\pi} \left[ \frac{100\ \Omega}{5\text{ mH}} + \frac{1}{(50\text{ k}\Omega)(50\text{ pF})} \right]$$

$$= \mathbf{66.85\text{ kHz}}$$

compares very favorably with the above solution.

d.  $V_p = IZ_{T_p} = (2\text{ mA})(R_s \parallel R_p) = (2\text{ mA})(47.62\text{ k}\Omega) = \mathbf{95.24\text{ V}}$

e. See Fig. 20.37.



**FIG. 20.37**

Resonance curve for the network in Fig. 20.36.

**EXAMPLE 20.10** Repeat Example 20.9, but ignore the effects of  $R_s$ , and compare results.

**Solutions:**

a.  $f_p$  is the same, **318.31 kHz**.

b. For  $R_s = \infty\ \Omega$ ,

$Q_p = Q_l = \mathbf{100}$  (versus 4.76)

c.  $BW = \frac{f_p}{Q_p} = \frac{318.31\text{ kHz}}{100} = \mathbf{3.18\text{ kHz}}$  (versus 66.87 kHz)



$$\begin{aligned} \text{d. } Z_{T_p} &= R_p = 1 \text{ M}\Omega \quad (\text{versus } 47.62 \text{ k}\Omega) \\ V_p &= IZ_{T_p} = (2 \text{ mA})(1 \text{ M}\Omega) = \mathbf{2000 \text{ V}} \quad (\text{versus } 95.24 \text{ V}) \end{aligned}$$

The results obtained clearly reveal that the source resistance can have a significant impact on the response characteristics of a parallel resonant circuit.

**EXAMPLE 20.11** Design a parallel resonant circuit to have the response curve in Fig. 20.38 using a 1 mH, 10  $\Omega$  inductor and a current source with an internal resistance of 40 k $\Omega$ .

**Solution:**

$$BW = \frac{f_p}{Q_p}$$

Therefore,

$$Q_p = \frac{f_p}{BW} = \frac{50,000 \text{ Hz}}{2500 \text{ Hz}} = \mathbf{20}$$

$$X_L = 2\pi f_p L = 2\pi(50 \text{ kHz})(1 \text{ mH}) = 314 \Omega$$

and 
$$Q_l = \frac{X_L}{R_l} = \frac{314 \Omega}{10 \Omega} = \mathbf{31.4}$$

$$R_p = Q_l^2 R = (31.4)^2 (10 \Omega) = \mathbf{9859.6 \Omega}$$

$$Q_p = \frac{R}{X_L} = \frac{R_s \parallel 9859.6 \Omega}{314 \Omega} = 20 \quad (\text{from above})$$

so that 
$$\frac{(R_s)(9859.6)}{R_s + 9859.6} = 6280$$

resulting in 
$$R_s = 17.298 \text{ k}\Omega$$

However, the source resistance was given as 40 k $\Omega$ . We must therefore add a parallel resistor ( $R'$ ) that will reduce the 40 k $\Omega$  to approximately 17.298 k $\Omega$ ; that is,

$$\frac{(40 \text{ k}\Omega)(R')}{40 \text{ k}\Omega + R'} = 17.298 \text{ k}\Omega$$

Solving for  $R'$ :

$$R' = \mathbf{30.48 \text{ k}\Omega}$$

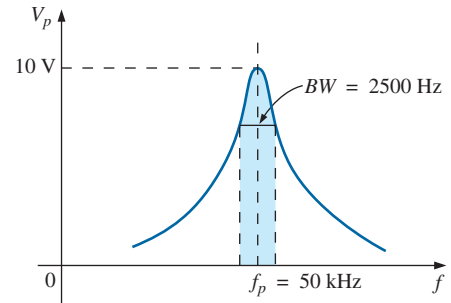
The closest commercial value is **30 k $\Omega$** . At resonance,  $X_L = X_C$ , and

$$\begin{aligned} X_C &= \frac{1}{2\pi f_p C} \\ C &= \frac{1}{2\pi f_p X_C} = \frac{1}{2\pi(50 \text{ kHz})(314 \Omega)} \end{aligned}$$

and  $C \cong \mathbf{0.01 \mu\text{F}}$  (commercially available)

$$\begin{aligned} Z_{T_p} &= R_s \parallel Q_l^2 R_l \\ &= 17.298 \text{ k}\Omega \parallel 9859.6 \Omega \\ &= 6.28 \text{ k}\Omega \end{aligned}$$

with 
$$V_p = IZ_{T_p}$$

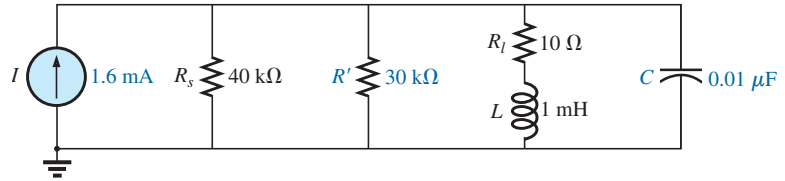


**FIG. 20.38**  
Example 20.11.



and 
$$I = \frac{V_p}{Z_{r_p}} = \frac{10 \text{ V}}{6.28 \text{ k}\Omega} \cong 1.6 \text{ mA}$$

The network appears in Fig. 20.39.



**FIG. 20.39**

*Network designed to meet the criteria in Fig. 20.38.*

## 20.13 APPLICATIONS

### Stray Resonance

Stray resonance, like stray capacitance and inductance and unexpected resistance levels, can occur in totally unexpected situations and can severely affect the operation of a system. All that is required to produce stray resonance is, for example, a level of capacitance introduced by parallel wires or copper leads on a printed circuit board, or simply two parallel conductive surfaces with residual charge and inductance levels associated with any conductor or components such as tape recorder heads, transformers, and so on, that provide the elements necessary for a resonance effect. In fact, this resonance effect is a very common effect in a cassette tape recorder. The play/record head is a coil that can act like an inductor and an antenna. Combine this factor with the stray capacitance and real capacitance in the network to form the tuning network, and the tape recorder with the addition of a semiconductor diode can respond like an AM radio. As you plot the frequency response of any transformer, you normally find a region where the response has a peaking effect (look ahead at Fig. 25.21). This peaking is due solely to the inductance of the coils of the transformer and the stray capacitance between the wires.

In general, any time you see an unexpected peaking in the frequency response of an element or a system, it is normally caused by a resonance condition. If the response has a detrimental effect on the overall operation of the system, a redesign may be in order, or a filter can be added that will block the frequencies that result in the resonance condition. Of course, when you add a filter composed of inductors and/or capacitors, you must be careful that you don't add another unexpected resonance condition. It is a problem that can be properly weighed only by constructing the system and exposing it to the full range of tests.

### Graphic and Parametric Equalizers

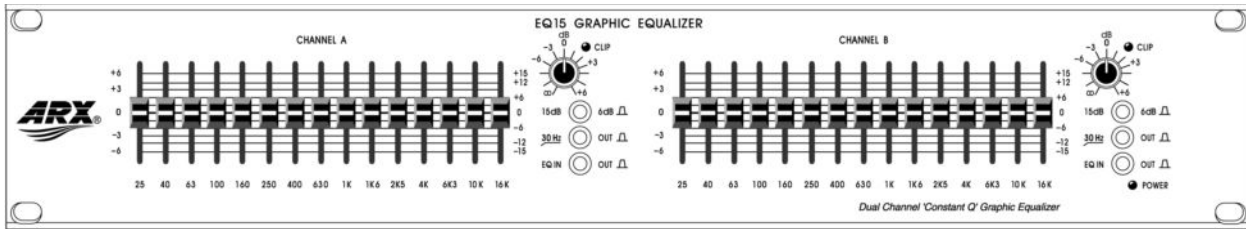
We have all noticed at one time or another that the music we hear in a concert hall doesn't quite sound the same when we play a recording of it on our home entertainment center. Even after we check the specifications of the speakers and amplifiers and find that both are nearly perfect (and the most expensive we can afford), the sound is still not what it should be. In general, we are experiencing the effects of the local environmental characteristics on the sound waves. Some typical problems are hard walls



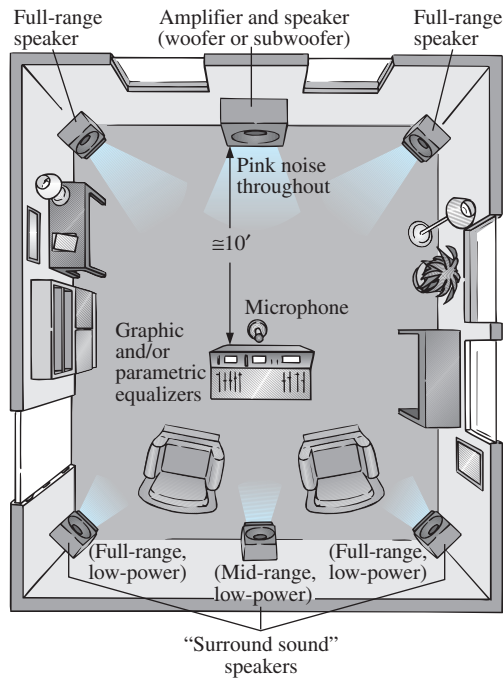


or floors (stone, cement) that make high frequencies sound louder. Curtains and rugs, on the other hand, absorb high frequencies. The shape of the room and the placement of the speakers and furniture also affect the sound that reaches our ears. Another criterion is the echo or reflection of sound that occurs in the room. Concert halls are designed very carefully with their vaulted ceilings and curved walls to allow a certain amount of echo. Even the temperature and humidity characteristics of the surrounding air affect the quality of the sound. It is certainly impossible, in most cases, to redesign your listening area to match a concert hall, but with the proper use of electronic systems you can develop a response that has all the qualities that you want from a home entertainment center.

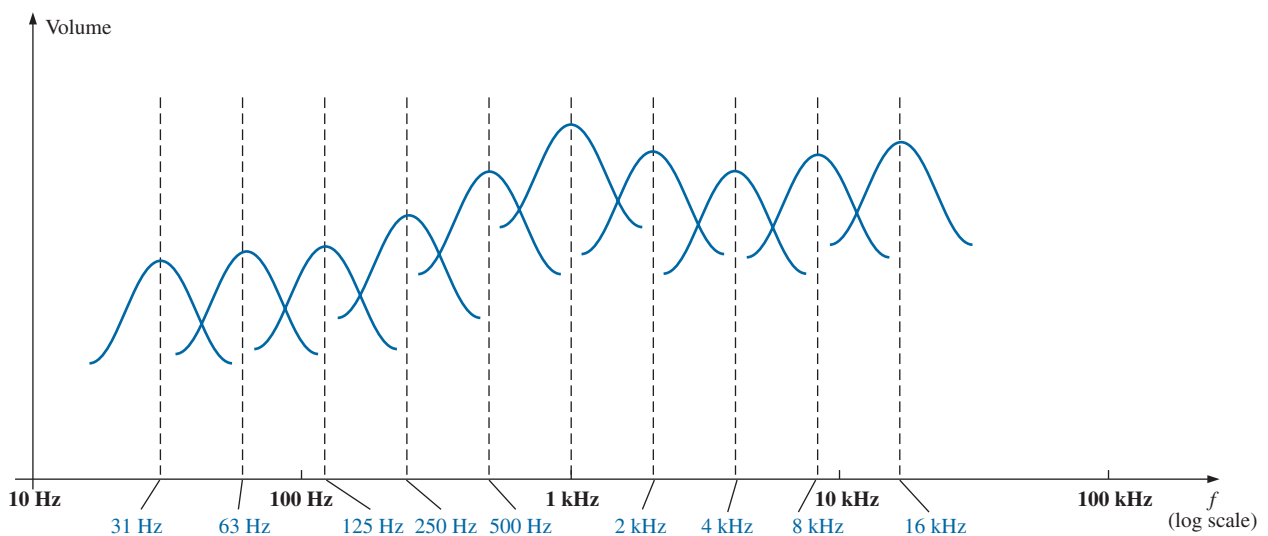
For a quality system, a number of steps can be taken: *characterization and digital delay (surround sound)* and *proper speaker and amplifier selection and placement*. Characterization is a process whereby a thorough sound absorption check of the room is performed and the frequency response determined. A *graphic equalizer* such as appearing in Fig. 20.40(a) is then used to make the response “flat” for the full range of frequencies. In other words, the room is made to appear as though all the frequencies receive equal amplification in the listening area. For instance, if the room is fully carpeted with full draping curtains, there is a considerable amount of high-frequency absorption, requiring that the high frequencies have additional amplification to match the sound levels of the mid and low frequencies. To *characterize* the typical rectangular-shaped room, a setup such as shown in Fig. 20.40(b) may be used. The amplifier and speakers are placed in the center of one wall, with additional speakers in the corners of the room facing the reception area. A mike is then placed in the reception area about 10 ft from the amplifier and centered between the two other speakers. A *pink noise* is then sent out from a spectrum analyzer (often an integral part of the graphic equalizer) to the amplifier and speakers. Pink noise is actually a square-wave signal whose amplitude and frequency can be controlled. A square-wave signal was chosen because a Fourier breakdown of a square-wave signal results in a broad range of frequencies for the system to check. You will find in Chapter 24 that a square wave can be constructed of an infinite series of sine waves of different frequencies. Once the proper volume of pink noise is established, the spectrum analyzer can be used to set the response of each slide band to establish the desired flat response. The center frequencies for the slides of the graphic equalizer in Fig. 20.40(a) are provided in Fig. 20.40(c), along with the frequency response for a number of adjoining frequencies evenly spaced on a logarithmic scale. Note that each center frequency is actually the resonant frequency for that slide. The design is such that each slide can control the volume associated with that frequency, but the bandwidth and frequency response stay fairly constant. A good spectrum analyzer has each slide set against a decibel (dB) scale (decibels are discussed in detail in Chapter 21). The decibel scale simply establishes a scale for the comparison of audio levels. At a normal listening level, usually a change of about 3 dB is necessary for the audio change to be detectable by the human ear. At low levels of sound, a 2 dB change may be detectable, but at loud sounds probably a 4 dB change would be necessary for the change to be noticed. These are not strict laws but guidelines commonly used by audio technicians. For the room in question, the mix of settings may be as shown in Fig. 20.40(c). Once set, the slides are not touched again. A flat response has been established for the room for the full audio range so that every sound or type of music is covered.



(a)



(b)



(c)

**FIG. 20.40**

(a) Dual-channel 15-band “Constant  $Q$ ” graphic equalizer (Courtesy of ARX Systems.); (b) setup; (c) frequency response.

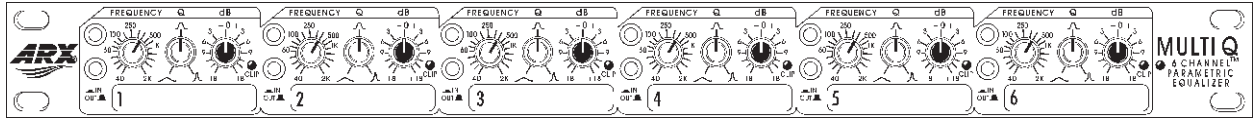


FIG. 20.41

Six-channel parametric equalizer.  
(Courtesy of ARX Systems.)

A *parametric equalizer* such as appearing in Fig. 20.41 is similar to a *graphic equalizer*; but instead of separate controls for the individual frequency ranges, it uses three basic controls over three or four broader frequency ranges. The typical controls—the *gain*, *center frequency*, and *bandwidth*—are typically available for the *low*-, *mid*-, and *high*-frequency ranges. Each is fundamentally an independent control; that is, a change in one can be made without affecting the other two. For the parametric equalizer in Fig. 20.41, each of the six channels has a frequency control switch which, in conjunction with the  $f \times 10$  switch, gives a range of center frequencies from 40 Hz through 16 kHz. It has controls for *BW* (“*Q*”) from 3 octaves to  $\frac{1}{20}$  octave, and  $\pm 18$  dB cut and boost. Some like to refer to the parametric equalizer as a *sophisticated tone control* and actually use them to enrich the sound after the flat response has been established by the graphic equalizer. The effect achieved with a standard tone control knob is sometimes referred to as “boring” compared to the effect established by a good parametric equalizer, primarily because the former can control only the volume and not the bandwidth or center frequency. In general, graphic equalizers establish the important *flat response* while parametric equalizers are adjusted to provide the *type* and *quality* of sound you like to hear. You can “notch out” the frequencies that bother you and remove tape “hiss” and the “sharpness” often associated with CDs.

One characteristic of concert halls that is more difficult to fake is the fullness of sound that concert halls are able to provide. In the concert hall, you have the direct sound from the instruments and the reflection of sound off the walls and the vaulted ceilings which were all carefully designed expressly for this purpose. Any reflection results in a delay in the sound waves reaching the ear, creating the fullness effect. Through digital delay, speakers can be placed to the back and side of a listener to establish the surround sound effect. In general, the delay speakers are much lower in wattage, with 20 W speakers typically used with a 100 W system. The echo response is one reason that people often like to play their stereos louder than they should for normal hearing. By playing the stereo louder, they create more echo and reflection off the walls, bringing into play some of the fullness heard at concert halls.

It is probably safe to say that any system composed of quality components, a graphic and parametric equalizer, and surround sound will have all the components necessary to have a quality reproduction of the concert hall effect.

## 20.14 COMPUTER ANALYSIS

### PSpice

**Series Resonance** This chapter provides an excellent opportunity to demonstrate what computer software programs can do for us. Imagine having to plot a detailed resonance curve with all the calculations



required for each frequency. At every frequency, the reactance of the inductive and capacitive elements changes, and the phasor operations would have to be repeated—a long and arduous task. However, with PSpice, taking a few moments to enter the circuit and establish the desired simulation results in a detailed plot in a few seconds that can have plot points every microsecond!

For the first time, the horizontal axis is in the frequency domain rather than in the time domain as in all the previous plots. For the series resonant circuit in Fig. 20.42, the magnitude of the source was chosen to produce a maximum current of  $I = 400 \text{ mV}/40 \Omega = 10 \text{ mA}$  at resonance, and the reactive elements establish a resonant frequency of

$$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(30 \text{ mH})(0.1 \mu\text{F})}} \cong 2.91 \text{ kHz}$$

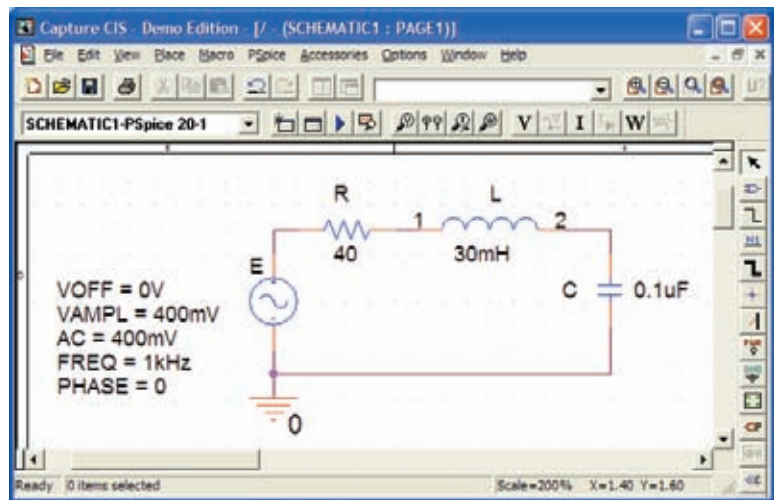


FIG. 20.42

Series resonant circuit to be analyzed using PSpice.

The quality factor is

$$Q_l = \frac{X_L}{R_l} = \frac{546.64 \Omega}{40 \Omega} \cong 13.7$$

which is relatively high and should give us a nice sharp response.

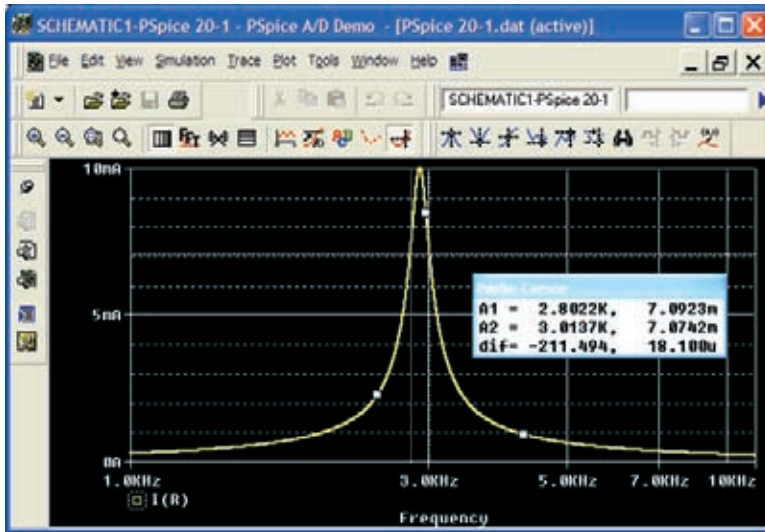
The bandwidth is

$$BW = \frac{f_s}{Q_l} = \frac{2.91 \text{ kHz}}{13.7} \cong 212 \text{ Hz}$$

which will be verified using our cursor options.

For the ac source, choose **VSIN**. All the parameters are set by double-clicking on the source symbol and entering the values in the **Property Editor** dialog box. For each, select **Name and Value** under **Display** followed by **Apply** before leaving the dialog box.

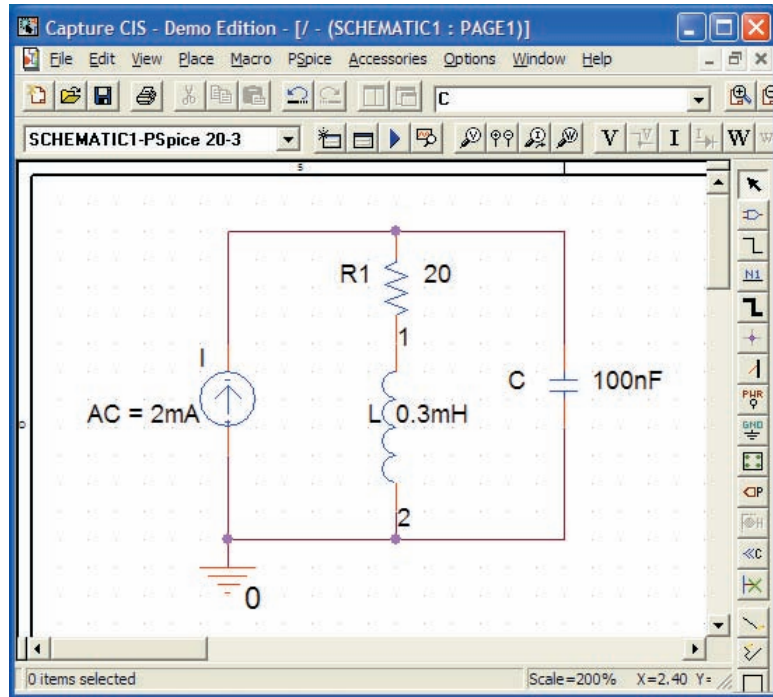
In the **Simulation Settings** dialog box, select **AC Sweep/Noise** and set the **Start Frequency** at 1 kHz, the **End Frequency** at 10 kHz, and the **Points/Decade** at 10,000. The **Logarithmic scale** and **Decade** settings remain at their default values. Choose 10,000 for **Points/Decade** to ensure a number of data points near the peak value. When the **SCHEMATIC1**


**FIG. 20.43**

Resonance curve for the current of the circuit in Fig. 20.42.

screen in Fig. 20.43 appears, **Trace-Add Trace-I(R)-OK** results in a logarithmic plot that peaks just to the left of 3 kHz. The spacing between grid lines on the **X-axis** should be increased, so select **Plot-Axis Settings-X Grid-unable Automatic-Spacing-Log-0.1-OK**. Next, select the **Toggle cursor** icon, and right-click to move the right cursor as close to 7.07 mA as possible (0.707 of the peak value to define the bandwidth) to obtain **A1** with a frequency of 2.80 kHz at a level of 7.09 mA—the best we can do with the 10,000 data points per decade. Now left-click, and place the left cursor as close to the same level as possible. The result is 3.01 kHz at a level of 7.07 mA for **A2**. The cursors were set in the order above to obtain a positive answer for the difference of the two as appearing in the third line of the **Probe Cursor** box. The resulting 211.49 Hz is an excellent match with the calculated value of 212 Hz.

**Parallel Resonance** Let us now investigate the parallel resonant circuit in Fig. 20.33 and compare the results with the longhand solution. The network appears in Fig. 20.44 using **ISRC** as the ac source voltage. Under the **Property Editor** heading, set the following values: **DC** = 0 A, **AC** = 2 mA, and **TRAN** = 0. Under **Display**, select **Do Not Display** for both **DC** and **TRAN** since they do not play a part in our analysis. In the **Simulation Settings** dialog box, select **AC Sweep/Noise**, and select the **Start Frequency** at 10 kHz since we know that it will resonate near 30 kHz. Choose the **End Frequency** as 100 kHz for a first run to see the results. Set the **Points/Decade** at 10,000 to ensure a good number of data points for the peaking region. After simulation, **Trace-Add Trace-V(C:1)-OK** results in the plot in Fig. 20.45 with a resonant frequency near 30 kHz. The selected range appears to be a good one, but the initial plot needed more grid lines on the **x-axis**, so use **Plot-Axis Settings-X-Grid-unenable Automatic-Spacing-Log-0.1-OK** to obtain a grid line at 10 kHz intervals. Next select the **Toggle cursor** pad and a left-click cursor is established on the screen. Choose the **Cursor Peak** pad to find the peak value of the curve. The result is **A1** = 319.45 mV at 28.94 kHz which is a very close match with the calculated value of 318.68 mV at 28.57 kHz for the maximum value of  $V_C$ . The bandwidth is defined at a level of 0.707(319.45 mV)

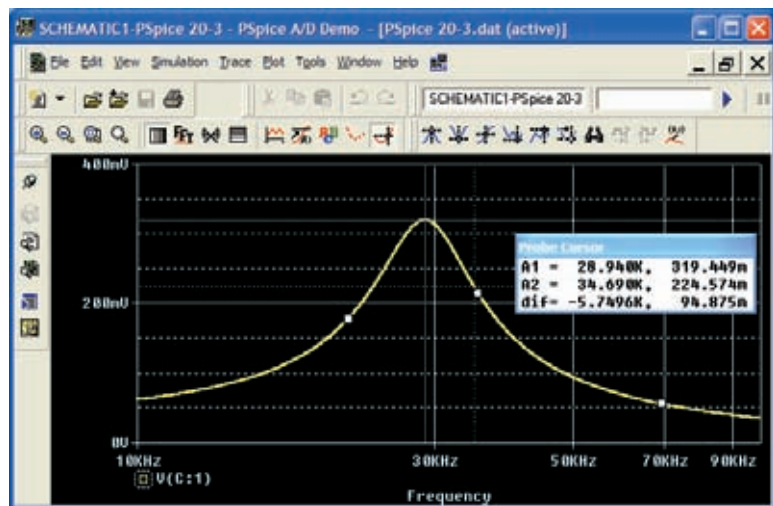


**FIG. 20.44**

*Parallel resonant network to be analyzed using PSpice.*

= 225.85 mV. Using the right-click cursor, you find that the closest value is 224.57 mV for the 10,000 points of data per decade. The resulting frequency is 34.69 kHz as shown in the **Probe Cursor** box in Fig. 20.45.

Now use the left-click cursor to find the same level to the left of the peak value so that you can determine the bandwidth. The closest that the left-click cursor can come to 225.85 mV is 224.96 mV at a frequency of 23.97 kHz. The bandwidth then appears as 10.72 kHz in the **Probe Cursor** box, comparing very well with the longhand solution of 10.68 kHz in Example 20.7.



**FIG. 20.45**

*Resonance curve for the voltage across the capacitor in Fig. 20.44.*

You can now look at the phase angle of the voltage across the parallel network to find the frequency when the network appears resistive and the phase angle is  $0^\circ$ . First use **Trace-Delete All Traces**, and call up **P(V(C:1))** followed by **OK**. The result is the plot in Fig. 20.46, revealing that the phase angle is close to  $-90^\circ$  at very high frequencies as the capacitive element with its decreasing reactance takes over the characteristics of the parallel network. At 10 kHz, the inductive element has a lower reactance than the capacitive element, and the network has a positive phase angle. Using the cursor option, move the left cursor along the horizontal axis until the phase angle is at its minimum value. As shown in Fig. 20.46, the smallest angle available with the determined data points is 49.86 mdegrees =  $0.05^\circ$  which is certainly very close to  $0^\circ$ . The corresponding frequency is 27.046 kHz which is essentially an exact match with the longhand solution of 27.051 kHz. Clearly, therefore, the frequency at which the phase angle is zero and the total impedance appears resistive is less than the frequency at which the output voltage is a maximum.

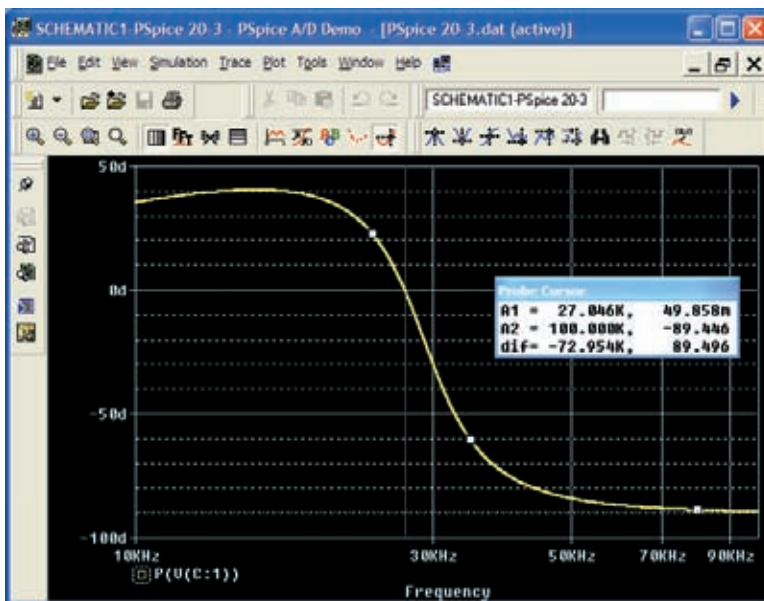


FIG. 20.46

Phase plot for the voltage  $v_C$  for the parallel resonant network in Fig. 20.44.

## Multisim

The results of Example 20.9 are now confirmed using Multisim. The network in Fig. 20.36 appears as shown in Fig. 20.47 after all the elements have been placed as described in earlier chapters. In particular, note that the frequency assigned to the 2 mA ac current source is 100 kHz. Since we have some idea that the resonant frequency is a few hundred kilohertz, it seems appropriate that the starting frequency for the plot begins at 100 kHz and extends to 1 MHz. Also, be sure that the **AC Magnitude** is set to 2 mA in the **Analysis Setup** within the **AC Current** dialog box.

For simulation, first select the sequence **Simulate-Analyses-AC Analysis** to obtain the **AC Analysis** dialog box. Set the **Start frequency** at 100 kHz, the **Stop frequency** at 1 MHz, **Sweep type** at **Decade**, **Number of points per decade** at 1000, and the **Vertical scale** at **Linear**. Under **Output variables**, select node number 1 as a **Variable for analysis**

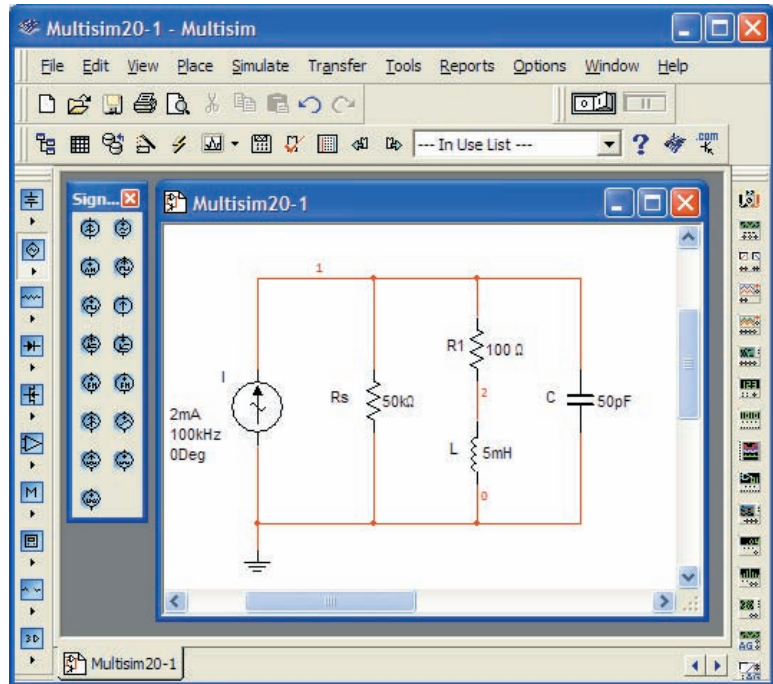


FIG. 20.47

Using Multisim to confirm the results of Example 20.9.

followed by **Simulate** to run the program. The results are the magnitude and phase plots in Fig. 20.48. Starting with the **Voltage** plot, select the **Show/Hide Grid** key, **Show/Hide Legend** key, and **Show/Hide Cursors** key. You will immediately note under the **AC Analysis** cursor box that the maximum value is 95.24 V and the minimum value is 6.94 V. By moving the cursor until you reach 95.24 V (**y1**), you can find the resonant fre-

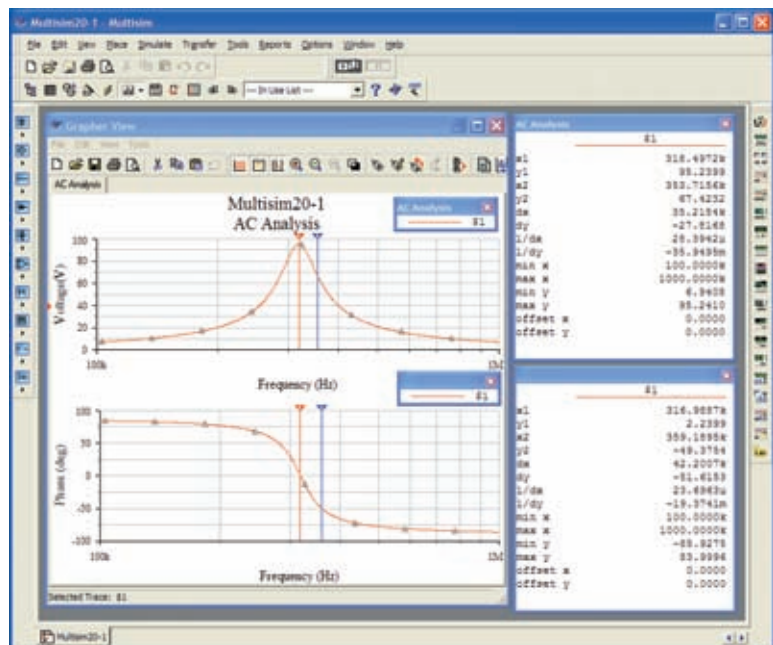


FIG. 20.48

Magnitude and phase plots for the voltage  $v_C$  of the network in Fig. 20.47.





quency. As shown in the top cursor dialog box in Fig. 20.48, this is achieved at 318.97 kHz (**x1**). The other (blue) cursor can be used to define the high cutoff frequency for the bandwidth by first calculating the 0.707 level of the output voltage. The result is  $0.707(95.24 \text{ V}) = 67.33 \text{ V}$ . The closest you can come to this level with the cursor is 67.42 V (**y2**) which defines a frequency of 353.71 kHz (**x2**). If you now use the red cursor to find the corresponding level below the resonant frequency, you find a level of 67.49 V (**y1**) at 287.08 kHz (**x1**). The resulting bandwidth is therefore  $353.71 \text{ kHz} - 287.08 \text{ kHz} = 66.63 \text{ kHz}$ .

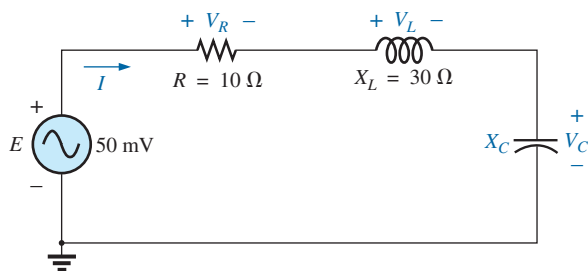
You can now determine the resonant frequency if you define resonance as that frequency that results in a phase angle of  $0^\circ$  for the output voltage. By repeating the process described above for the phase plot, set the red cursor as close to  $0^\circ$  as possible. The result is  $2.24^\circ$  (**y1**) at 316.98 kHz (**x1**), clearly revealing that the resonant frequency defined by the phase angle is less than that defined by the peak voltage. However, with a  $Q_i$  of about 100, the difference of 1.99 kHz is not significant. Also note that when the second cursor was set on 359.19 kHz, the phase angle of  $-49.37^\circ$  is close to the  $45^\circ$  expected at the cutoff frequency.

Again, the computer solution is a very close match with the longhand solution in Example 20.9 with a perfect match of 95.24 V for the peak value and only a small difference in bandwidth with 66.87 kHz in Example 20.9 and 66.63 kHz here. For the high cutoff frequency, the computer generated a result of 353.71 kHz, while the theoretical solution was 351.7 kHz. For the low cutoff frequency, the computer responded with 287.08 kHz compared to a theoretical solution of 284 kHz.

## PROBLEMS

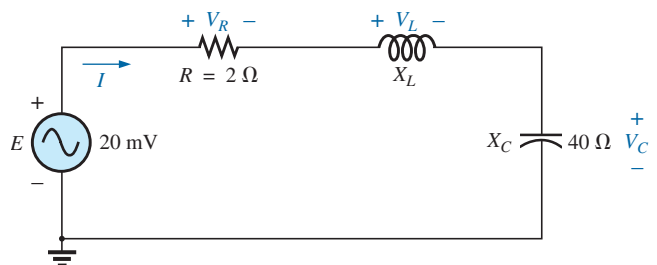
### SECTIONS 20.2 THROUGH 20.7 Series Resonance

- Find the resonant  $\omega_s$  and  $f_s$  for the series circuit with the following parameters:
  - $R = 10 \Omega$ ,  $L = 1 \text{ H}$ ,  $C = 16 \mu\text{F}$
  - $R = 300 \Omega$ ,  $L = 0.5 \text{ H}$ ,  $C = 0.16 \mu\text{F}$
  - $R = 20 \Omega$ ,  $L = 0.28 \text{ mH}$ ,  $C = 7.46 \mu\text{F}$
- For the series circuit in Fig. 20.49:
  - Find the value of  $X_C$  for resonance.
  - Determine the total impedance of the circuit at resonance.
  - Find the magnitude of the current  $I$ .
  - Calculate the voltages  $V_R$ ,  $V_L$ , and  $V_C$  at resonance. How are  $V_L$  and  $V_C$  related? How does  $V_R$  compare to the applied voltage  $E$ ?
  - What is the quality factor of the circuit? Is it a high or low  $Q$  circuit?
  - What is the power dissipated by the circuit at resonance?



**FIG. 20.49**  
Problem 2.

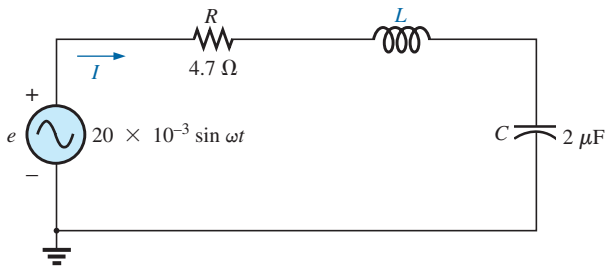
- For the series circuit in Fig. 20.50:
  - Find the value of  $X_L$  for resonance.
  - Determine the magnitude of the current  $I$  at resonance.
  - Find the voltages  $V_R$ ,  $V_L$ , and  $V_C$  at resonance, and compare their magnitudes.
  - Determine the quality factor of the circuit. Is it a high or low  $Q$  circuit?
  - If the resonant frequency is 5 kHz, determine the value of  $L$  and  $C$ .
  - Find the bandwidth of the response if the resonant frequency is 5 kHz.
  - What are the low and high cutoff frequencies?



**FIG. 20.50**  
Problem 3.



4. For the circuit in Fig. 20.51:
  - a. Find the value of  $L$  in millihenries if the resonant frequency is 1800 Hz.
  - b. Calculate  $X_L$  and  $X_C$ . How do they compare?
  - c. Find the magnitude of the current  $I_{rms}$  at resonance.
  - d. Find the power dissipated by the circuit at resonance.
  - e. What is the apparent power delivered to the system at resonance?
  - f. What is the power factor of the circuit at resonance?
  - g. Calculate the  $Q$  of the circuit and the resulting bandwidth.
  - h. Find the cutoff frequencies, and calculate the power dissipated by the circuit at these frequencies.



**FIG. 20.51**  
Problem 4.

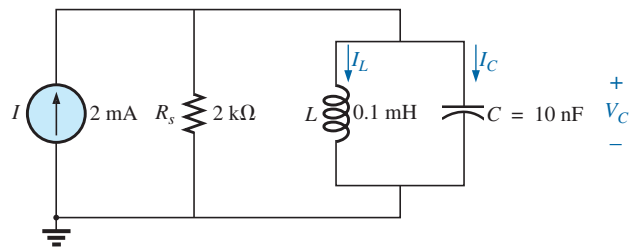
5. a. Find the bandwidth of a series resonant circuit having a resonant frequency of 6000 Hz and a  $Q_s$  of 15.
- b. Find the cutoff frequencies.
- c. If the resistance of the circuit at resonance is 3  $\Omega$ , what are the values of  $X_L$  and  $X_C$  in ohms?
- d. What is the power dissipated at the half-power frequencies if the maximum current flowing through the circuit is 0.5 A?
6. A series circuit has a resonant frequency of 10 kHz. The resistance of the circuit is 5  $\Omega$ , and  $X_C$  at resonance is 200  $\Omega$ .
  - a. Find the bandwidth.
  - b. Find the cutoff frequencies.
  - c. Find  $Q_s$ .
  - d. If the input voltage is 30 V  $\angle 0^\circ$ , find the voltage across the coil and capacitor in phasor form.
  - e. Find the power dissipated at resonance.
7. a. The bandwidth of a series resonant circuit is 200 Hz. If the resonant frequency is 2000 Hz, what is the value of  $Q_s$  for the circuit?
- b. If  $R = 2 \Omega$ , what is the value of  $X_L$  at resonance?
- c. Find the value of  $L$  and  $C$  at resonance.
- d. Find the cutoff frequencies.
8. The cutoff frequencies of a series resonant circuit are 5400 Hz and 6000 Hz.
  - a. Find the bandwidth of the circuit.
  - b. If  $Q_s$  is 9.5, find the resonant frequency of the circuit.
  - c. If the resistance of the circuit is 2  $\Omega$ , find the value of  $X_L$  and  $X_C$  at resonance.
  - d. Find the value of  $L$  and  $C$  at resonance.
- \*9. Design a series resonant circuit with an input voltage of 5 V  $\angle 0^\circ$  to have the following specifications:
  - a. A peak current of 500 mA at resonance
  - b. A bandwidth of 120 Hz

- c. A resonant frequency of 8400 Hz
- d. Find the value of  $L$  and  $C$  and the cutoff frequencies.

- \*10. Design a series resonant circuit to have a bandwidth of 400 Hz using a coil with a  $Q_l$  of 20 and a resistance of 2  $\Omega$ . Find the values of  $L$  and  $C$  and the cutoff frequencies.
- \*11. A series resonant circuit is to resonate at  $\omega_s = 2\pi \times 10^6$  rad/s and draw 20 W from a 120 V source at resonance. If the fractional bandwidth is 0.16:
  - a. Determine the resonant frequency in hertz.
  - b. Calculate the bandwidth in hertz.
  - c. Determine the values of  $R$ ,  $L$ , and  $C$ .
  - d. Find the resistance of the coil if  $Q_l = 80$ .
- \*12. A series resonant circuit will resonate at a frequency of 1 MHz with a fractional bandwidth of 0.2. If the quality factor of the coil at resonance is 12.5 and its inductance is 100  $\mu$ H, determine the following:
  - a. The resistance of the coil.
  - b. The additional resistance required to establish the indicated fractional bandwidth.
  - c. The required value of capacitance.

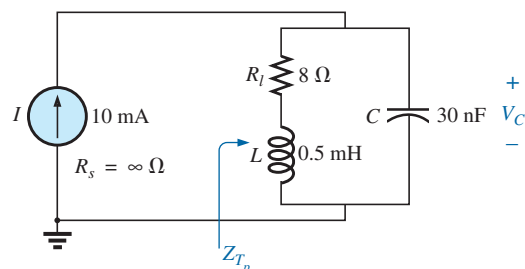
**SECTIONS 20.8 THROUGH 20.12 Parallel Resonance**

13. For the “ideal” parallel resonant circuit in Fig. 20.52:
  - a. Determine the resonant frequency ( $f_p$ ).
  - b. Find the voltage  $V_C$  at resonance.
  - c. Determine the currents  $I_L$  and  $I_C$  at resonance.
  - d. Find  $Q_p$ .



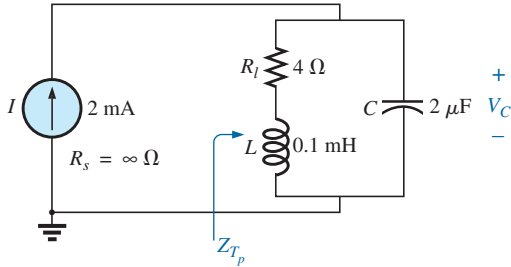
**FIG. 20.52**  
Problem 13.

14. For the parallel resonant network in Fig. 20.53:
  - a. Calculate  $f_s$ .
  - b. Determine  $Q_l$  using  $f = f_s$ . Can the approximate approach be applied?
  - c. Determine  $f_p$  and  $f_m$ .
  - d. Calculate  $X_L$  and  $X_C$  using  $f_p$ . How do they compare?



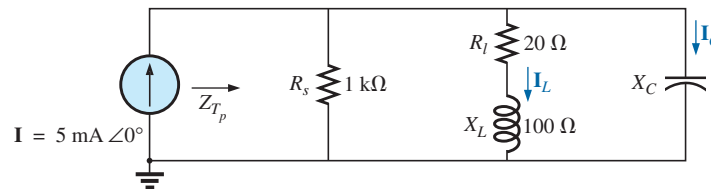
**FIG. 20.53**  
Problem 14.

- e. Find the total impedance at resonance ( $f_p$ ).
  - f. Calculate  $V_C$  at resonance ( $f_p$ ).
  - g. Determine  $Q_p$  and the  $BW$  using  $f_p$ .
  - h. Calculate  $I_L$  and  $I_C$  at  $f_p$ .
15. Repeat Problem 14 for the network in Fig. 20.54.



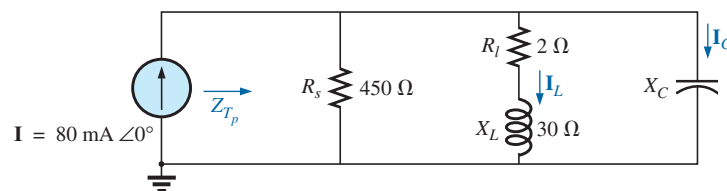
**FIG. 20.54**  
Problem 15.

16. For the network in Fig. 20.55:
- a. Find the value of  $X_C$  at resonance ( $f_p$ ).
  - b. Find the total impedance  $Z_{T_p}$  at resonance ( $f_p$ ).
  - c. Find the currents  $I_L$  and  $I_C$  at resonance ( $f_p$ ).
  - d. If the resonant frequency is 20,000 Hz, find the value of  $L$  and  $C$  at resonance.
  - e. Find  $Q_p$  and the  $BW$ .



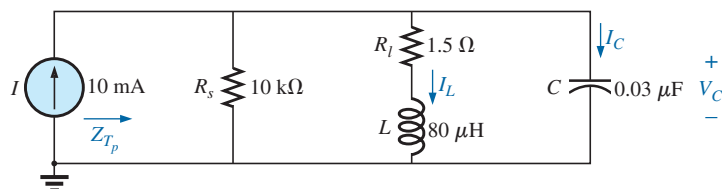
**FIG. 20.55**  
Problem 16.

17. Repeat Problem 16 for the network in Fig. 20.56.



**FIG. 20.56**  
Problem 17.

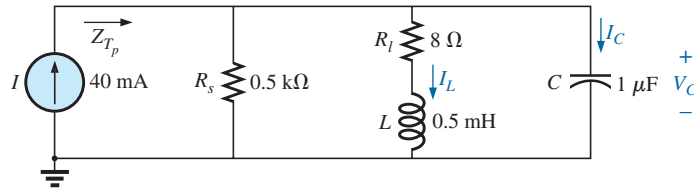
18. For the network in Fig. 20.57:



**FIG. 20.57**  
Problem 18.

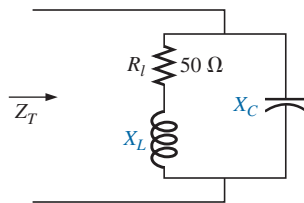


- a. Find the resonant frequencies  $f_s$ ,  $f_p$ , and  $f_m$ . What do the results suggest about the  $Q_p$  of the network?
  - b. Find the values of  $X_L$  and  $X_C$  at resonance ( $f_p$ ). How do they compare?
  - c. Find the impedance  $Z_{T_p}$  at resonance ( $f_p$ ).
  - d. Calculate  $Q_p$  and the  $BW$ .
  - e. Find the magnitude of currents  $I_L$  and  $I_C$  at resonance ( $f_p$ ).
  - f. Calculate the voltage  $V_C$  at resonance ( $f_p$ ).
- \*19. Repeat Problem 18 for the network in Fig. 20.58.



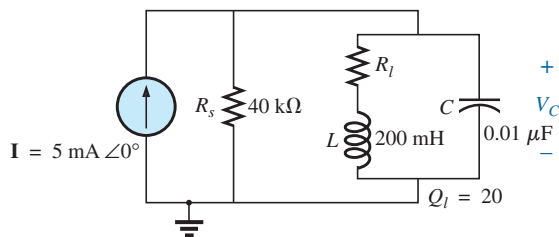
**FIG. 20.58**  
Problems 19 and 29.

20. It is desired that the impedance  $Z_T$  of the high  $Q$  circuit in Fig. 20.59 be  $50\text{ k}\Omega \angle 0^\circ$  at resonance ( $f_p$ ).
- a. Find the value of  $X_L$ .
  - b. Compute  $X_C$ .
  - c. Find the resonant frequency ( $f_p$ ) if  $L = 16\text{ mH}$ .
  - d. Find the value of  $C$ .



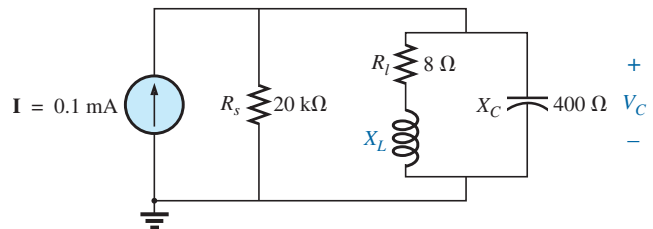
**FIG. 20.59**  
Problem 20.

21. For the network in Fig. 20.60:
- a. Find  $f_p$ .
  - b. Calculate the magnitude of  $V_C$  at resonance ( $f_p$ ).
  - c. Determine the power absorbed at resonance.
  - d. Find the  $BW$ .



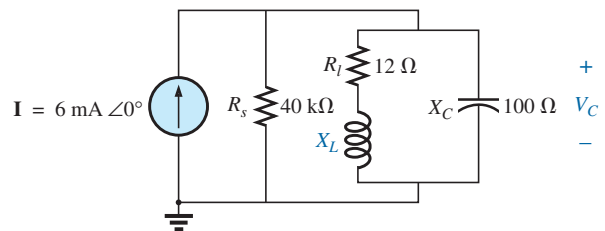
**FIG. 20.60**  
Problem 21.

- \*22. For the network in Fig. 20.61:
- a. Find the value of  $X_L$  for resonance.
  - b. Find  $Q_l$ .
  - c. Find the resonant frequency ( $f_p$ ) if the bandwidth is 1 kHz.
  - d. Find the maximum value of the voltage  $V_C$ .
  - e. Sketch the curve of  $V_C$  versus frequency. Indicate its peak value, resonant frequency, and band frequencies.



**FIG. 20.61**  
Problem 22.

- \*23. Repeat Problem 22 for the network in Fig. 20.62.



**FIG. 20.62**  
Problem 23.

- \*24. For the network in Fig. 20.63:
- Find  $f_s$ ,  $f_p$ , and  $f_m$ .
  - Determine  $Q_l$  and  $Q_p$  at  $f_p$  after a source conversion is performed.
  - Find the input impedance  $Z_{T_p}$ .
  - Find the magnitude of the voltage  $V_C$ .
  - Calculate the bandwidth using  $f_p$ .
  - Determine the magnitude of the currents  $I_C$  and  $I_L$ .

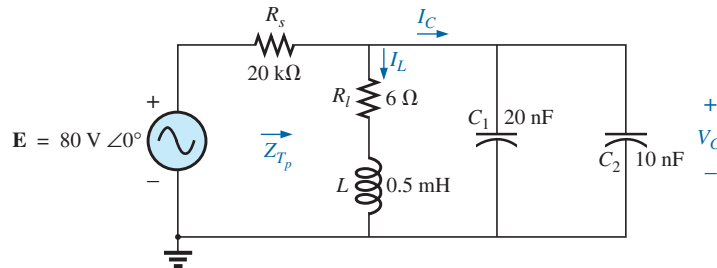


FIG. 20.63  
Problem 24.

- \*25. For the network in Fig. 20.64, the following are specified:

$$\begin{aligned} f_p &= 20 \text{ kHz} \\ BW &= 1.8 \text{ kHz} \\ L &= 2 \text{ mH} \\ Q_l &= 80 \end{aligned}$$

Find  $R_s$  and  $C$ .

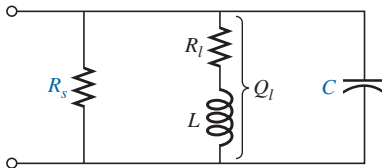


FIG. 20.64  
Problem 25.

- \*26. Design the network in Fig. 20.65 to have the following characteristics:

- $BW = 500 \text{ Hz}$
- $Q_p = 30$
- $V_{C_{\max}} = 1.8 \text{ V}$

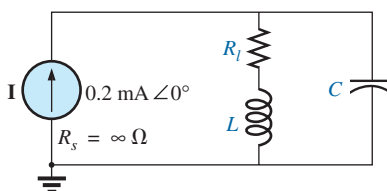


FIG. 20.65  
Problem 26.

- \*27. For the parallel resonant circuit in Fig. 20.66:

- Determine the resonant frequency.
- Find the total impedance at resonance.
- Find  $Q_p$ .
- Calculate the  $BW$ .
- Repeat parts (a) through (d) for  $L = 20 \mu\text{H}$  and  $C = 20 \text{ nF}$ .
- Repeat parts (a) through (d) for  $L = 0.4 \text{ mH}$  and  $C = 1 \text{ nF}$ .
- For the network in Fig. 20.66 and the parameters of parts (e) and (f), determine the ratio  $L/C$ .
- Do your results confirm the conclusions in Fig. 20.28 for changes in the  $L/C$  ratio?

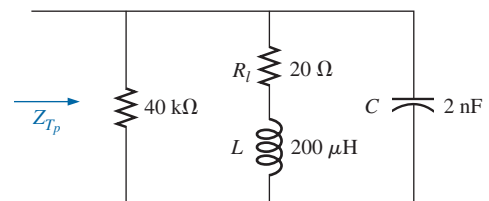


FIG. 20.66  
Problem 27.

## SECTION 20.14 Computer Analysis

### PSpice or Multisim

- Verify the results in Example 20.8. That is, show that the resonant frequency is 40 kHz, the cutoff frequencies are as calculated, and the bandwidth is 1.85 kHz.
- Find  $f_p$  and  $f_m$  for the parallel resonant network in Fig. 20.58, and comment on the resulting bandwidth as it relates to the quality factor of the network.



## GLOSSARY

**Band (cutoff, half-power, corner) frequencies** Frequencies that define the points on the resonance curve that are 0.707 of the peak current or voltage value. In addition, they define the frequencies at which the power transfer to the resonant circuit will be half the maximum power level.

**Bandwidth (*BW*)** The range of frequencies between the band, cutoff, or half-power frequencies.

**Quality factor (*Q*)** A ratio that provides an immediate indication of the sharpness of the peak of a resonance curve. The higher the *Q*, the sharper the peak and the more quickly it drops off to the right and left of the resonant frequency.

**Resonance** A condition established by the application of a particular frequency (the resonant frequency) to a series or parallel *R-L-C* network. The transfer of power to the system is a maximum, and, for frequencies above and below, the power transfer drops off to significantly lower levels.

**Selectivity** A characteristic of resonant networks directly related to the bandwidth of the resonant system. High selectivity is associated with small bandwidth (high *Q*'s), and low selectivity with larger bandwidths (low *Q*'s).