Methods of Analysis and Selected Topics (ac)

17

Objectives

- *Understand the differences between independent and dependent sources and how the magnitude and angle of a controlled source is determined by the dependent variable.*

- *Be able to convert between voltage and current sources and vice versa in the ac domain.*

- *Become proficient in the application of mesh and nodal analysis to ac networks with independent and controlled sources.*

- *Be able to define the relationship between the elements of an ac bridge network that will establish a balance condition.*

17.1 INTRODUCTION

For networks with two or more sources that are not in series or parallel, the methods described in the last two chapters cannot be applied. Rather, methods such as mesh analysis or nodal analysis must be used. Since these methods were discussed in detail for dc circuits in Chapter 8, this chapter considers the variations required to apply these methods to ac circuits. Dependent sources are also introduced for both mesh and nodal analysis.

The branch-current method is not discussed again because it falls within the framework of mesh analysis. In addition to the methods mentioned above, the bridge network and Δ-Y, Y-Δ conversions are also discussed for ac circuits.

Before we examine these topics, however, we must consider the subject of independent and controlled sources.

17.2 INDEPENDENT VERSUS DEPENDENT (CONTROLLED) SOURCES

In the previous chapters, each source appearing in the analysis of dc or ac networks was an **independent source,** such as E and I (or **E** and **I**) in Fig. 17.1.

The term independent specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.

A dependent or controlled source is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.
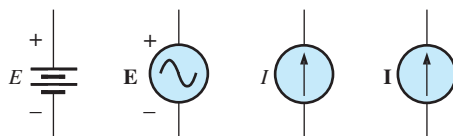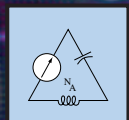

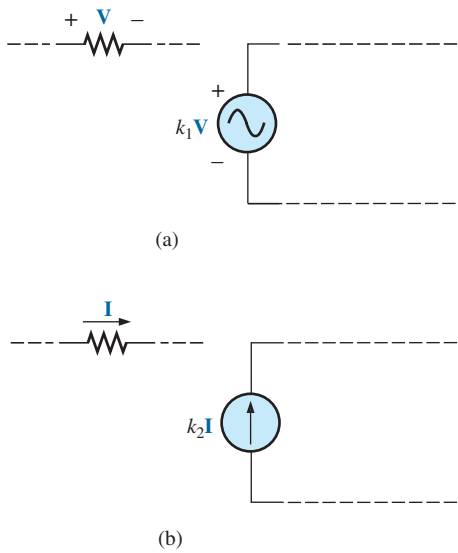
FIG. 17.1
Independent sources.

FIG. 17.2
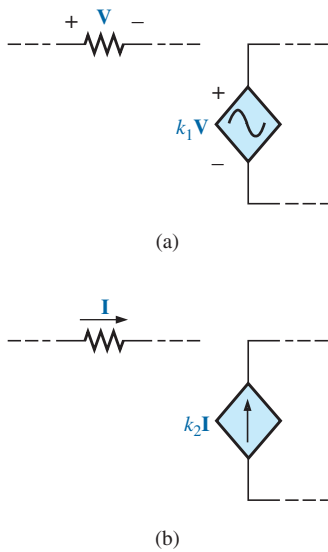
Controlled or dependent sources.

FIG. 17.3

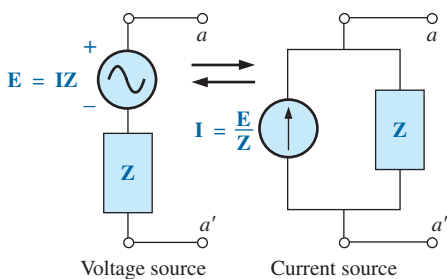*Special notation for controlled or dependent sources.*

FIG. 17.5

Source conversion.

Currently two symbols are used for controlled sources. One simply uses the independent symbol with an indication of the controlling element, as shown in Fig. 17.2. In Fig. 17.2(a), the magnitude and phase of the voltage are controlled by a voltage V elsewhere in the system, with the magnitude further controlled by the constant k_1. In Fig. 17.2(b), the magnitude and phase of the current source are controlled by a current I elsewhere in the system, with the magnitude further controlled by the constant k_2. To distinguish between the dependent and independent sources, the notation in Fig. 17.3 was introduced. In recent years, many respected publications on circuit analysis have accepted the notation in Fig. 17.3, although a number of excellent publications in the area of electronics continue to use the symbol in Fig. 17.2, especially in the circuit modeling for a variety of electronic devices such as the transistor and FET. This text uses the symbols in Fig. 17.3.

Possible combinations for controlled sources are indicated in Fig. 17.4. Note that the magnitude of current sources or voltage sources can be controlled by a voltage and a current, respectively. Unlike with the independent source, isolation such that V or $I = 0$ in Fig. 17.4(a) results in the short-circuit or open-circuit equivalent as indicated in Fig. 17.4(b). Note that the type of representation under these conditions is controlled by whether it is a current source or a voltage source, not by the controlling agent (V or I).
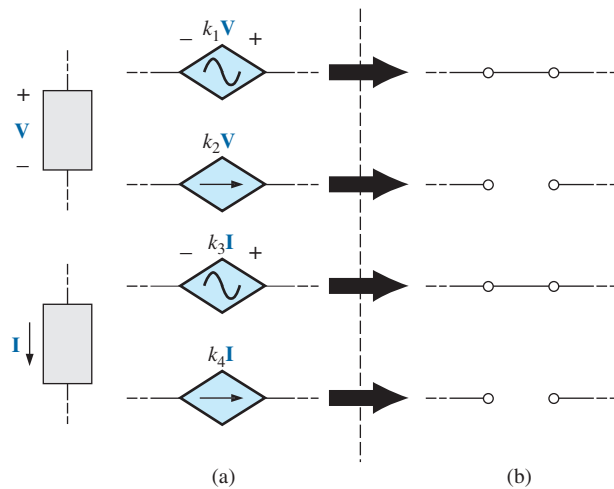

FIG. 17.4

Conditions of $V = 0$ V and $I = 0$ A for a controlled source.

17.3 SOURCE CONVERSIONS

When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source, or a voltage source to a current source. This **source conversion** can be accomplished in much the same manner as for dc circuits, except now we shall be dealing with phasors and impedances instead of just real numbers and resistors.

Independent Sources

In general, the format for converting one type of independent source to another is as shown in Fig. 17.5.

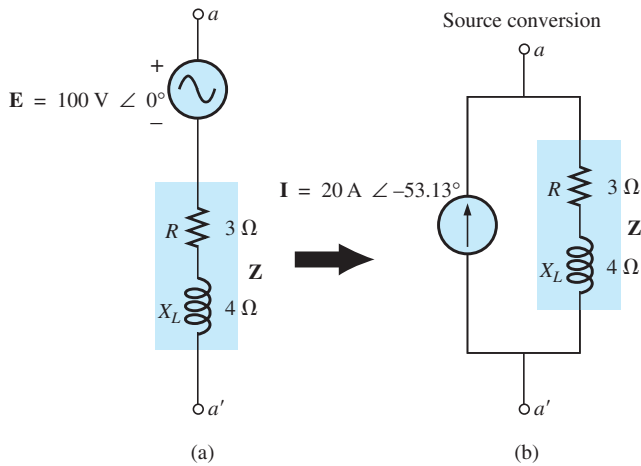**EXAMPLE 17.1** Convert the voltage source in Fig. 17.6(a) to a current source.

FIG. 17.6
Example 17.1.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100 \text{ V} \angle 0°}{5 \ \Omega \ \angle 53.13°}$$
$$= \mathbf{20 \text{ A}} \angle \mathbf{-53.13°} \qquad [\text{Fig. 17.6(b)}]$$

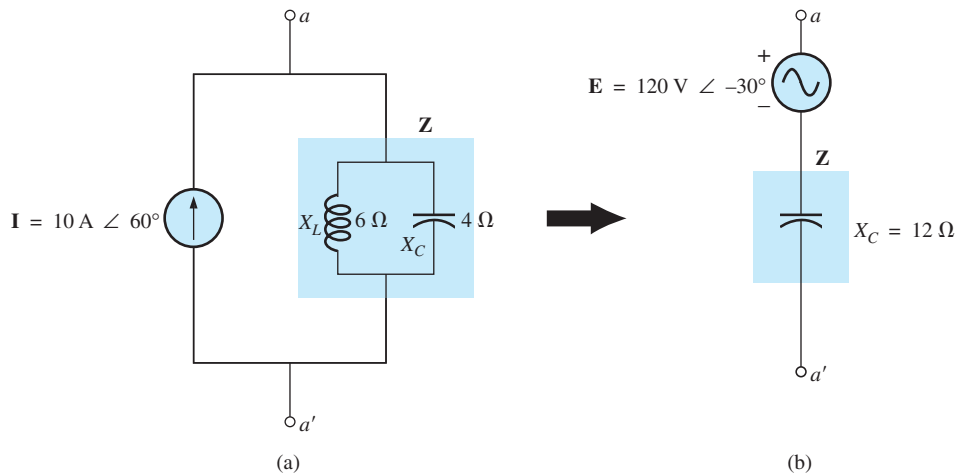**EXAMPLE 17.2** Convert the current source in Fig. 17.7(a) to a voltage source.

FIG. 17.7
Example 17.2.

Solution:

$$\mathbf{Z} = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j\,X_C + j\,X_L}$$

$$= \frac{(4\,\Omega \angle -90°)(6\,\Omega \angle 90°)}{-j\,4\,\Omega + j\,6\,\Omega} = \frac{24\,\Omega \angle 0°}{2 \angle 90°}$$

$$= \mathbf{12\,\Omega \angle -90°}$$

$$\mathbf{E} = \mathbf{IZ} = (10\,\text{A} \angle 60°)(12\,\Omega \angle -90°)$$

$$= \mathbf{120\,V\angle -30°} \qquad [\text{Fig. 17.7(b)}]$$

Dependent Sources

For dependent sources, the direct conversion in Fig. 17.5 can be applied if the controlling variable (**V** or **I** in Fig. 17.4) is not determined by a portion of the network to which the conversion is to be applied. For example, in Figs. 17.8 and 17.9, **V** and **I**, respectively, are controlled by an external portion of the network. Conversions of the other kind, where **V** and **I** are controlled by a portion of the network to be converted, are considered in Sections 18.3 and 18.4.

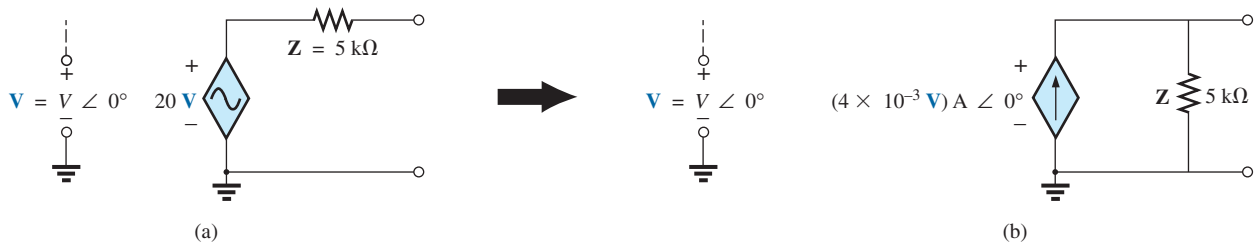**EXAMPLE 17.3** Convert the voltage source in Fig. 17.8(a) to a current source.

(a)   (b)

FIG. 17.8
Source conversion with a voltage-controlled voltage source.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{(20\mathbf{V})\,\text{V}\angle 0°}{5\,\text{k}\Omega \angle 0°}$$

$$= (\mathbf{4 \times 10^{-3}\,V})\mathbf{A} \angle \mathbf{0°} \qquad [\text{Fig. 17.8(b)}]$$

EXAMPLE 17.4 Convert the current source in Fig. 17.9(a) to a voltage source.

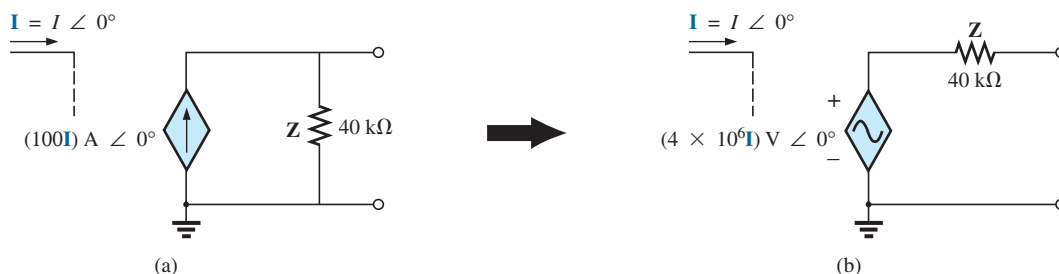(a)   (b)

FIG. 17.9
Source conversion with a current-controlled current source.

Solution:

$$\mathbf{E} = \mathbf{IZ} = [(100\mathbf{I}) \text{ A } \angle 0°][40 \text{ k}\Omega \angle 0°]$$
$$= (\mathbf{4 \times 10^6 I}) \text{ V } \angle \mathbf{0°} \qquad [\text{Fig. 17.9(b)}]$$

17.4 MESH ANALYSIS

General Approach

Independent Voltage Sources Before examining the application of the method to ac networks, the student should first review the appropriate sections on **mesh analysis** in Chapter 8 since the content of this section will be limited to the general conclusions of Chapter 8.

The general approach to mesh analysis for independent sources includes the same sequence of steps appearing in Chapter 8. In fact, throughout this section the only change from the dc coverage is to substitute impedance for resistance and admittance for conductance in the general procedure.

1. *Assign a distinct current in the clockwise direction to each independent closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. However, it eliminates the need to have to choose a direction for each application. Any direction can be chosen for each loop current with no loss in accuracy as long as the remaining steps are followed properly.*
2. *Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.*
3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the format approach to follow.*
   a. *If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents passing through in the opposite direction.*
   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*
4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

The technique is applied as above for all networks with independent sources or for networks with *dependent sources where the controlling variable is not a part of the network under investigation.* If the controlling variable is part of the network being examined, a method to be described shortly must be applied.

EXAMPLE 17.5 Using the general approach to mesh analysis, find the current $\mathbf{I}_1$ in Fig. 17.10.

Solution: When applying these methods to ac circuits, it is good practice to represent the resistors and reactances (or combinations thereof) by subscripted impedances. When the total solution is found in terms of these subscripted impedances, the numerical values can be substituted to find the unknown quantities.
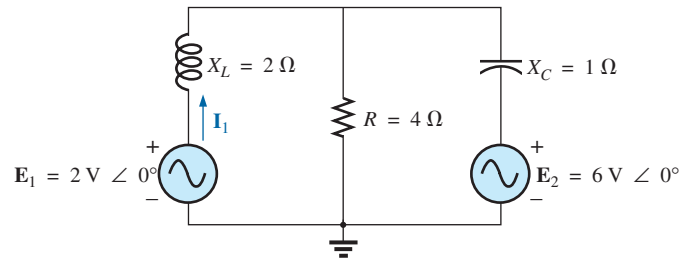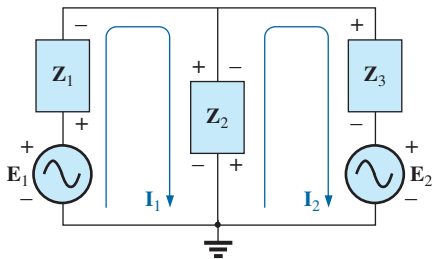
FIG. 17.10

Example 17.5.

FIG. 17.11

Assigning the mesh currents and subscripted impedances for the network in Fig. 17.10.

The network is redrawn in Fig. 17.11 with subscripted impedances:

$$\mathbf{Z}_1 = +jX_L = +j\,2\,\Omega \qquad \mathbf{E}_1 = 2\text{ V} \angle 0°$$
$$\mathbf{Z}_2 = R = 4\,\Omega \qquad\qquad \mathbf{E}_2 = 6\text{ V} \angle 0°$$
$$\mathbf{Z}_3 = -jX_C = -j\,1\,\Omega$$

Steps 1 and 2 are as indicated in Fig. 17.11.

Step 3:

$$+\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

or

$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

so that

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

which are rewritten as

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\quad = \mathbf{E}_1$$
$$-\mathbf{I}_1\mathbf{Z}_2 \qquad\quad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = -\mathbf{E}_2$$

Step 4: Using determinants, we obtain

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$

$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_1 = \frac{(2\text{ V} - 6\text{ V})(4\,\Omega) + (2\text{ V})(-j\,1\,\Omega)}{(+j\,2\,\Omega)(4\,\Omega) + (+j\,2\,\Omega)(-j\,2\,\Omega) + (4\,\Omega)(-j\,2\,\Omega)}$$

$$= \frac{-16 - j\,2}{j\,8 - j^2\,2 - j\,4} = \frac{-16 - j\,2}{2 + j\,4} = \frac{16.12\text{ A} \angle -172.87°}{4.47 \angle 63.43°}$$

$$= \textbf{3.61 A} \angle \textbf{-236.30°} \ \text{ or } \ \textbf{3.61 A} \angle \textbf{123.70°}$$

Dependent Voltage Sources For dependent voltage sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: Treat each dependent source like an independent source when Kirchhoff's voltage law is applied to each independent loop. However, once the equation is written, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen mesh currents.
3. Step 4 is as before.

EXAMPLE 17.6 Write the mesh currents for the network in Fig. 17.12 having a dependent voltage source.

Solution:

Steps 1 and 2 are defined in Fig. 17.12.

Step 3: $\quad E_1 - I_1R_1 - R_2(I_1 - I_2) = 0$
$\qquad\quad R_2(I_2 - I_1) + \mu V_x - I_2R_3 = 0$

Then substitute $V_x = (I_1 - I_2)R_2$
$\quad$ The result is two equations and two unknowns.

$$E_1 - I_1R_1 - R_2(I - I_2) = 0$$
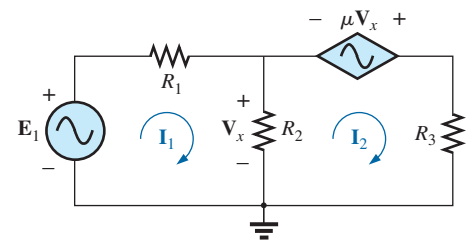$$\underline{R_2(I_2 - I_1) + \mu R_2(I_1 - I_2) - I_2R_3 = 0}$$

FIG. 17.12

Applying mesh analysis to a network with a voltage-controlled voltage source.

Independent Current Sources For independent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each current source as an open circuit (recall the *supermesh* designation in Chapter 8), and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited to the mesh currents.
3. Step 4 is as before.

EXAMPLE 17.7 Write the mesh currents for the network in Fig. 17.13 having an independent current source.

Solution:

Steps 1 and 2 are defined in Fig. 17.13.

Step 3: $\quad E_1 - I_1Z_1 + E_2 - I_2Z_2 = 0\quad$ (only remaining independent
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ path)

with $\qquad I_1 + I = I_2$

The result is two equations and two unknowns.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
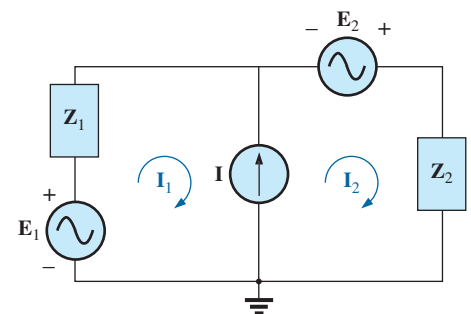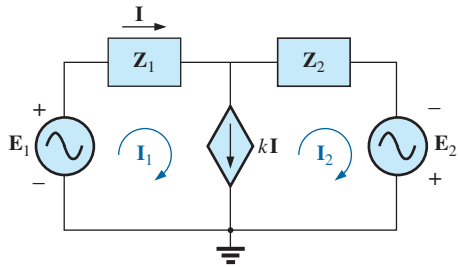

FIG. 17.13

Applying mesh analysis to a network with an independent current source.

2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent current sources, except now the dependent sources have to be defined in terms of the chosen mesh currents to ensure that the final equations have only mesh currents as the unknown quantities.
3. Step 4 is as before.

FIG. 17.14

Applying mesh analysis to a network with a current-controlled current source.

EXAMPLE 17.8 Write the mesh currents for the network in Fig. 17.14 having a dependent current source.

Solution:

Steps 1 and 2 are defined in Fig. 17.14.

Step 3: $\qquad\qquad \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0$

and $\qquad\qquad k\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$

Now $\mathbf{I} = \mathbf{I}_1$ so that $\qquad k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2 \quad$ or $\quad \mathbf{I}_2 = \mathbf{I}_1(1 - k)$

The result is two equations and two unknowns.

Format Approach

The format approach was introduced in Section 8.8. The steps for applying this method are repeated here with changes for its use in ac circuits:

1. *Assign a loop current to each independent closed loop (as in the previous section) in a clockwise direction.*
2. *The number of required equations is equal to the number of chosen independent closed loops. Column 1 of each equation is formed by summing the impedance values of those impedances through which the loop current of interest passes and multiplying the result by that loop current.*
3. *We must now consider the mutual terms that are always subtracted from the terms in the first column. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each mutual term is the product of the mutual impedance and the other loop current passing through the same element.*
4. *The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. Negative signs are assigned to those potentials for which the reverse is true.*
5. *Solve the resulting simultaneous equations for the desired loop currents.*

The technique is applied as above for all networks with independent sources or for networks with dependent sources where the controlling variable is not a part of the network under investigation. If the controlling variable is part of the network being examined, additional care must be taken when applying the above steps.

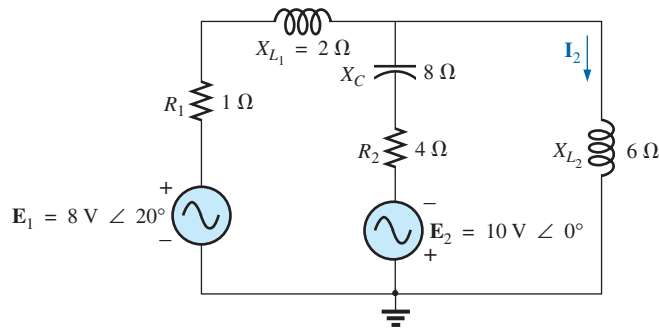**EXAMPLE 17.9** Using the format approach to mesh analysis, find the current $\mathbf{I}_2$ in Fig. 17.15.

FIG. 17.15
Example 17.9.

Solution: The network is redrawn in Fig. 17.16:

$$\mathbf{Z}_1 = R_1 + j\,X_{L_1} = 1\,\Omega + j\,2\,\Omega \qquad \mathbf{E}_1 = 8\text{ V} \angle 20°$$
$$\mathbf{Z}_2 = R_2 - j\,X_C = 4\,\Omega - j\,8\,\Omega \qquad \mathbf{E}_2 = 10\text{ V} \angle 0°$$
$$\mathbf{Z}_3 = +j\,X_{L_2} = +j\,6\,\Omega$$

Note the reduction in complexity of the problem with the substitution of the subscripted impedances.

Step 1 is as indicated in Fig. 17.16.

Steps 2 to 4:

FIG. 17.16
Assigning the mesh currents and subscripted impedances for the network in Fig. 17.15.

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 + \mathbf{E}_2$$
$$\underline{\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2}$$

which are rewritten as

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad = \mathbf{E}_1 + \mathbf{E}_2$$
$$\underline{-\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = -\mathbf{E}_2}$$

Step 5: Using determinants, we have

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{Z}_2 & -\mathbf{E}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{-(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{E}_2 + \mathbf{Z}_2(\mathbf{E}_1 + \mathbf{E}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2^2}$$

$$= \frac{\mathbf{Z}_2\mathbf{E}_1 - \mathbf{Z}_1\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_2 = \frac{(4\,\Omega - j\,8\,\Omega)(8\text{ V} \angle 20°) - (1\,\Omega + j\,2\,\Omega)(10\text{ V} \angle 0°)}{(1\,\Omega + j\,2\,\Omega)(4\,\Omega - j\,8\,\Omega) + (1\,\Omega + j\,2\,\Omega)(+j\,6\,\Omega) + (4\,\Omega - j\,8\,\Omega)(+j\,6\,\Omega)}$$

$$= \frac{(4 - j\,8)(7.52 + j\,2.74) - (10 + j\,20)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$

$$= \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\text{ A} \angle -58.74°}{63.53 \angle 28.18°}$$

$$= \mathbf{1.27\text{ A}} \ \angle -\mathbf{86.92°}$$

Calculator Solution: The calculator can be an effective tool in performing the long, laborious calculations involved with the final equation appearing above. However, you must be very careful to use brackets to define the order of the arithmetic operations (remember that each open bracket must be followed by a close bracket). Using the TI-89 calculator, the sequence in Fig. 17.17 provides the solution for $\mathbf{I}_2$.

$$((4-8i)\times(8\angle20°)-(1+2i)\times(10\angle0°))\div((1+2i)\times(4-8i)+(1+2i)\times(6i)+(4-8i)\times(6i)) \blacktriangleright \text{Polar} \qquad 1.27\text{E}0\angle-86.95\text{E}0$$

FIG. 17.17
Determining I_2 using the TI-89 calculator.

Mathcad Solution: This example provides an excellent opportunity to demonstrate the power of Mathcad. First, define the impedances and parameters for the equations to follow as shown in Fig. 17.18. Then enter the **guess** values of the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Enter the label **Given** followed by the equations for the network. Note that in this example, you are not continuing with the analysis until the matrix is defined—you are working directly from the network equations. Once the equations have been properly entered, enter **Find(I1,I2)**. Then selecting the equal sign results in the single-column matrix with the results in rectangular form. Conversion to polar form requires defining a variable **A** and then calling for the magnitude and angle using the definitions entered earlier in the listing and both the **Calculator** and **Greek** toolbars. The result for $\mathbf{I}_2$ is 1.274 A $\angle-86.94°$, which is an excellent match with the theoretical solution.
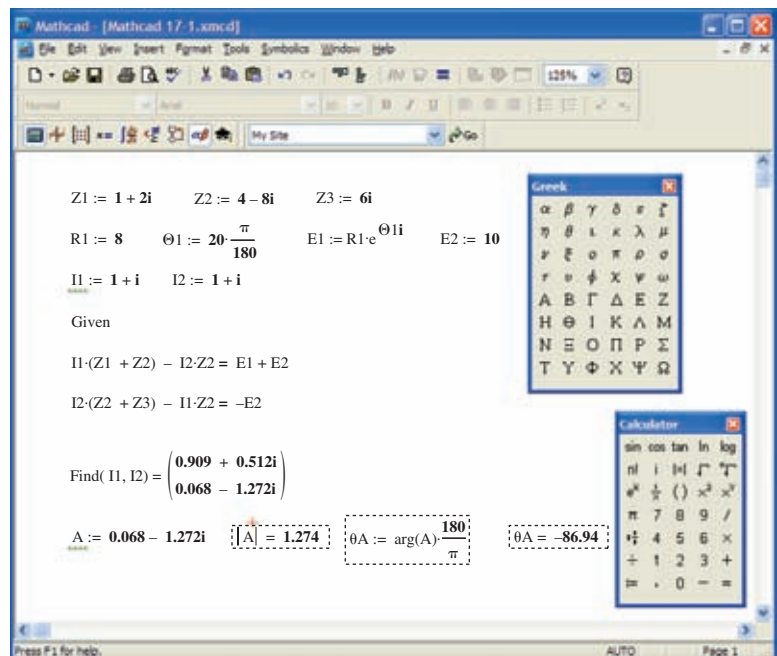

FIG. 17.18
Using Mathcad to verify the results in Example 17.9.

EXAMPLE 17.10 Write the mesh equations for the network in Fig. 17.19. Do not solve.
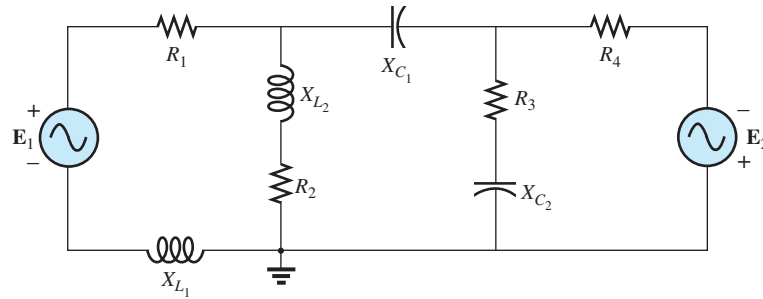
FIG. 17.19
Example 17.10.

Solution: The network is redrawn in Fig. 17.20. Again note the re-
duced complexity and increased clarity provided by the use of sub-
scripted impedances:

$$\mathbf{Z}_1 = R_1 + j X_{L_1} \qquad \mathbf{Z}_4 = R_3 - j X_{C_2}$$
$$\mathbf{Z}_2 = R_2 + j X_{L_2} \qquad \mathbf{Z}_5 = R_4$$
$$\mathbf{Z}_3 = j X_{C_1}$$

and

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 = 0$$
$$\mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_4 = \mathbf{E}_2$$

or

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2(\mathbf{Z}_2) \qquad\qquad + 0 \qquad\qquad = \mathbf{E}_1$$
$$\mathbf{I}_1\mathbf{Z}_2 \qquad - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_3(\mathbf{Z}_4) \qquad = 0$$
$$0 \qquad\qquad - \mathbf{I}_2(\mathbf{Z}_4) \qquad\qquad - \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) = \mathbf{E}_2$$
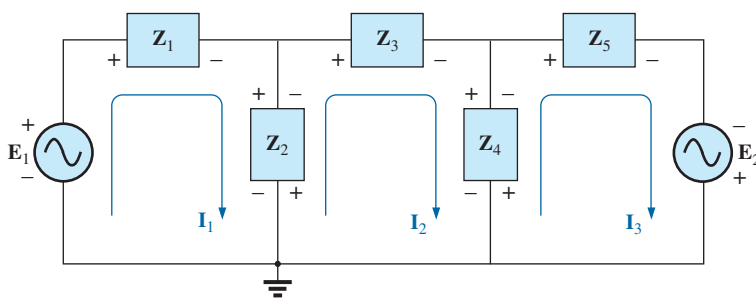

FIG. 17.20
*Assigning the mesh currents and subscripted impedances for the network in
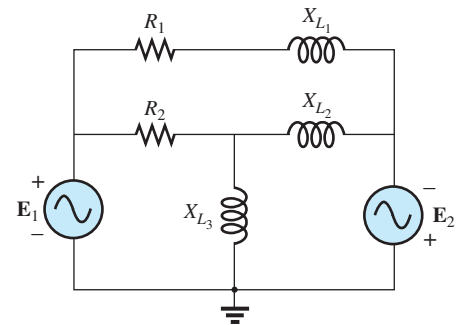Fig. 17.19.*

FIG. 17.21
Example 17.11.

EXAMPLE 17.11 Using the format approach, write the mesh equations
for the network in Fig. 17.21.

Solution: The network is redrawn as shown in Fig. 17.22, where

$$\mathbf{Z}_1 = R_1 + j X_{L_1} \quad \mathbf{Z}_3 = j X_{L_2}$$
$$\mathbf{Z}_2 = R_2 \qquad\qquad \mathbf{Z}_4 = j X_{L_3}$$

and

$$\mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_3 = 0$$
$$\mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_1\mathbf{Z}_4 = \mathbf{E}_2$$

or

$$\mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad - \mathbf{I}_3\mathbf{Z}_4 \qquad\qquad = \mathbf{E}_1$$
$$-\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_3\mathbf{Z}_3 \qquad = 0$$
$$-\mathbf{I}_1\mathbf{Z}_4 \qquad\qquad - \mathbf{I}_2\mathbf{Z}_3 \qquad\qquad + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) = \mathbf{E}_2$$
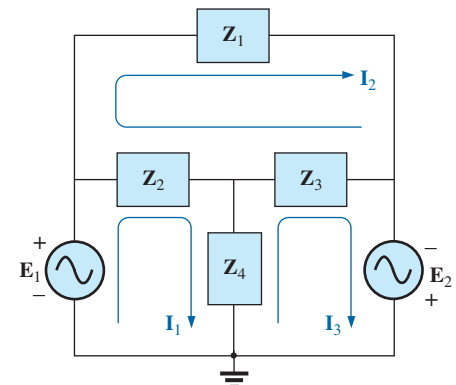

FIG. 17.22
*Assigning the mesh currents and subscripted
impedances for the network in Fig. 17.21.*

Note the symmetry *about* the diagonal axis; that is, note the location of $-Z_2$, $-Z_4$, and $-Z_3$ off the diagonal.

17.5 NODAL ANALYSIS

General Approach

Independent Sources Before examining the application of the method to ac networks, a review of the appropriate sections on **nodal analysis** in Chapter 8 is suggested since the content of this section is limited to the general conclusions of Chapter 8.

The fundamental steps are the following:

1. *Determine the number of nodes within the network.*
2. *Pick a reference node and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*
3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.*
4. *Solve the resulting equations for the nodal voltages.*

A few examples will refresh your memory about the content of Chapter 8 and the general approach to a nodal-analysis solution.

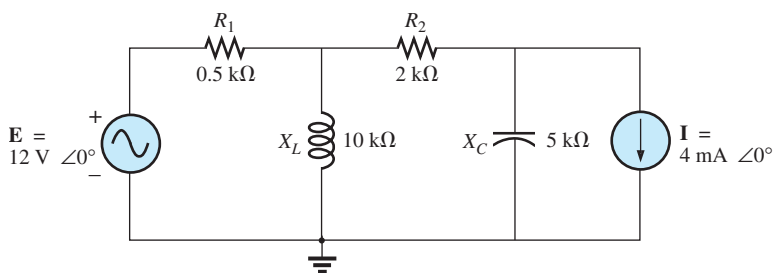**EXAMPLE 17.12** Determine the voltage across the inductor for the network in Fig. 17.23.

FIG. 17.23
Example 17.12.

Solution:

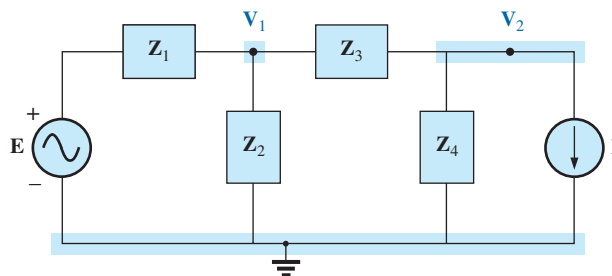Steps 1 and 2 are as indicated in Fig. 17.24.

FIG. 17.24
Assigning the nodal voltages and subscripted impedances to the network in Fig. 17.23.

Step 3: Note Fig. 17.25 for the application of Kirchhoff's current law to node V_1:

$$\Sigma \, I_i = \Sigma \, I_o$$

$$0 = I_1 + I_2 + I_3$$

$$\frac{V_1 - E}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = 0$$

Rearranging terms:

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E_1}{Z_1} \qquad (17.1)$$

Note Fig. 17.26 for the application of Kirchoff's current law to node V_2.

$$0 = I_3 + I_4 + I$$

$$\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} + I = 0$$

Rearranging terms:

$$V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] - V_1\left[\frac{1}{Z_3}\right] = -I \qquad (17.2)$$

Grouping equations:

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E}{Z_1}$$

$$V_1\left[\frac{1}{Z_3}\right] \qquad\qquad - V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] = I$$

$$\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{0.5 \text{ k}\Omega} + \frac{1}{j \, 10 \text{ k}\Omega} + \frac{1}{2 \text{ k}\Omega} = 2.5 \text{ mS} \angle -2.29°$$

$$\frac{1}{Z_3} + \frac{1}{Z_4} = \frac{1}{2 \text{ k}\Omega} + \frac{1}{-j \, 5 \text{ k}\Omega} = 0.539 \text{ mS} \angle 21.80°$$

and

$$V_1[2.5 \text{ mS} \angle -2.29°] - V_2[0.5 \text{ mS} \angle 0°] = 24 \text{ mA} \angle 0°$$

$$V_1[0.5 \text{ mS} \angle 0°] \quad - V_2[0.539 \text{ mS} \angle 21.80°] = 4 \text{ mA} \angle 0°$$

with

$$V_1 = \frac{\begin{vmatrix} 24 \text{ mA} \angle 0° & -0.5 \text{ mS} \angle 0° \\ 4 \text{ mA} \angle 0° & -0.539 \text{ mS} \angle 21.80° \end{vmatrix}}{\begin{vmatrix} 2.5 \text{ mS} \angle -2.29° & -0.5 \text{ mS} \angle 0° \\ 0.5 \text{ mS} \angle 0° & -0.539 \text{ mS} \angle 21.80° \end{vmatrix}}$$

$$= \frac{(24 \text{ mA} \angle 0°)(-0.539 \text{ mS} \angle 21.80°) + (0.5 \text{ mS} \angle 0°)(4 \text{ mA} \angle 0°)}{(2.5 \text{ mS} \angle -2.29°)(-0.539 \text{ mS} \angle 21.80°) + (0.5 \text{ mS} \angle 0°)(0.5 \text{ mS} \angle 0°)}$$

$$= \frac{-12.94 \times 10^{-6} \text{ V} \angle 21.80° + 2 \times 10^{-6} \text{ V} \angle 0°}{-1.348 \times 10^{-6} \angle 19.51° + 0.25 \times 10^{-6} \angle 0°}$$

$$= \frac{-(12.01 + j \, 4.81) \times 10^{-6} \text{ V} + 2 \times 10^{-6} \text{ V}}{-(1.271 + j \, 0.45) \times 10^{-6} + 0.25 \times 10^{-6}}$$

$$= \frac{-10.01 \text{ V} - j \, 4.81 \text{ V}}{-1.021 - j \, 0.45} = \frac{11.106 \text{ V} \angle -154.33°}{1.116 \angle -156.21°}$$

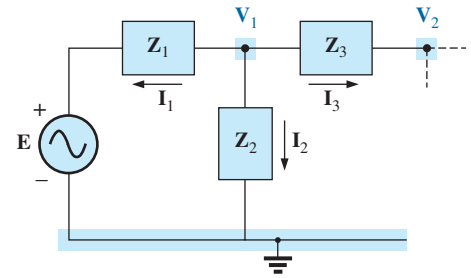$$V_1 = 9.95 \text{ V} \angle 1.88°$$

FIG. 17.25

Applying Kirchhoff's current law to the node V_1 in Fig. 17.24.
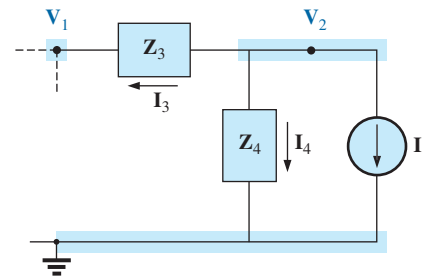

FIG. 17.26

Applying Kirchhoff's current law to the node V_2 in Fig. 17.24.
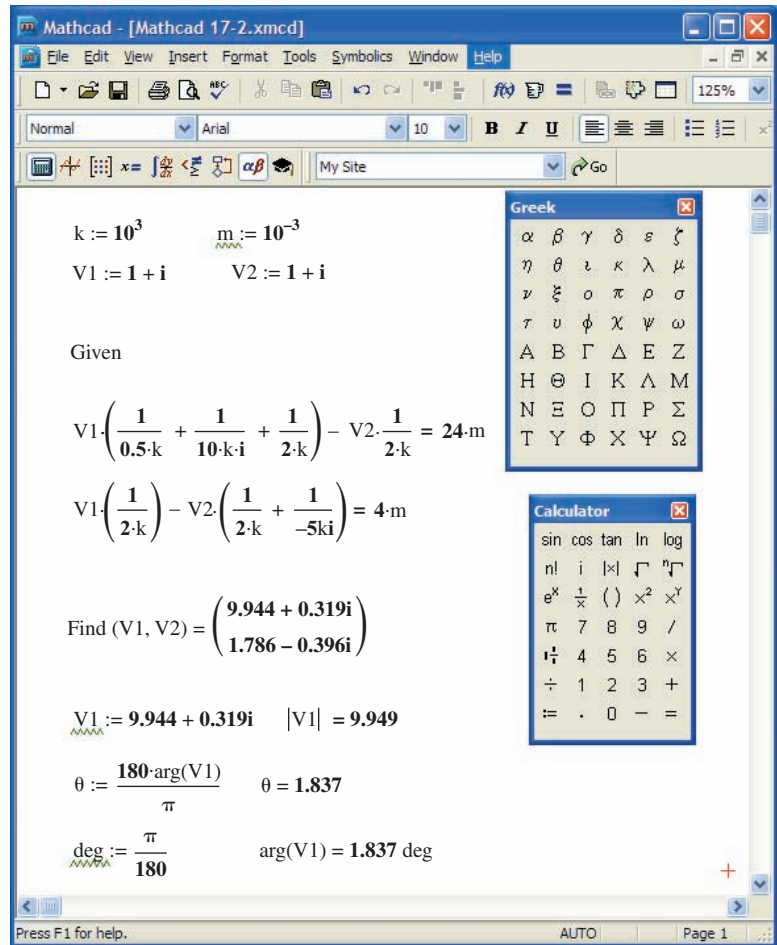
FIG. 17.27
Using Mathcad to verify the results of Example 17.12.

Mathcad Solution: The length and the complexity of the above mathematical development strongly suggest the use of an alternative approach such as Mathcad. The printout in Fig. 17.27 first defines the letters **k** and **m** to specific numerical values so that the power-of-ten format did not have to be included in the equations. Thus, the results are cleaner and easier to review. When entering the equations, remember that the *j* is entered as 1*j without* the multiplication sign between the 1 and the *j*. A multiplication sign between the two would define the *j* as another variable. Also be sure that the multiplication process is inserted between the nodal variables and the brackets. If an error signal continues to surface, it is often best to simply reenter the entire listing—errors are often not easy to spot simply by looking at the resulting equations.

Finally the results are obtained and converted to polar form for comparison with the theoretical solution. The solution of 9.949 A $\angle 1.837°$ is a very close confirmation of the longhand solution.

Before leaving this example, let's look at another method for obtaining the polar form of the solution. The method appears in the bottom of Fig. 17.27. First define **deg** as shown, and then pick up **arg** from the **Insert-*f(x)*-Insert Function-arg** sequence. Next enter **V1**; the result is in radian form but with a small black rectangle in the place where the units normally appear. Click on that black rectangle, and the bracket ap-

pears where you can type **deg.** When you select the equal sign, the angle in degrees appears.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each dependent current source like an independent source when Kirchhoff's current law is applied to each defined node. However, once the equations are established, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen nodal voltages.
3. Step 4 is as before.

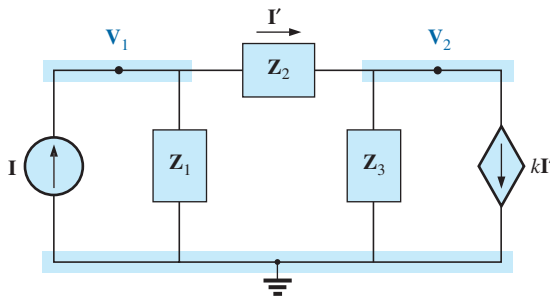**EXAMPLE 17.13** Write the nodal equations for the network in Fig. 17.28 having a dependent current source.

FIG. 17.28
Applying nodal analysis to a network with a current-controlled current source.

Solution:

Steps 1 and 2 are as defined in Fig. 17.28.

Step 3: At node $\mathbf{V}_1$,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}$$

At node $\mathbf{V}_2$,

$$\mathbf{I}_2 + \mathbf{I}_3 + k\mathbf{I} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\left[\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}\right] = 0$$

and

$$\mathbf{V}_1\left[\frac{1 - k}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1 - k}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] = 0$$

resulting in two equations and two unknowns.

Independent Voltage Sources between Assigned Nodes For independent voltage sources between assigned nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each source between defined nodes as a short circuit (recall the *supernode* classification in Chapter 8), and write the nodal equations for each remaining independent node. Then relate the chosen nodal voltages to the independent voltage source to ensure that the unknowns of the final equations are limited solely to the nodal voltages.
3. Step 4 is as before.

EXAMPLE 17.14 Write the nodal equations for the network in Fig. 17.29 having an independent source between two assigned nodes.
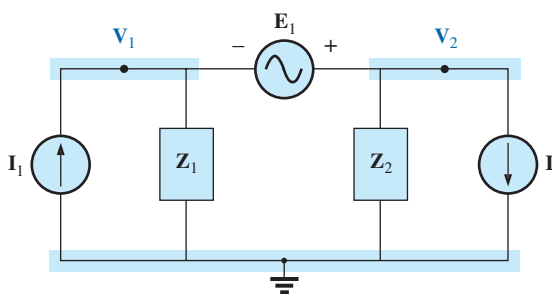
FIG. 17.29
Applying nodal analysis to a network with an independent voltage source between defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.29.

Step 3: Replacing the independent source $\mathbf{E}_1$ with a short-circuit equivalent results in a supernode that generates the following equation when Kirchhoff's current law is applied to node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2}{\mathbf{Z}_2} + \mathbf{I}_2$$

with $$\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{E}_1$$

and we have two equations and two unknowns.

Dependent Voltage Sources between Defined Nodes For dependent voltage sources between defined nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent voltage sources, except now the dependent sources have to be defined in terms of the chosen nodal voltages to ensure that the final equations have only nodal voltages as their unknown quantities.
3. Step 4 is as before.

EXAMPLE 17.15 Write the nodal equations for the network in Fig. 17.30 having a dependent voltage source between two defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.30.

Step 3: Replacing the dependent source μV_x with a short-circuit equivalent results in the following equation when Kirchhoff's current law is applied at node V_1:

$$I = I_1 + I_2$$

$$\frac{V_1}{Z_1} + \frac{(V_1 - V_2)}{Z_2} - I = 0$$

and

$$V_2 = \mu V_x = \mu[V_1 - V_2]$$

or

$$V_2 = \frac{\mu}{1 + \mu} V_1$$

resulting in two equations and two unknowns. Note that because the impedance Z_3 is in parallel with a voltage source, it does not appear in the analysis. It will, however, affect the current through the dependent voltage source.
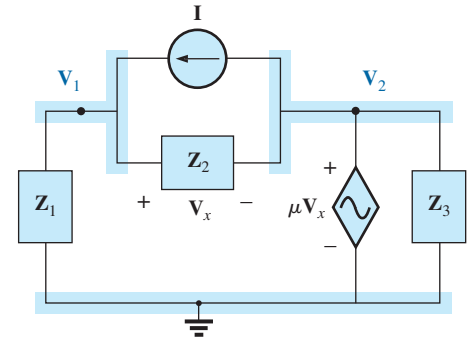
FIG. 17.30
Applying nodal analysis to a network with a voltage-controlled voltage source.

Format Approach

A close examination of Eqs. (17.1) and (17.2) in Example 17.12 reveals that they are the same equations that would have been obtained using the format approach introduced in Chapter 8. Recall that the approach required that the voltage source first be converted to a current source, but the writing of the equations was quite direct and minimized any chances of an error due to a lost sign or missing term.

The sequence of steps required to apply the format approach is the following:

1. *Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining independent nodes of the network.*
2. *The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the admittances tied to the node of interest and multiplying the result by that subscripted nodal voltage.*
3. *The mutual terms are always subtracted from the terms of the first column. It is possible to have more than one mutual term if the nodal voltage of interest has an element in common with more than one other nodal voltage. Each mutual term is the product of the mutual admittance and the other nodal voltage tied to that admittance.*
4. *The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node, and a negative sign if it draws current from the node.*
5. *Solve resulting simultaneous equations for the desired nodal voltages. The comments offered for mesh analysis regarding independent and dependent sources apply here also.*

EXAMPLE 17.16 Using the format approach to nodal analysis, find the voltage across the 4 Ω resistor in Fig. 17.31.
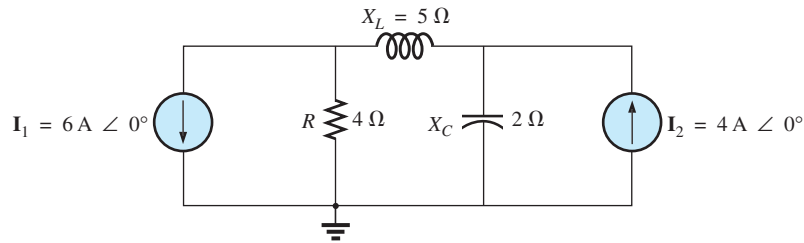

FIG. 17.31
Example 17.16.

Solution: Choosing nodes (Fig. 17.32) and writing the nodal equations, we have

$$\mathbf{Z}_1 = R = 4\ \Omega \quad \mathbf{Z}_2 = j\,X_L = j\,5\ \Omega \quad \mathbf{Z}_3 = -j\,X_C = -j\,2\ \Omega$$
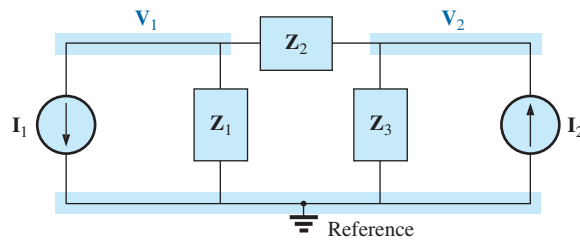

FIG. 17.32
Assigning the nodal voltages and subscripted impedances for the network in Fig. 17.31.

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) = -\mathbf{I}_1$$
$$\mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{V}_1(\mathbf{Y}_2) = +\mathbf{I}_2$$

or

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) \qquad = -\mathbf{I}_1$$
$$-\mathbf{V}_1(\mathbf{Y}_2) \qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) = +\mathbf{I}_2$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3}$$

Using determinants yields

$$\mathbf{V}_1 = \frac{\begin{vmatrix} -\mathbf{I}_1 & -\mathbf{Y}_2 \\ +\mathbf{I}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Y}_1 + \mathbf{Y}_2 & -\mathbf{Y}_2 \\ -\mathbf{Y}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}} = \frac{\begin{vmatrix} -6\ \text{A} + j\,0.2\ \text{S} \\ +4\ \text{A} + j\,0.3\ \text{S} \end{vmatrix}}{\begin{vmatrix} 0.25\ \text{S} - j\,0.2\ \text{S} & +j\,0.2\ \text{S} \\ +j\,0.2\ \text{S} & +j\,0.3\ \text{S} \end{vmatrix}}$$

$$= \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{(\mathbf{Y}_1 + \mathbf{Y}_2)(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{Y}_2^2}$$

$$= \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_2\mathbf{Y}_3 + \mathbf{Y}_1\mathbf{Y}_2}$$

Substituting numerical values, we have

$$V_1 = \frac{-[(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)]6\,A\,\angle 0° + 4\,A\,\angle 0°\,(1/j\,5\,\Omega)}{(1/4\,\Omega)(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)(1/-j\,2\,\Omega) + (1/4\,\Omega)(1/j\,5\,\Omega)}$$

$$= \frac{-(+j\,0.5 - j\,0.2)6\,\angle 0° + 4\,\angle 0°\,(-j\,0.2)}{(1/-j\,8) + (1/10) + (1/j\,20)}$$

$$= \frac{(-0.3\,\angle 90°)(6\,\angle 0°) + (4\,\angle 0°)(0.2\,\angle -90°)}{j\,0.125 + 0.1 - j\,0.05}$$

$$= \frac{-1.8\,\angle 90° + 0.8\,\angle -90°}{0.1 + j\,0.075}$$

$$= \frac{2.6\,V\,\angle -90°}{0.125\,\angle 36.87°}$$

$$V_1 = \mathbf{20.80\,V\,\angle -126.87°}$$

Calculator Solution: Using the TI-89 calculator, enter the parameters for the determinant form for V_1 in the rectangular form as shown by the sequence in Fig. 17.33. Note the different negative signs used to enter the data.

det ([(−) 6 , ▪ 2 i ; 4 , ▪ 3 i]) ÷

det ([▪ 2 5 − ▪ 2 i , ▪ 2 i ; ▪ 2 i ,

▪ 3 i]) ▶ Polar ENTER 20.80E0 ∠ −126.8E0
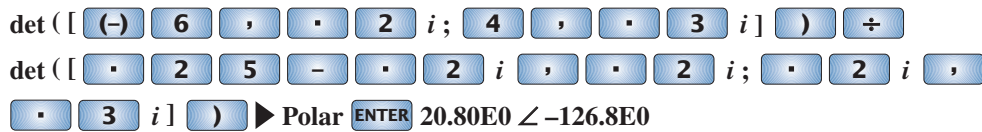
FIG. 17.33
Determining V_1 using the TI-89 calculator.

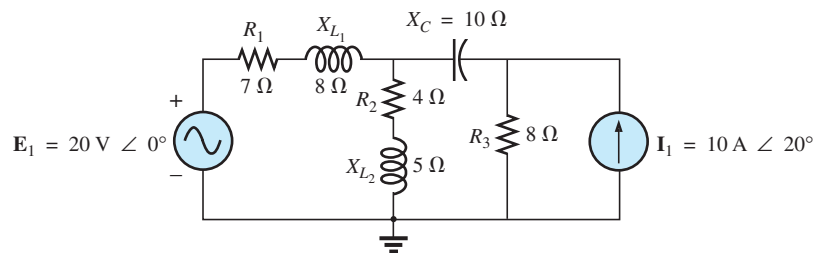**EXAMPLE 17.17** Using the format approach, write the nodal equations for the network in Fig. 17.34.

FIG. 17.34
Example 17.17.

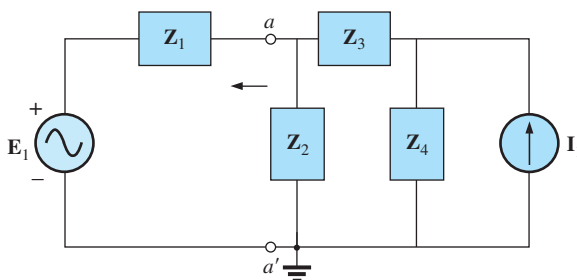***Solution:*** The circuit is redrawn in Fig. 17.35, where

FIG. 17.35
Assigning the subscripted impedances for the network in Fig. 17.34.

$$\begin{aligned}\mathbf{Z}_1 &= R_1 + j\,X_{L_1} = 7\,\Omega + j\,8\,\Omega \qquad \mathbf{E}_1 = 20\,\text{V}\,\angle 0° \\ \mathbf{Z}_2 &= R_2 + j\,X_{L_2} = 4\,\Omega + j\,5\,\Omega \qquad \mathbf{I}_1 = 10\,\text{A}\,\angle 20° \\ \mathbf{Z}_3 &= -j\,X_C = -j\,10\,\Omega \\ \mathbf{Z}_4 &= R_3 = 8\,\Omega\end{aligned}$$

Converting the voltage source to a current source and choosing nodes, we obtain Fig. 17.36. Note the "neat" appearance of the network using the subscripted impedances. Working directly with Fig. 17.34 would be more difficult and could produce errors.
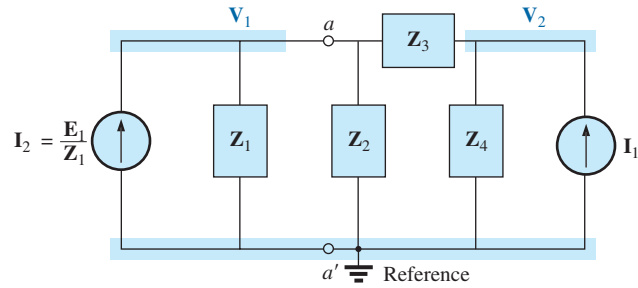
FIG. 17.36
Converting the voltage source in Fig. 17.35 to a current source and defining the nodal voltages.

Write the nodal equations:

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) = +\mathbf{I}_2$$
$$\mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_3) = +\mathbf{I}_1$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \quad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \quad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} \quad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4}$$

which are rewritten as

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) \qquad\quad = +\mathbf{I}_2$$
$$-\mathbf{V}_1(\mathbf{Y}_3) \qquad\qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) = +\mathbf{I}_1$$

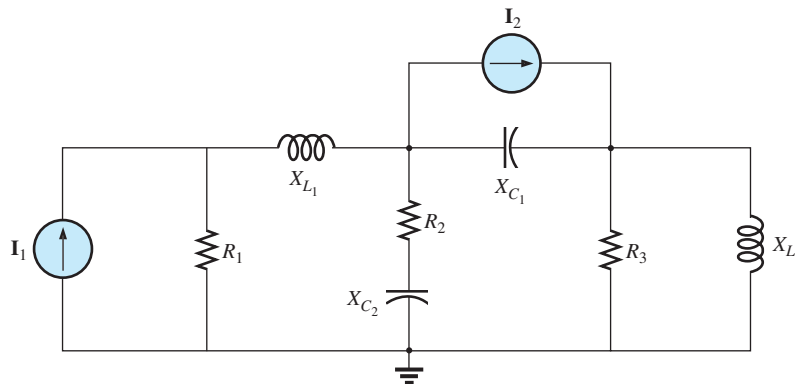**EXAMPLE 17.18** Write the nodal equations for the network in Fig. 17.37. Do not solve.

FIG. 17.37
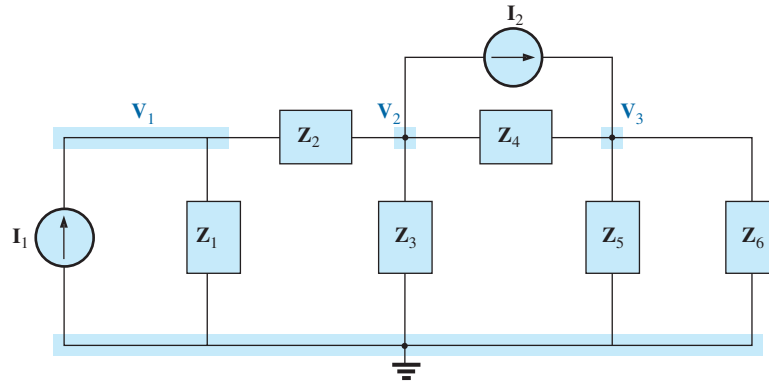*Example 17.18.*

Solution: Choose nodes (Fig. 17.38):

FIG. 17.38

Assigning the nodal voltages and subscripted impedances for the network in Fig. 17.35.

$$\mathbf{Z}_1 = R_1 \qquad \mathbf{Z}_2 = j\,X_{L_1} \quad \mathbf{Z}_3 = R_2 - j\,X_{C_2}$$
$$\mathbf{Z}_4 = -j\,X_{C_1} \quad \mathbf{Z}_5 = R_3 \qquad \mathbf{Z}_6 = j\,X_{L_2}$$

and write the nodal equations:

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) = +\mathbf{I}_1$$
$$\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_2) - \mathbf{V}_3(\mathbf{Y}_4) = -\mathbf{I}_2$$
$$\mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) - \mathbf{V}_2(\mathbf{Y}_4) = +\mathbf{I}_2$$

which are rewritten as

$$
\begin{array}{lll}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) & +\,0 & = +\mathbf{I}_1 \\
-\mathbf{V}_1(\mathbf{Y}_2) & +\,\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_3(\mathbf{Y}_4) & = -\mathbf{I}_2 \\
0 & -\,\mathbf{V}_2(\mathbf{Y}_4) \quad +\,\mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) & = +\mathbf{I}_2
\end{array}
$$

$$\mathbf{Y}_1 = \frac{1}{R_1} \qquad \mathbf{Y}_2 = \frac{1}{j\,X_{L_1}} \quad \mathbf{Y}_3 = \frac{1}{R_2 - j\,X_{C_2}}$$
$$\mathbf{Y}_4 = \frac{1}{-j\,X_{C_1}} \quad \mathbf{Y}_5 = \frac{1}{R_3} \qquad \mathbf{Y}_6 = \frac{1}{j\,X_{L_2}}$$

Note the symmetry about the diagonal for this example and those preceding it in this section.

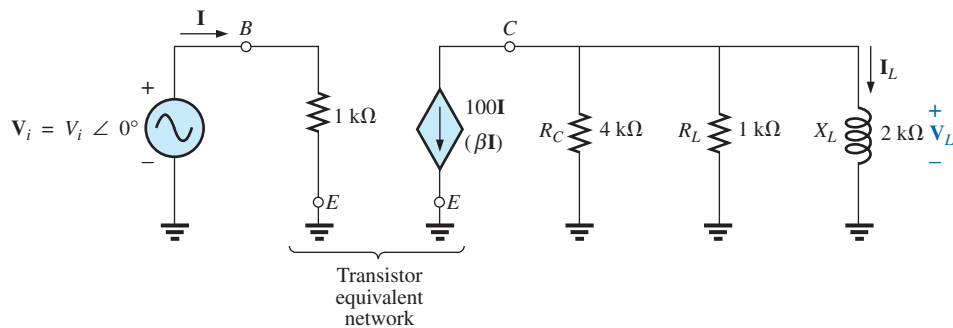**EXAMPLE 17.19** Apply nodal analysis to the network in Fig. 17.39. Determine the voltage $\mathbf{V}_L$.
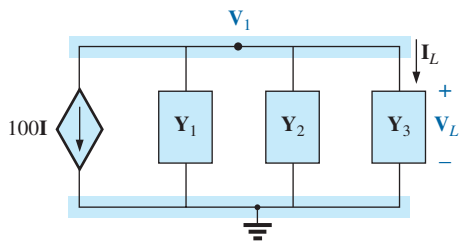

FIG. 17.39

Example 17.19.

FIG. 17.40

Assigning the nodal voltage and subscripted impedances for the network in Fig. 17.39.

Solution: In this case, there is no need for a source conversion. The network is redrawn in Fig. 17.40 with the chosen nodal voltage and subscripted impedances.

Apply the format approach:

$$Y_1 = \frac{1}{Z_1} = \frac{1}{4\ k\Omega} = 0.25\ mS\ \angle 0° = G_1\ \angle 0°$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{1\ k\Omega} = 1\ mS\ \angle 0° = G_2\ \angle 0°$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{2\ k\Omega\ \angle 90°} = 0.5\ mS\ \angle -90°$$

$$= -j\,0.5\ mS = -j\,B_L$$

$$V_1:\ (Y_1 + Y_2 + Y_3)V_1 = -100I$$

and

$$V_1 = \frac{-100I}{Y_1 + Y_2 + Y_3}$$

$$= \frac{-100I}{0.25\ mS + 1\ mS - j\,0.5\ mS}$$

$$= \frac{-100 \times 10^3 I}{1.25 - j\,0.5} = \frac{-100 \times 10^3 I}{1.3463\ \angle -21.80°}$$

$$= -74.28 \times 10^3 I\ \angle 21.80°$$

$$= -74.28 \times 10^3 \left(\frac{V_i}{1\ k\Omega}\right) \angle 21.80°$$

$$V_1 = V_L = -(74.28V_i)V\ \angle 21.80°$$

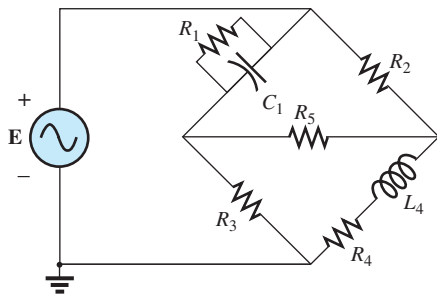## 17.6 BRIDGE NETWORKS (ac)

The basic bridge configuration was discussed in some detail in Section 8.11 for dc networks. We now continue to examine **bridge networks** by considering those that have reactive components and a sinusoidal ac voltage or current applied.

We first analyze various familiar forms of the bridge network using mesh analysis and nodal analysis (the format approach). The balance conditions are investigated throughout the section.

Apply **mesh analysis** to the network in Fig. 17.41. The network is redrawn in Fig. 17.42, where

$$Z_1 = \frac{1}{Y_1} = \frac{1}{G_1 + j\,B_C} = \frac{G_1}{G_1^2 + B_C^2} - j\,\frac{B_C}{G_1^2 + B_C^2}$$

$$Z_2 = R_2 \quad Z_3 = R_3 \quad Z_4 = R_4 + j\,X_L \quad Z_5 = R_5$$

Applying the format approach:

$$(Z_1 + Z_3)I_1 - (Z_1)I_2 - (Z_3)I_3 = E$$
$$(Z_1 + Z_2 + Z_5)I_2 - (Z_1)I_1 - (Z_5)I_3 = 0$$
$$(Z_3 + Z_4 + Z_5)I_3 - (Z_3)I_1 - (Z_5)I_2 = 0$$

which are rewritten as

$$I_1(Z_1 + Z_3) - I_2 Z_1 \qquad - I_3 Z_3 \qquad = E$$
$$-I_1 Z_1 \qquad + I_2(Z_1 + Z_2 + Z_5) - I_3 Z_5 \qquad = 0$$
$$-I_1 Z_3 \qquad - I_2 Z_5 \qquad + I_3(Z_3 + Z_4 + Z_5) = 0$$

FIG. 17.41
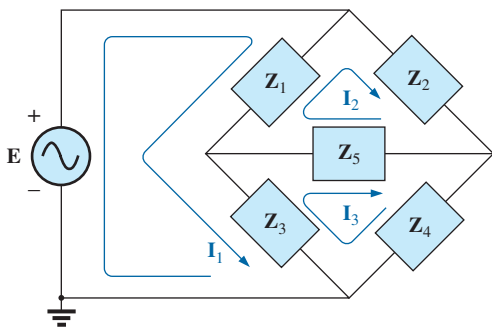
Maxwell bridge.

FIG. 17.42

Assigning the mesh currents and subscripted impedances for the network in Fig. 17.41.

Note the symmetry about the diagonal of the above equations. For balance, $\mathbf{I}_{Z_5} = 0$ A , and

$$\mathbf{I}_{Z_5} = \mathbf{I}_2 - \mathbf{I}_3 = 0$$

From the above equations,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & \mathbf{E} & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & 0 & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & 0 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & -\mathbf{Z}_1 & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & -\mathbf{Z}_5 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}$$

$$= \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_4 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

where Δ signifies the determinant of the denominator (or coefficients). Similarly,

$$\mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

and

$$\mathbf{I}_{Z_5} = \mathbf{I}_2 - \mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_4 - \mathbf{Z}_3\mathbf{Z}_2)}{\Delta}$$

For $\mathbf{I}_{Z_5} = 0$, the following must be satisfied (for a finite Δ not equal to zero):

$$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{I}_{Z_5} = 0 \qquad \qquad (17.3)$$

This condition is analyzed in greater depth later in this section.

Applying **nodal analysis** to the network in Fig. 17.43 results in the configuration in Fig. 17.44, where

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{R_1 - jX_C} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{R_2}$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{R_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4} = \frac{1}{R_4 + jX_L} \qquad \mathbf{Y}_5 = \frac{1}{R_5}$$

and

$$(\mathbf{Y}_1 + \mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_1)\mathbf{V}_2 - (\mathbf{Y}_2)\mathbf{V}_3 = \mathbf{I}$$
$$(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)\mathbf{V}_2 - (\mathbf{Y}_1)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_3 = 0$$
$$(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\mathbf{V}_3 - (\mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_2 = 0$$
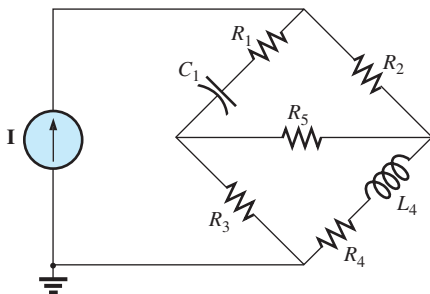

FIG. 17.43
Hay bridge.

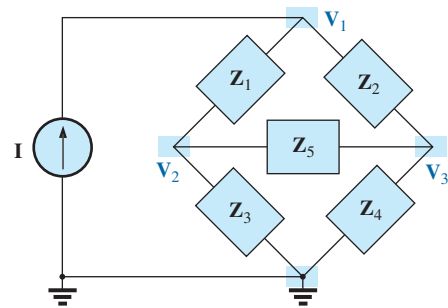**FIG. 17.44**
Assigning the nodal voltages and subscripted impedances for the network in Fig. 17.43.

which are rewritten as

$$\begin{aligned}
V_1(Y_1 + Y_2) - V_2Y_1 & & -V_3Y_2 & = I \\
-V_1Y_2 & + V_2(Y_1 + Y_3 + Y_5) - V_3Y_5 & & = 0 \\
-V_1Y_2 & - V_2Y_5 & + V_3(Y_2 + Y_4 + Y_5) & = 0
\end{aligned}$$

Again, note the symmetry about the diagonal axis. For balance, $V_{Z_5} = 0$ V, and

$$V_{Z_5} = V_2 - V_3 = 0$$

From the above equations,

$$V_2 = \frac{\begin{vmatrix} Y_1 + Y_2 & I & -Y_2 \\ -Y_1 & 0 & -Y_5 \\ -Y_2 & 0 & (Y_2 + Y_4 + Y_5) \end{vmatrix}}{\begin{vmatrix} Y_1 + Y_2 & -Y_1 & -Y_2 \\ -Y_1 & (Y_1 + Y_3 + Y_5) & -Y_5 \\ -Y_2 & -Y_5 & (Y_2 + Y_4 + Y_5) \end{vmatrix}}$$

$$= \frac{I(Y_1Y_3 + Y_1Y_4 + Y_1Y_5 + Y_3Y_5)}{\Delta}$$

Similarly,

$$V_3 = \frac{I(Y_1Y_3 + Y_3Y_2 + Y_1Y_5 + Y_3Y_5)}{\Delta}$$

Note the similarities between the above equations and those obtained for mesh analysis. Then

$$V_{Z_5} = V_2 - V_3 = \frac{I(Y_1Y_4 - Y_3Y_2)}{\Delta}$$

For $V_{Z_5} = 0$, the following must be satisfied for a finite Δ not equal to zero:

$$\boxed{Y_1Y_4 = Y_3Y_2} \qquad V_{Z_5} = 0 \qquad (17.4)$$

However, substituting $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$, $Y_3 = 1/Z_3$, and $Y_4 = 1/Z_4$, we have

$$\frac{1}{Z_1Z_4} = \frac{1}{Z_3Z_2}$$

or $\qquad \boxed{Z_1Z_4 = Z_3Z_2} \qquad V_{Z_5} = 0$

corresponding with Eq. (17.3) obtained earlier.

Let us now investigate the balance criteria in more detail by considering the network in Fig. 17.45, where it is specified that I and $V = 0$.

Since $I = 0$,

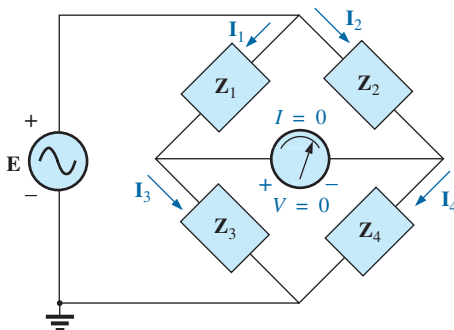$$\boxed{I_1 = I_3} \qquad\qquad (17.5a)$$

and $\qquad \boxed{I_2 = I_4} \qquad\qquad (17.5b)$

FIG. 17.45
Investigating the balance criteria for an ac bridge configuration.

In addition, for $\mathbf{V} = 0$,

$$\boxed{\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2} \tag{17.5c}$$

and

$$\boxed{\mathbf{I}_3\mathbf{Z}_3 = \mathbf{I}_4\mathbf{Z}_4} \tag{17.5d}$$

Substituting the preceding current relations into Eq. (17.5d), we have

$$\mathbf{I}_1\mathbf{Z}_3 = \mathbf{I}_2\mathbf{Z}_4$$

and

$$\mathbf{I}_2 = \frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1$$

Substituting this relationship for $\mathbf{I}_2$ into Eq. (17.5c) yields

$$\mathbf{I}_1\mathbf{Z}_1 = \left(\frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1\right)\mathbf{Z}_2$$

and

$$\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$$

as obtained earlier. Rearranging, we have

$$\boxed{\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}} \tag{17.6}$$

corresponding with Eq. (8.2) for dc resistive networks.

For the network in Fig. 17.43, which is referred to as a **Hay bridge** when $\mathbf{Z}_5$ is replaced by a sensitive galvanometer,

$$\mathbf{Z}_1 = R_1 - j\,X_C$$
$$\mathbf{Z}_2 = R_2$$
$$\mathbf{Z}_3 = R_3$$
$$\mathbf{Z}_4 = R_4 + j\,X_L$$

This particular network is used for measuring the resistance and inductance of coils in which the resistance is a small fraction of the reactance X_L.

Substitute into Eq. (17.6) in the following form:

$$\mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_4\mathbf{Z}_1$$
$$R_2R_3 = (R_4 + j\,X_L)(R_1 - j\,X_C)$$

or

$$R_2R_3 = R_1R_4 + j\,(R_1X_L - R_4X_C) + X_C\,X_L$$

so that

$$R_2R_3 + j\,0 = (R_1R_4 + X_C\,X_L) + j\,(R_1X_L - R_4X_C)$$

For the equations to be equal, *the real and imaginary parts must be equal.* Therefore, for a balanced Hay bridge,

$$\boxed{R_2R_3 = R_1R_4 + X_C X_L} \tag{17.7a}$$

and

$$\boxed{0 = R_1X_L - R_4X_C} \tag{17.7b}$$

or substituting

$$X_L = \omega L \quad \text{and} \quad X_C = \frac{1}{\omega C}$$

we have
$$X_C X_L = \left(\frac{1}{\omega C}\right)(\omega L) = \frac{L}{C}$$

and
$$R_2 R_3 = R_1 R_4 + \frac{L}{C}$$

with
$$R_1 \omega L = \frac{R_4}{\omega C}$$

Solving for R_4 in the last equation yields
$$R_4 = \omega^2 L C R_1$$

and substituting into the previous equation, we have
$$R_2 R_3 = R_1(\omega^2 L C R_1) + \frac{L}{C}$$

Multiply through by C and factor:
$$C R_2 R_3 = L(\omega^2 C^2 R_1^2 + 1)$$

and
$$\boxed{L = \frac{C R_2 R_3}{1 + \omega^2 C^2 R_1^2}} \tag{17.8a}$$

With additional algebra this yields:

$$\boxed{R_4 = \frac{\omega^2 C^2 R_1 R_2 R_3}{1 + \omega^2 C^2 R_1^2}} \tag{17.8b}$$

Eqs. (17.7) and (17.8) are the balance conditions for the Hay bridge. Note that each is frequency dependent. For different frequencies, the resistive and capacitive elements must vary for a particular coil to achieve balance. For a coil placed in the Hay bridge as shown in Fig. 17.43, the resistance and inductance of the coil can be determined by Eqs. (17.8a) and (17.8b) when balance is achieved.

The bridge in Fig. 17.41 is referred to as a **Maxwell bridge** when Z_5 is replaced by a sensitive galvanometer. This setup is used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Application of Eq. (17.6) in the form:
$$\mathbf{Z_2 Z_3} = \mathbf{Z_4 Z_1}$$

and substituting

$$\mathbf{Z_1} = R_1 \angle 0° \parallel X_{C_1} \angle -90° = \frac{(R_1 \angle 0°)(X_{C_1} \angle -90°)}{R_1 - j X_{C_1}}$$

$$= \frac{R_1 X_{C_1} \angle -90°}{R_1 - j X_{C_1}} = \frac{-j R_1 X_{C_1}}{R_1 - j X_{C_1}}$$

$$\mathbf{Z_2} = R_2$$
$$\mathbf{Z_3} = R_3$$
and $$\mathbf{Z_4} = R_4 + j X_{L_4}$$

we have
$$(R_2)(R_3) = (R_4 + j X_{L_4})\left(\frac{-j R_1 X_{C_1}}{R_1 - j X_{C_1}}\right)$$

$$R_2 R_3 = \frac{-j R_1 R_4 X_{C_1} + R_1 X_{C_1} X_{L_4}}{R_1 - j X_{C_1}}$$

or $\qquad (R_2R_3)(R_1 - j X_{C_1}) = R_1 X_{C_1} X_{L_4} - j R_1 R_4 X_{C_1}$

and $\qquad R_1R_2R_3 - j R_2R_3X_{C_1} = R_1 X_{C_1} X_{L_4} - j R_1 R_4 X_{C_1}$

so that for balance

$$\cancel{R_1}R_2R_3 = \cancel{R_1}X_{C_1}X_{L_4}$$

$$R_2 R_3 = \left(\frac{1}{\cancel{2\pi f}C_1} \right) (\cancel{2\pi f}L_4)$$

and $\qquad \boxed{L_4 = C_1R_2R_3} \qquad\qquad$ **(17.9)**

and $\qquad R_2R_3 \cancel{X}_{C_1} = R_1 R_4 \cancel{X}_{C_1}$

so that $\qquad \boxed{R_4 = \dfrac{R_2R_3}{R_1}} \qquad\qquad$ **(17.10)**

Note the absence of frequency in Eqs. (17.9) and (17.10).

One remaining popular bridge is the **capacitance comparison bridge** of Fig. 17.46. An unknown capacitance and its associated resistance can be determined using this bridge. Application of Eq. (17.6) yields the following results:

$$\boxed{C_4 = C_3\frac{R_1}{R_2}} \qquad\qquad \textbf{(17.11)}$$

$$\boxed{R_4 = \frac{R_2R_3}{R_1}} \qquad\qquad \textbf{(17.12)}$$
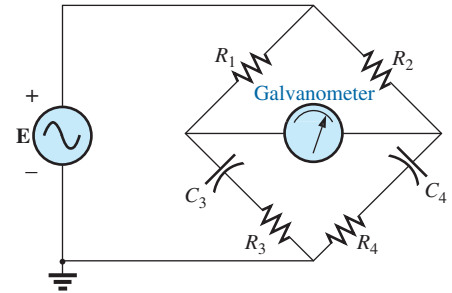
FIG. 17.46
Capacitance comparison bridge.

The derivation of these equations appears as a problem at the end of the chapter.

17.7 Δ-Y, Y-Δ CONVERSIONS

The Δ-Y, Y-Δ (or π-T, T-π as defined in Section 8.12) conversions for ac circuits are not derived here since the development corresponds exactly with that for dc circuits. Taking the **Δ-Y configuration** shown in Fig. 17.47, we find the general equations for the impedances of the Y in terms of those for the Δ:

$$\boxed{\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}} \qquad\qquad \textbf{(17.13)}$$

$$\boxed{\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}} \qquad\qquad \textbf{(17.14)}$$

$$\boxed{\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}} \qquad\qquad \textbf{(17.15)}$$
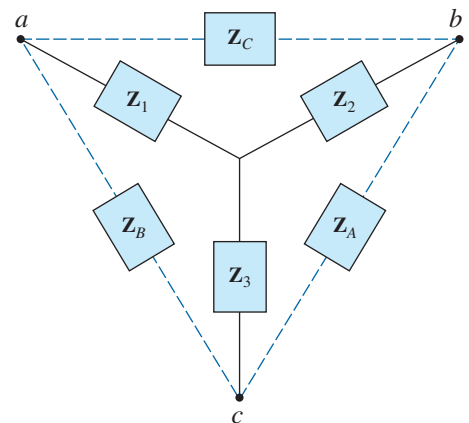

FIG. 17.47
Δ-Y configuration.

For the impedances of the Δ in terms of those for the Y, the equations are

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2}$$

(17.16)

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}$$

(17.17)

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3}$$

(17.18)

Note that each impedance of the Y is equal to the product of the impedances in the two closest branches of the Δ, divided by the sum of the impedances in the Δ.

Further, the value of each impedance of the Δ is equal to the sum of the possible product combinations of the impedances of the Y, divided by the impedances of the Y farthest from the impedance to be determined.

Drawn in different forms (Fig. 17.48), they are also referred to as the T and π configurations.
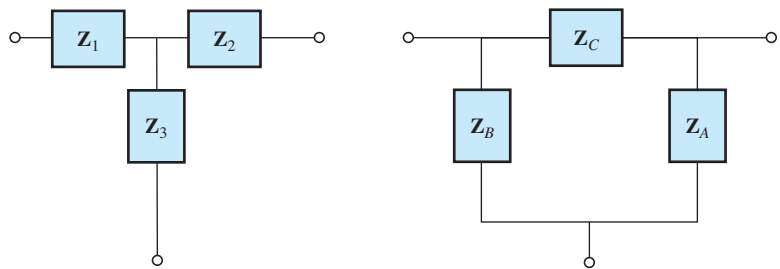
FIG. 17.48
The T and π configurations.

In the study of dc networks, we found that if all of the resistors of the Δ or Y were the same, the conversion from one to the other could be accomplished using the equation

$$R_\Delta = 3R_Y \quad \text{or} \quad R_Y = \frac{R_\Delta}{3}$$

For ac networks,

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

(17.19)

Be careful when using this simplified form. It is not sufficient for all the impedances of the Δ or Y to be of the same magnitude: *The angle associated with each must also be the same.*

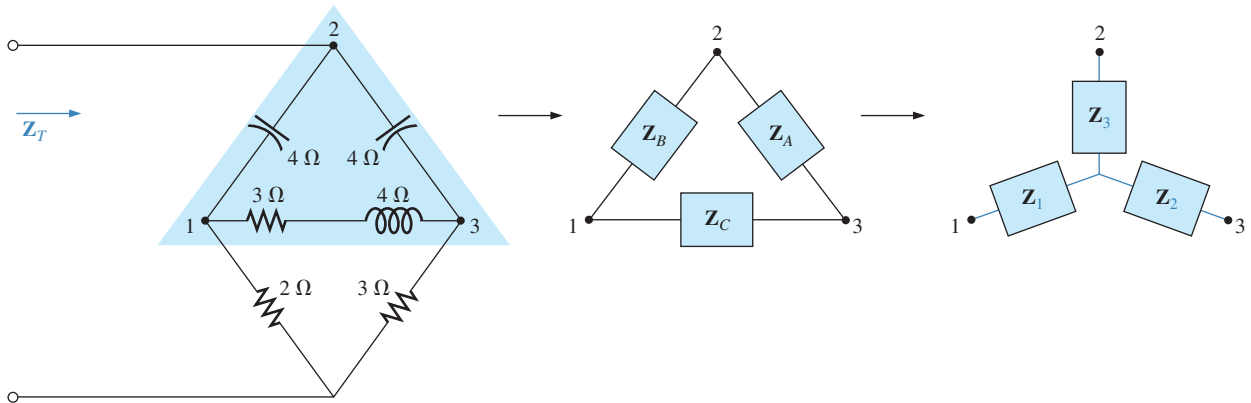**EXAMPLE 17.20** Find the total impedance $\mathbf{Z}_T$ of the network in Fig. 17.49.

FIG. 17.49

Converting the upper Δ of a bridge configuration to a Y.

Solution:

$$\mathbf{Z}_B = -j\,4 \qquad \mathbf{Z}_A = -j\,4 \qquad \mathbf{Z}_C = 3 + j\,4$$

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(3\,\Omega + j\,4\,\Omega)}{(-j\,4\,\Omega) + (-j\,4\,\Omega) + (3\,\Omega + j\,4\,\Omega)}$$

$$= \frac{(4\,\angle-90°)(5\,\angle53.13°)}{3 - j\,4} = \frac{20\,\angle-36.87°}{5\,\angle-53.13°}$$

$$= 4\,\Omega\,\angle16.13° = 3.84\,\Omega + j\,1.11\,\Omega$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(3\,\Omega + j\,4\,\Omega)}{5\,\Omega\,\angle-53.13°}$$

$$= 4\,\Omega\,\angle16.13° = 3.84\,\Omega + j\,1.11\,\Omega$$

Recall from the study of dc circuits that if two branches of the Y or Δ are the same, the corresponding Δ or Y, respectively, will also have two similar branches. In this example, $\mathbf{Z}_A = \mathbf{Z}_B$. Therefore, $\mathbf{Z}_1 = \mathbf{Z}_2$, and

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(-j\,4\,\Omega)}{5\,\Omega\,\angle-53.13°}$$

$$= \frac{16\,\Omega\,\angle-180°}{5\,\angle-53.13°} = 3.2\,\Omega\,\angle-126.87° = -1.92\,\Omega - j\,2.56\,\Omega$$

Replace the Δ by the Y (Fig. 17.50):

$$\mathbf{Z}_1 = 3.84\,\Omega + j\,1.11\,\Omega \qquad \mathbf{Z}_2 = 3.84\,\Omega + j\,1.11\,\Omega$$
$$\mathbf{Z}_3 = -1.92\,\Omega - j\,2.56\,\Omega \qquad \mathbf{Z}_4 = 2\,\Omega$$
$$\mathbf{Z}_5 = 3\,\Omega$$

Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_4$ are in series:

$$\mathbf{Z}_{T_1} = \mathbf{Z}_1 + \mathbf{Z}_4 = 3.84\,\Omega + j\,1.11\,\Omega + 2\,\Omega = 5.84\,\Omega + j\,1.11\,\Omega$$
$$= 5.94\,\Omega\,\angle10.76°$$

Impedances $\mathbf{Z}_2$ and $\mathbf{Z}_5$ are in series:

$$\mathbf{Z}_{T_2} = \mathbf{Z}_2 + \mathbf{Z}_5 = 3.84\,\Omega + j\,1.11\,\Omega + 3\,\Omega = 6.84\,\Omega + j\,1.11\,\Omega$$
$$= 6.93\,\Omega\,\angle9.22°$$
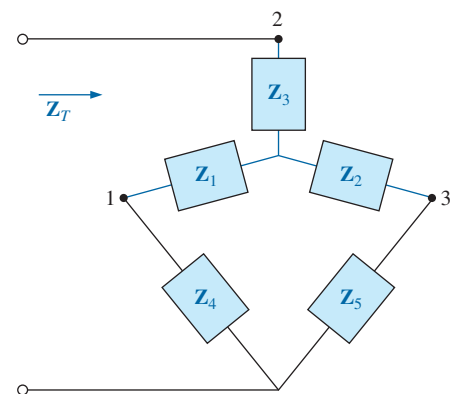
FIG. 17.50

The network in Fig. 17.49 following the substitution of the Y configuration.

Impedances $\mathbf{Z}_{T_1}$ and $\mathbf{Z}_{T_2}$ are in parallel:

$$\mathbf{Z}_{T_3} = \frac{\mathbf{Z}_{T_1}\mathbf{Z}_{T_2}}{\mathbf{Z}_{T_1} + \mathbf{Z}_{T_2}} = \frac{(5.94\ \Omega\ \angle 10.76°)(6.93\ \Omega\ \angle 9.22°)}{5.84\ \Omega\ + j\,1.11\ \Omega\ + 6.84\ \Omega\ + j\,1.11\ \Omega}$$

$$= \frac{41.16\ \Omega\ \angle 19.98°}{12.68 + j\,2.22} = \frac{41.16\ \Omega\ \angle 19.98°}{12.87\ \angle 9.93°} = 3.198\ \Omega\ \angle 10.05°$$

$$= 3.15\ \Omega\ + j\,0.56\ \Omega$$

Impedances $\mathbf{Z}_3$ and $\mathbf{Z}_{T_3}$ are in series. Therefore,

$$\mathbf{Z}_T = \mathbf{Z}_3 + \mathbf{Z}_{T_3} = -1.92\ \Omega\ - j\,2.56\ \Omega\ + 3.15\ \Omega\ + j\,0.56\ \Omega$$

$$= 1.23\ \Omega\ - j\,2.0\ \Omega\ = \mathbf{2.35\ \Omega\ \angle -58.41°}$$

EXAMPLE 17.21 Using both the Δ-Y and Y-Δ transformations, find the total impedance $\mathbf{Z}_T$ for the network in Fig. 17.51.
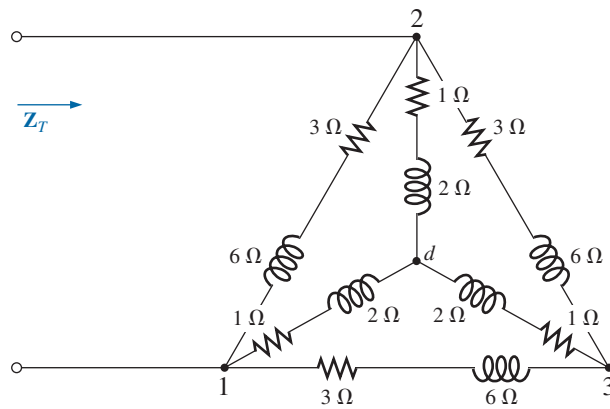
FIG. 17.51
Example 17.21.

Solution: *Using the Δ-Y transformation,* we obtain Fig. 17.52. In this case, since both systems are balanced (same impedance in each branch), the center point d' of the transformed Δ will be the same as point d of the original Y:

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = \frac{3\ \Omega\ + j\,6\ \Omega}{3} = 1\ \Omega\ + j\,2\ \Omega$$
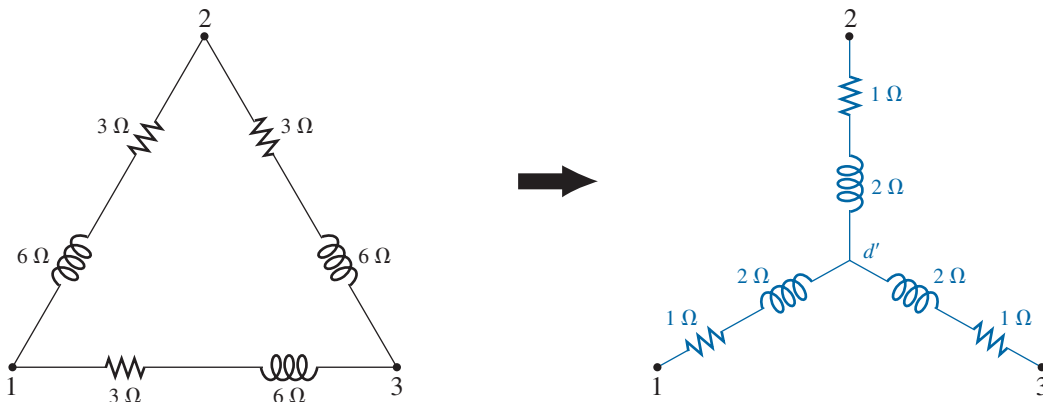
FIG. 17.52
Converting a Δ configuration to a Y configuration.

and (Fig. 17.53)

$$\mathbf{Z}_T = 2 \left(\frac{1\,\Omega + j\,2\,\Omega}{2} \right) = \mathbf{1\,\Omega + j\,2\,\Omega}$$
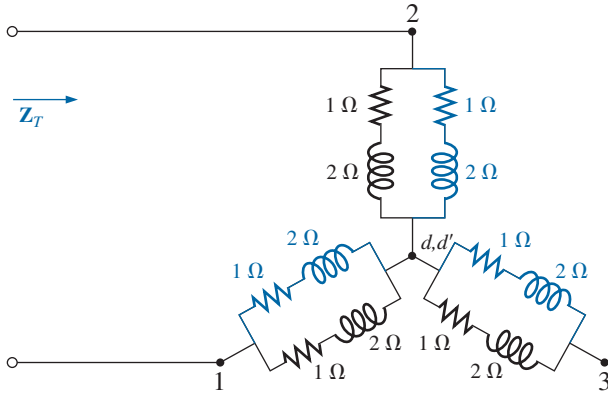
FIG. 17.53

Substituting the Y configuration in Fig. 17.52 into the network in Fig. 17.51.

Using the Y-Δ transformation (Fig. 17.54), we obtain

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(1\,\Omega + j\,2\,\Omega) = 3\,\Omega + j\,6\,\Omega$$
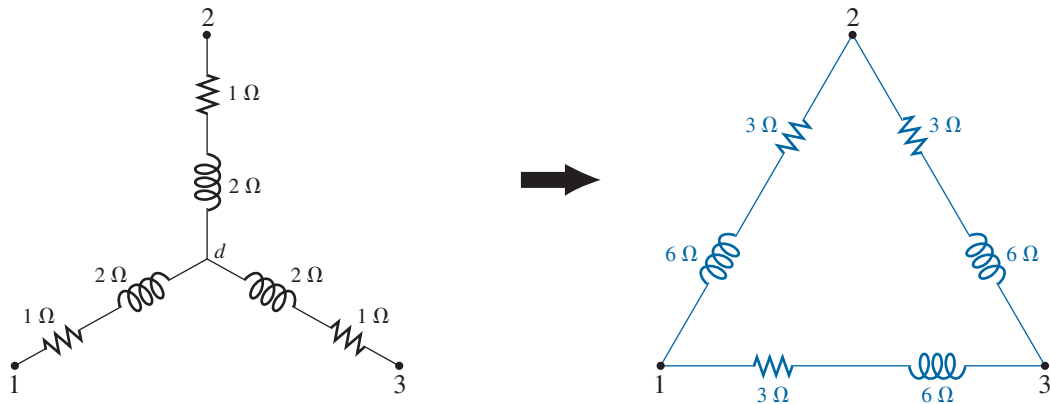
FIG. 17.54

Converting the Y configuration in Fig. 17.51 to a Δ.

Each resulting parallel combination in Fig. 17.55 will have the following impedance:

$$\mathbf{Z}' = \frac{3\,\Omega + j\,6\,\Omega}{2} = 1.5\,\Omega + j\,3\,\Omega$$

and

$$\mathbf{Z}_T = \frac{\mathbf{Z}'\,(2\mathbf{Z}')}{\mathbf{Z}' + 2\mathbf{Z}'} = \frac{2(\mathbf{Z}')^2}{3\mathbf{Z}'} = \frac{2\mathbf{Z}'}{3}$$

$$= \frac{2(1.5\,\Omega + j\,3\,\Omega)}{3} = \mathbf{1\,\Omega + j\,2\,\Omega}$$

which compares with the above result.
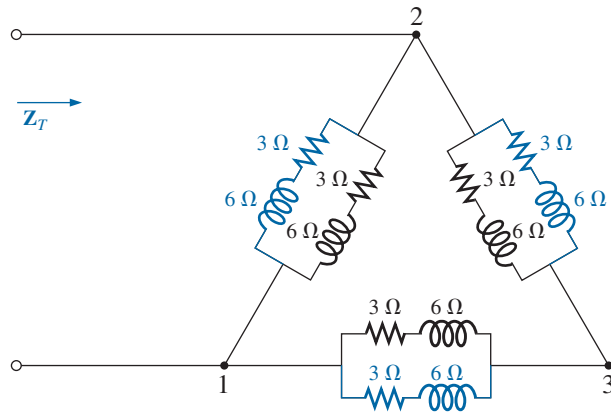
FIG. 17.55

Substituting the Δ configuration in Fig. 17.54 into the network in Fig. 17.51.

17.8 COMPUTER ANALYSIS

PSpice

Nodal Analysis The first application of PSpice is to determine the nodal voltages for the network in Example 17.16 and compare solutions. The network appears as shown in Fig. 17.56 using elements that were determined from the reactance level at a frequency of 1 kHz. There is no need to continually use 1 kHz. Any frequency will do, but remember to use the chosen frequency to find the network components and when setting up the simulation.

For the current sources, choose **ISIN** so that the phase angle can be specified (even though it is 0°), although the symbol does not have the arrow used in the text material. The direction must be recognized as pointing from the + to − sign of the source. That requires that the sources $\mathbf{I}_1$ and $\mathbf{I}_2$ be set as shown in Fig. 17.56. Reverse the source $\mathbf{I}_2$ by using the **Mirror Vertically** option obtained by right-clicking the source symbol on the screen. Setting up the **ISIN** source is the same as that used with the **VSIN** source. It can be found under the **SOURCE** library, and its attributes are the same as for the **VSIN** source. For each source, set **IOFF** to 0 A; the amplitude is the peak value of the source current. The frequency will be the same for each source. Then select **VPRINT1** from the
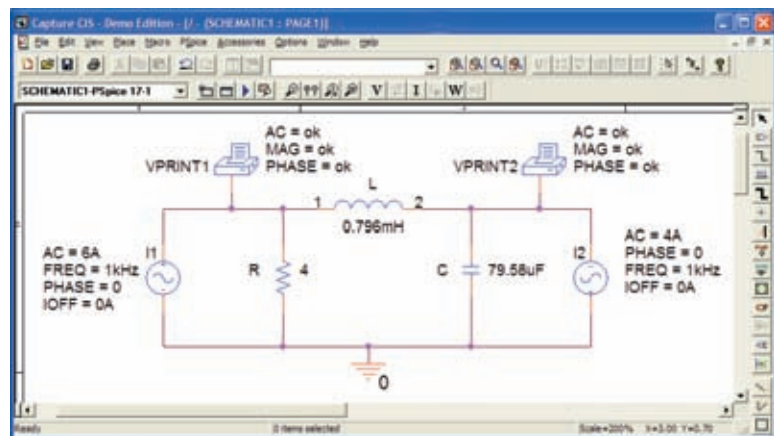
FIG. 17.56

Using PSpice to verify the results in Example 17.16.

SPECIAL library and place it to generate the desired nodal voltages. Finally add the remaining elements to the network as shown in Fig. 17.56. For each source, double-click the symbol to generate the **Property Editor** dialog box. Set **AC** at the 6 A level for the I_1 source and at 4 A for the I_2 source, followed by **Display** and **Name and Value** for each. It appears as shown in Fig. 17.56. Double-clicking on each **VPRINT1** option also provides the **Property Editor,** so **OK** can be added under **AC, MAG,** and **PHASE.** For each quantity, select **Display** followed by **Name and Value** and **OK.** Then select **Value** and **VPRINT1** is displayed as **Value** only. Selecting **Apply** and leaving the dialog box results in the listing next to each source in Fig. 17.56. For **VPRINT2,** first change the listing on **Value** from **VPRINT1** to **VPRINT2** before selecting **Display** and **Apply.**

Now select the **New Simulation Profile** icon, and enter **PSpice 17-1** as the **Name** followed by **Create.** In the **Simulation Settings** dialog box, select **AC Sweep,** and set the **Start Frequency** and **End Frequency** at 1 kHz with 1 for the **Points/Decade.** Click **OK,** and select the **Run PSpice** icon; a **SCHEMATIC1** screen results. Exiting (**X**) brings you back to the **Orcad Capture** window. Selecting **PSpice** followed by **View Output File** results in the display in Fig. 17.57, providing exactly the same results as obtained in Example 17.16 with $V_1 = 20.8 \text{ V} \angle -126.9°$. The other nodal voltage is $8.617 \text{ V} \angle -15.09°$.

```
** Profile: "SCHEMATIC1-PSpice 17-1" | C:\ICA11\PSpice\PSpice 17-1-
PSpiceFiles\SCHEMATIC1\PSpice 17-1.sim ]


****   AC ANALYSIS                 TEMPERATURE =  27.000 DEG C
****************************************************************************

FREQ          VM(N01900)    VP(N01900)
1.000E+03     2.080E+01     -1.269E+02

FREQ          VM(N01908)    VP(N01908)
1.000E+03     8.617E+00     -1.509E+01
```

FIG. 17.57

Output file for the nodal voltages for the network in Fig. 17.56.

Current-Controlled Current Source (CCCS) Our interest now turns to controlled sources in the PSpice environment. Controlled sources are not particularly difficult to apply once a few important elements of their use are understood. The network in Fig. 17.14 has a current-controlled current source in the center leg of the configuration. The magnitude of the current source is k times the current through resistor R_1, where k can be greater or less than 1. The resulting schematic, appearing in Fig. 17.58, seems quite complex in the area of the controlled source, but once you understand the role of each component, the schematic is not that difficult to understand. First, since it is the only new element in the schematic, let us concentrate on the controlled source. Current-controlled current sources (CCCS) are called up under the **ANALOG** library as **F** and appear as shown in the center in Fig. 17.58. Pay attention to the direction of the current in each part of the symbol. In particular, note that the sensing current of **F** has the same direction as the defining controlling current in Fig. 17.14. In addition, note that the controlled current source also has the same direction as the source in Fig. 17.14. If you double-click on the CCCS symbol, the **Property Editor** dialog box appears with the **GAIN** (k as described above) set at 1. In this example, the gain must be set at **0.7,** so click on the region below the **GAIN** label and enter **0.7.** Then select **Display** followed by **Name and Value-OK.** Exit the **Property Editor,** and **GAIN = 0.7** appears with the CCCS as shown in Fig. 17.58.
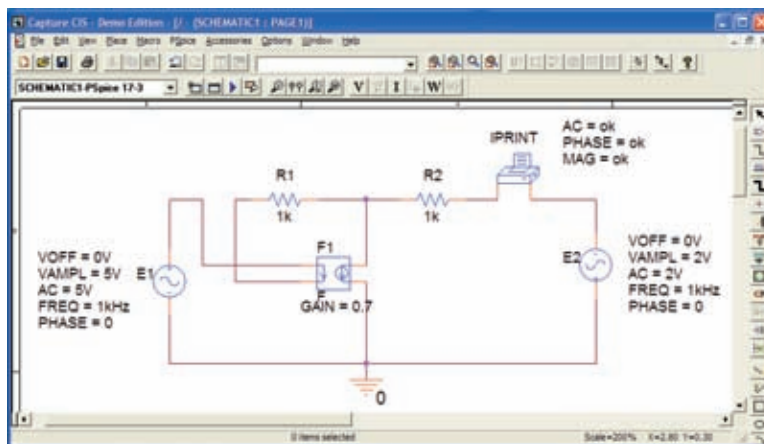
FIG. 17.58
Using PSpice to verify the results in Example 17.8.

The other new component in this schematic is **IPRINT;** it can be found in the **SPECIAL** library. It is used to tell the program to list the current in the branch of interest in the output file. If you fail to tell the program which output data you would like, it will simply run through the simulation and list specific features of the network but will not provide any voltages or currents. In this case, the current I_2 through the resistor R_2 is desired. Double-clicking on the **IPRINT** component results in the **Property Editor** dialog box with a number of elements that need to be defined—much like that for **VPRINT.** First enter **OK** beneath **AC** and follow with **Display-Name and Value-OK.** Repeat for **MAG** and **PHASE,** and then select **Apply** before leaving the dialog box. The **OK** tells the software program that these are the quantities that it is "ok" to generate and provide. The purpose of the **Apply** at the end of each visit to the **Property Editor** dialog box is to "apply" the changes made to the network under investigation. When you exit the **Property Editor,** the three chosen parameters appear on the schematic with the **OK** directive. You may find that the labels will appear all over the **IPRINT** symbol. No problem—just click on each, and move to a more convenient location.

The remaining components of the network should be fairly familiar, but don't forget to **Mirror Vertically** the voltage source **E2.** In addition, do not forget to call up the **Property Editor** for each source and set the level of **AC, FREQ, VAMPL,** and **VOFF** and be sure that the **PHASE** is set on the default value of 0°. The value appears with each parameter in Fig. 17.58 for each source. Always be sure to select **Apply** before leaving the **Property Editor.** After placing all the components on the screen, you must connect them with a **Place wire** selection. Normally, this is pretty straightforward. However, with controlled sources, it is often necessary to cross over wires *without* making a connection. In general, when you're placing a wire over another wire and you don't want a connection to be made, click a spot on one side of the wire to be crossed to create the temporary red square. Then cross the wire, and click again to establish another red square. If the connection is done properly, the crossed wire should not show a connection point (a small red dot). In this example, the top of the controlling current was connected first from the **E1** source. Then a wire was connected from the lower end of the sensing current to the point where a 90° turn up the page was to be made. The wire was clicked in place at this point before crossing the original wire and clicked again before making the right turn to resistor R_1. You will not find a small red dot where the wires cross.

Now for the simulation. In the **Simulation Settings** dialog box, select **AC Sweep/Noise** with a **Start** and **End Frequency** of 1 kHz. There will be 1 **Point/Decade.** Click **OK,** and select the **Run Spice** key; a **SCHEMATIC1** results that should be exited to obtain the **Orcad Capture** screen. Select **PSpice** followed by **View Output File,** and scroll down until you read **AC ANALYSIS** (see Fig. 17.59). The magnitude of the desired current is 1.615 mA with a phase angle of 0°, a perfect match with the theoretical analysis to follow. One would expect a phase angle of 0° since the network is composed solely of resistive elements.

```
** Profile: "SCHEMATIC1-PSpice 17-3"  | C:\ICA11\PSpice\PSpice 17-3-
PSpiceFiles\SCHEMATIC1\PSpice 17-3.sim ]

****    AC ANALYSIS                  TEMPERATURE =   27.000 DEG C
******************************************************************

 FREQ        IM(V_PRINT1)         IP(V_PRINT1)
 1.000E+03   1.615E-03            0.000E+00
```

FIG. 17.59

The output file for the mesh current I_2 in Fig. 17.14.

The equations obtained earlier using the supermesh approach were

$$\mathbf{E} - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0 \quad \text{or} \quad \mathbf{I}_1\mathbf{Z}_1 + \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 + \mathbf{E}_2$$

and

$$k\mathbf{I} = k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$$

resulting in

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{1-k} = \frac{\mathbf{I}_2}{1-0.7} = \frac{\mathbf{I}_2}{0.3} = 3.333\mathbf{I}_2$$

so that $\mathbf{I}_1(1\text{ k}\Omega) + \mathbf{I}_2(1\text{ k}\Omega) = 7\text{ V}$ (from above)

becomes $(3.333\mathbf{I}_2)1\text{ k}\Omega + \mathbf{I}_2(1\text{ k}\Omega) = 7\text{ V}$

or $(4.333\text{ k}\Omega)\mathbf{I}_2 = 7\text{ V}$

and

$$\mathbf{I}_2 = \frac{7\text{ V}}{4.333\text{ k}\Omega} = \textbf{1.615 mA} \angle\textbf{0°}$$

confirming the computer solution.

PROBLEMS

SECTION 17.2   Independent versus Dependent (Controlled) Sources

1. Discuss, in your own words, the difference between a controlled and an independent source.

SECTION 17.3   Source Conversions

2. Convert the voltage sources in Fig. 17.60 to current sources.

(a)

(b)
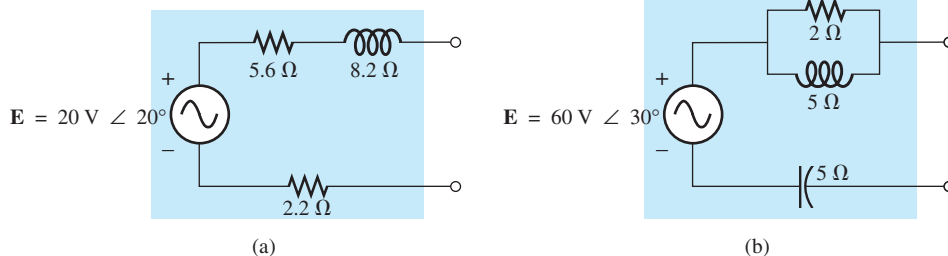
FIG. 17.60

Problem 2.

3. Convert the current sources in Fig. 17.61 to voltage sources.

(a)   (b)
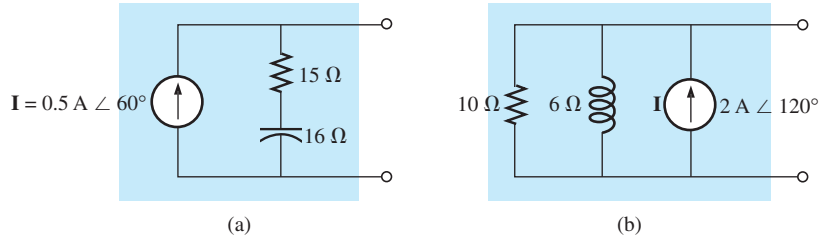
FIG. 17.61
Problem 3.

4. Convert the voltage source in Fig. 17.62(a) to a current source
and the current source in Fig. 17.62(b) to a voltage source.

(a)   (b)
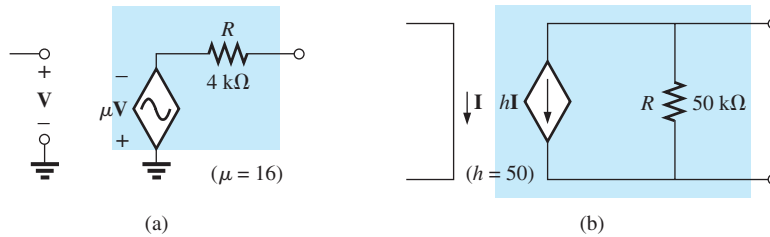
FIG. 17.62
Problem 4.

SECTION 17.4 Mesh Analysis

5. Write the mesh equations for the networks in Fig. 17.63. De-
termine the current through the resistor *R*.

(a)   (b)
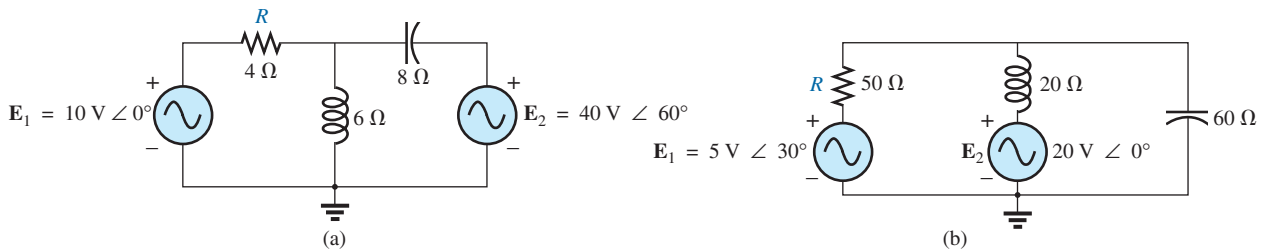
FIG. 17.63
Problems 5 and 34.

6. Write the mesh equations for the networks in Fig. 17.64. De-
termine the current through the resistor R_1.

(a)   (b)
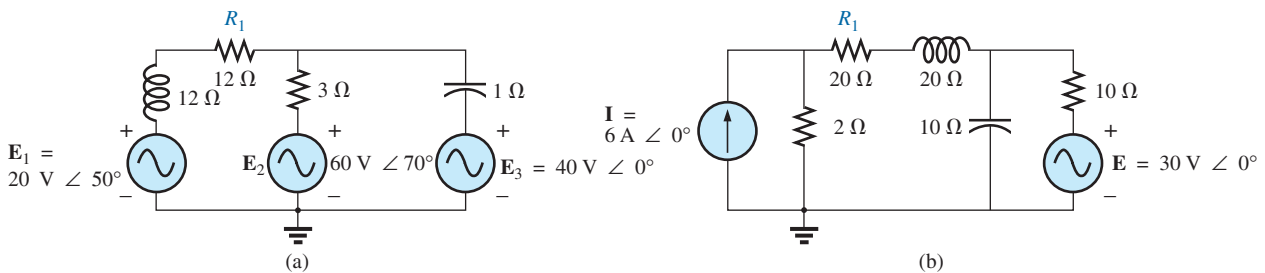
FIG. 17.64
Problems 6 and 16.

*7. Write the mesh equations for the networks in Fig. 17.65. Determine the current through the resistor R_1.

(a)

(b)
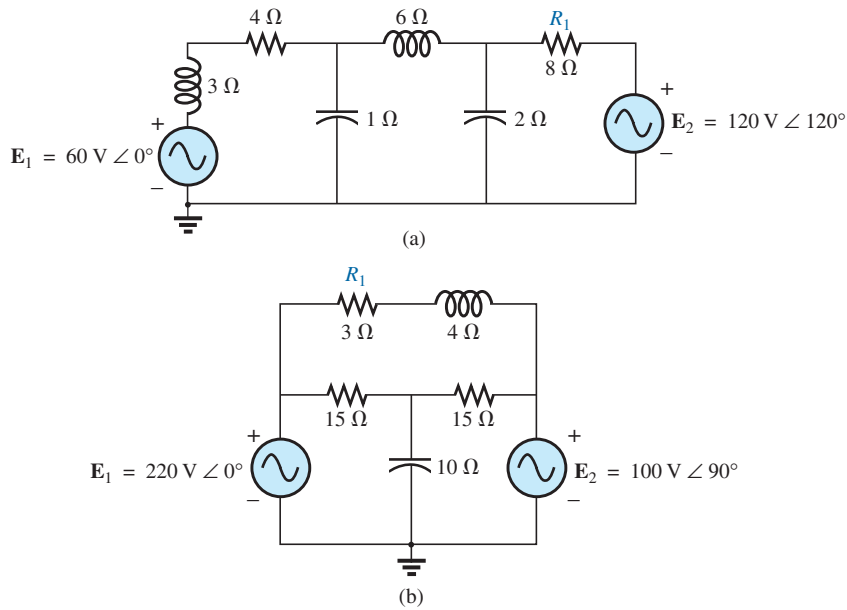
FIG. 17.65

Problems 7, 17, and 35.

*8. Write the mesh equations for the networks in Fig. 17.66. Determine the current through the resistor R_1.

(a)

(b)
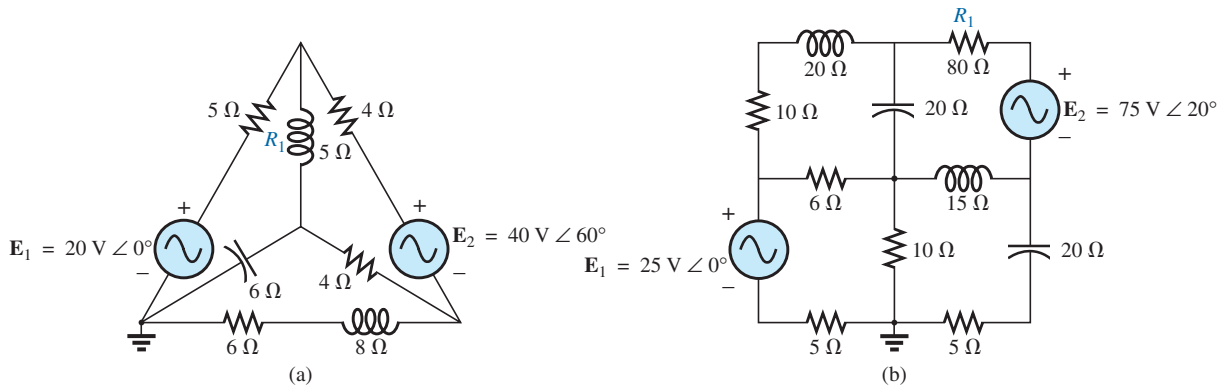
FIG. 17.66

Problems 8, 18, and 19.

9. Using mesh analysis, determine the current I_L (in terms of **V**) for the network in Fig. 17.67.
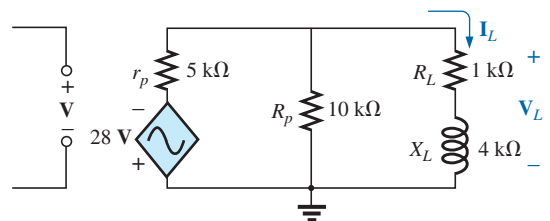
FIG. 17.67

Problem 9.

*10. Using mesh analysis, determine the current I_L (in terms of I) for the network in Fig. 17.68.
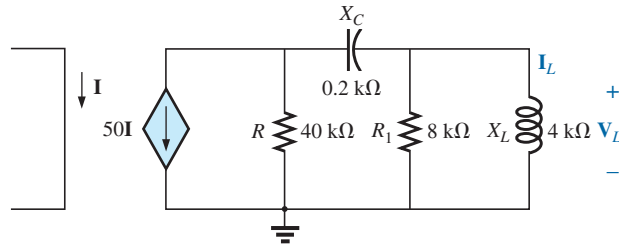
FIG. 17.68
Problem 10.

*11. Write the mesh equations for the network in Fig. 17.69, and determine the current through the 1 kΩ and 2 kΩ resistors.
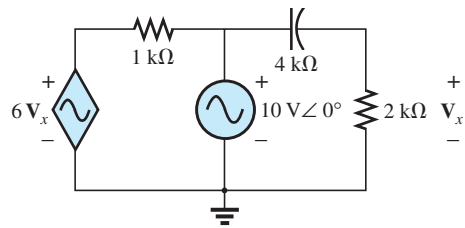
FIG. 17.69
Problems 11 and 36.

*12. Write the mesh equations for the network in Fig. 17.70, and determine the current through the 10 kΩ resistor.
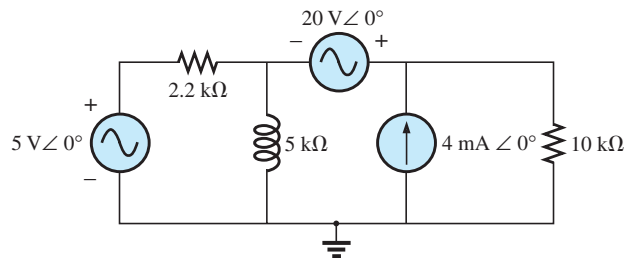
FIG. 17.70
Problems 12 and 37.

*13. Write the mesh equations for the network in Fig. 17.71, and determine the current through the inductive element.
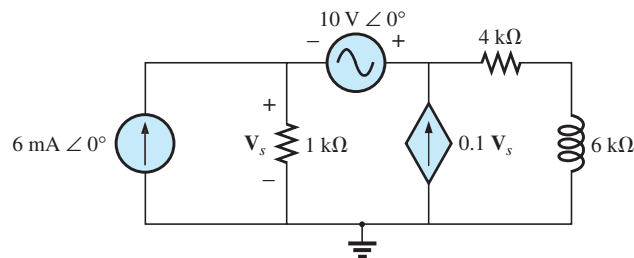
FIG. 17.71
Problems 13 and 38.

SECTION 17.5 Nodal Analysis

14. Determine the nodal voltages for the networks in Fig. 17.72.

(a)

(b)
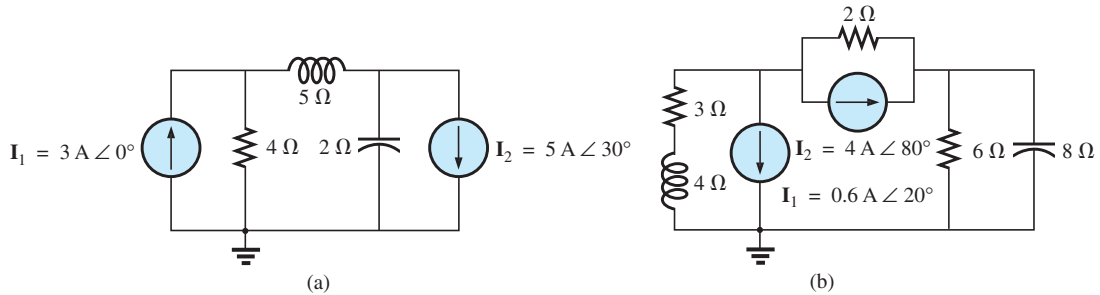
FIG. 17.72

Problems 14 and 39.

15. Determine the nodal voltages for the networks in Fig. 17.73.

(a)

(b)
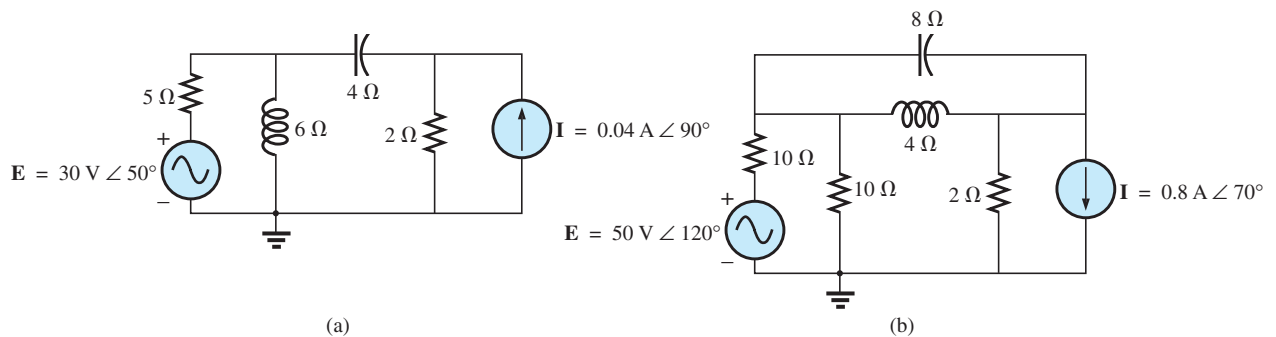
FIG. 17.73

Problem 15.

16. Determine the nodal voltages for the network in Fig. 17.64(b).

17. Determine the nodal voltages for the network in Fig. 17.65(b).

*****18.** Determine the nodal voltages for the network in Fig. 17.66(a).

*****19.** Determine the nodal voltages for the network in Fig. 17.66(b).

*****20.** Determine the nodal voltages for the networks in Fig. 17.74.

(a)

(b)
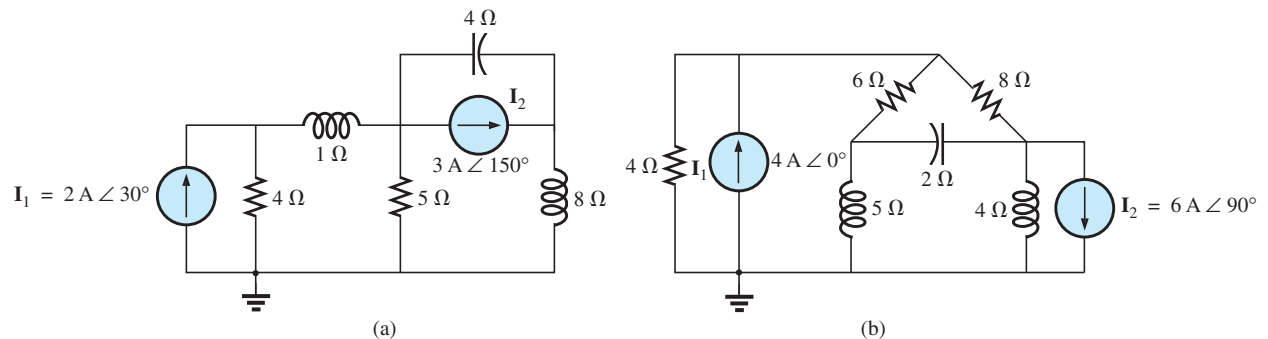
FIG. 17.74

Problem 20.

*21. Write the nodal equations for the network in Fig. 17.75, and find the voltage across the 1 kΩ resistor.
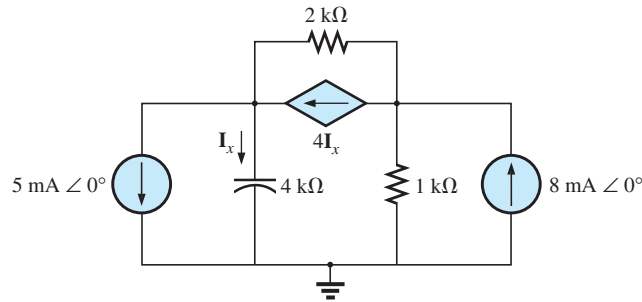
FIG. 17.75
Problems 21 and 40.

*22. Write the nodal equations for the network in Fig. 17.76, and find the voltage across the capacitive element.
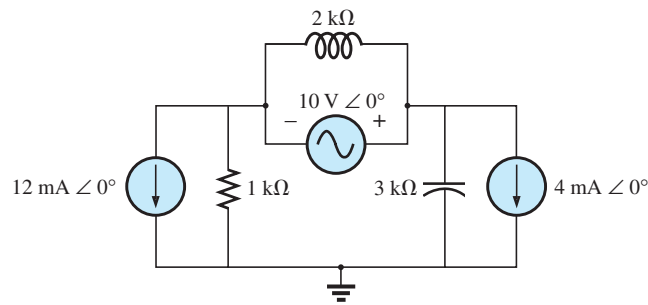
FIG. 17.76
Problems 22 and 41.

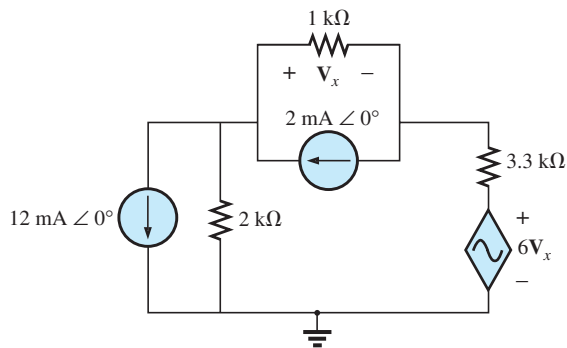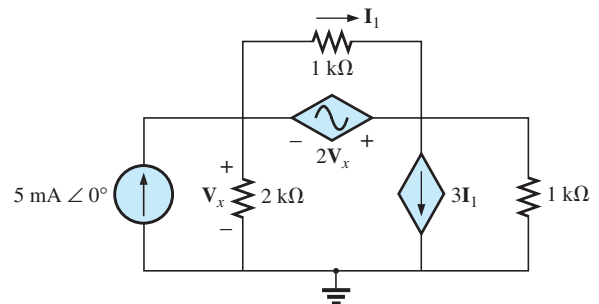*23. Write the nodal equations for the network in Fig. 17.77, and find the voltage across the 2 kΩ resistor.

*24. Write the nodal equations for the network in Fig. 17.78, and find the voltage across the 2 kΩ resistor.

FIG. 17.77
Problems 23 and 42.

FIG. 17.78
Problems 24 and 43.

*25. For the network in Fig. 17.79, determine the voltage V_L in terms of the voltage E_i.
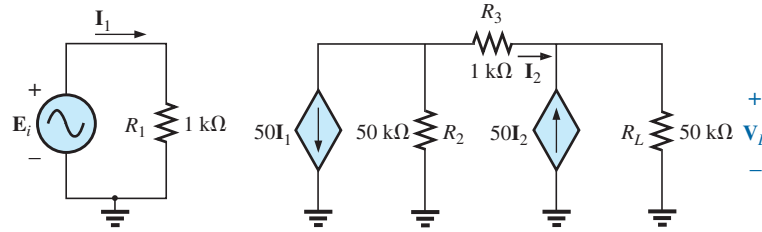
FIG. 17.79
Problem 25.

SECTION 17.6 Bridge Networks (ac)

26. For the bridge network in Fig. 17.80:
    a. Is the bridge balanced?
    b. Using mesh analysis, determine the current through the capacitive reactance.
    c. Using nodal analysis, determine the voltage across the capacitive reactance.
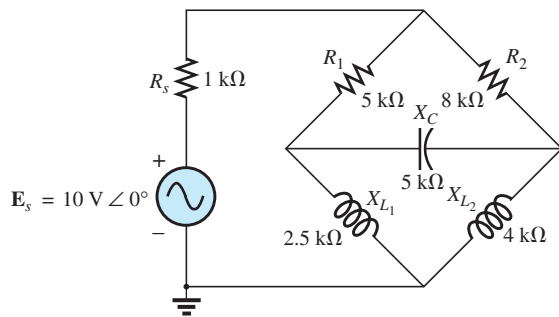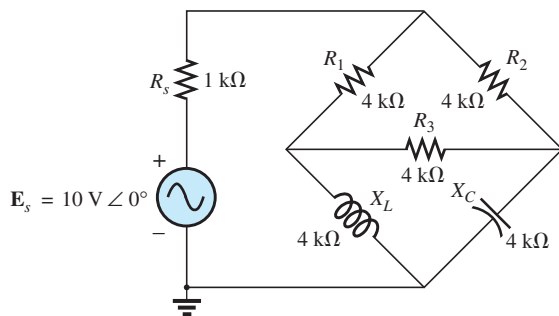

FIG. 17.80
Problem 26.

27. For the bridge network in Fig. 17.81:
    a. Is the bridge balanced?
    b. Using mesh analysis, determine the current through the capacitive reactance.
    c. Using nodal analysis, determine the voltage across the capacitive reactance.

FIG. 17.81
Problem 27.

28. The Hay bridge in Fig. 17.82 is balanced. Using Eq. (17.3), determine the unknown inductance L_x and resistance R_x.
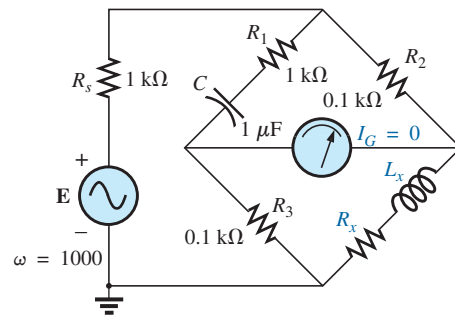

FIG. 17.82
Problem 28.

29. Determine whether the Maxwell bridge in Fig. 17.83 is balanced ($\omega = 1000$ rad/s).
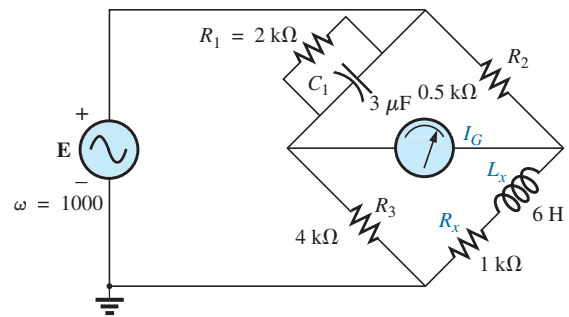

FIG. 17.83
Problem 29.

30. Derive the balance Eqs. (17.11) and (17.12) for the capacitance comparison bridge.

31. Determine the balance equations for the inductance bridge
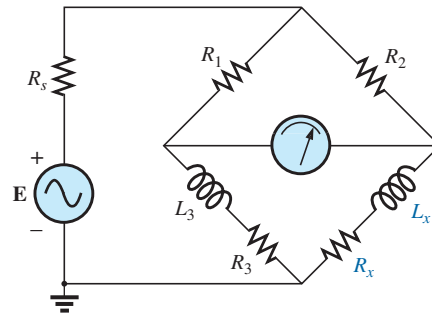in Fig. 17.84.

FIG. 17.84
Problem 31.

SECTION 17.7 Δ-Y, Y-Δ Conversions

32. Using the Δ-Y or Y-Δ conversion, determine the current **I** for
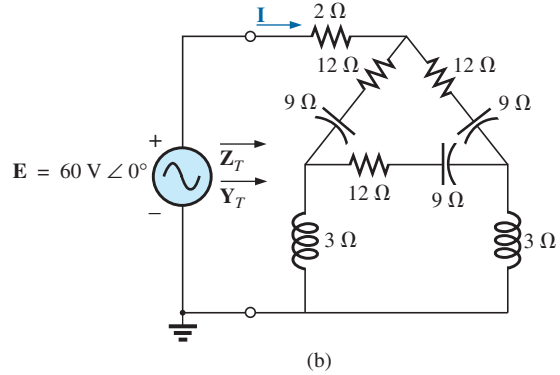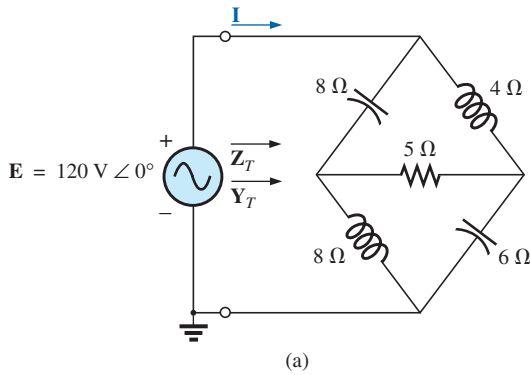the networks in Fig. 17.85.

(a)

(b)

FIG. 17.85
Problem 32.

33. Using the Δ-Y or Y-Δ conversion, determine the current **I** for
the networks in Fig. 17.86. (**E** = 100 V ∠0° in each case.)
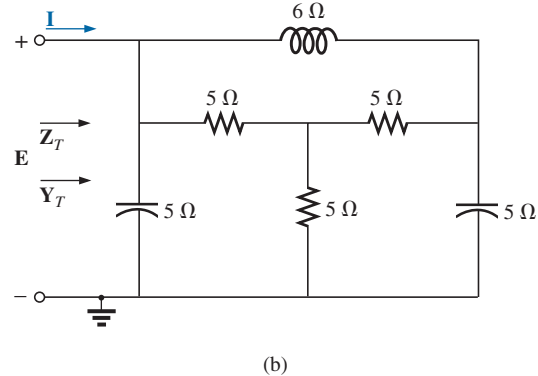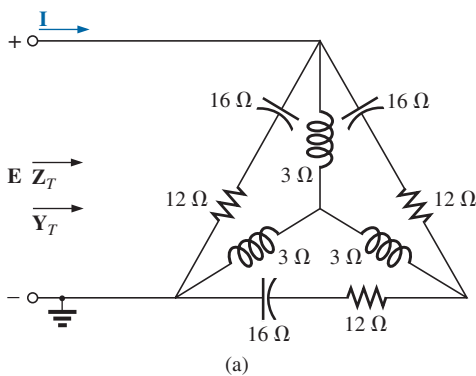

(a)

(b)

FIG. 17.86
Problem 33.

SECTION 17.8   Computer Analysis
PSpice or Multisim

34. Determine the mesh currents for the network in Fig. 17.63(a).

35. Determine the mesh currents for the network in Fig. 17.65(a).

*36. Determine the mesh currents for the network in Fig. 17.69.

*37. Determine the mesh currents for the network in Fig. 17.70.

*38. Determine the mesh currents for the network in Fig. 17.71.

39. Determine the nodal voltages for the network in Fig. 17.72(b).

*40. Determine the nodal voltages for the network in Fig. 17.75.

*41. Determine the nodal voltages for the network in Fig. 17.76.

*42. Determine the nodal voltages for the network in Fig. 17.77.

*43. Determine the nodal voltages for the network in Fig. 17.78.

GLOSSARY

Bridge network   A network configuration having the appearance of a diamond in which no two branches are in series or parallel.

Capacitance comparison bridge   A bridge configuration having a galvanometer in the bridge arm that is used to determine an unknown capacitance and associated resistance.

Delta (Δ) configuration   A network configuration having the appearance of the capital Greek letter *delta.*

Dependent (controlled) source   A source whose magnitude and/or phase angle is determined (controlled) by a current or voltage of the system in which it appears.

Hay bridge   A bridge configuration used for measuring the resistance and inductance of coils in those cases where the resistance is a small fraction of the reactance of the coil.

Independent source   A source whose magnitude is independent of the network to which it is applied. It displays its terminal characteristics even if completely isolated.

Maxwell bridge   A bridge configuration used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Mesh analysis   A method through which the loop (or mesh) currents of a network can be determined. The branch currents of the network can then be determined directly from the loop currents.

Nodal analysis   A method through which the nodal voltages of a network can be determined. The voltage across each element can then be determined through application of Kirchhoff's voltage law.

Source conversion   The changing of a voltage source to a current source, or vice versa, which will result in the same terminal behavior of the source. In other words, the external network is unaware of the change in sources.

Wye (Y) configuration   A network configuration having the appearance of the capital letter *Y.*