13

Sinusoidal Alternating Waveforms

Objectives

- *Become familiar with the characteristics of a sinusoidal waveform including its general format, average value, and effective value.*
- *Be able to determine the phase relationship between two sinusoidal waveforms of the same frequency.*
- *Understand how to calculate the average and effective values of any waveform.*
- *Become familiar with the use of instruments designed to measure ac quantities.*

13.1 INTRODUCTION

The analysis thus far has been limited to dc networks, networks in which the currents or voltages are fixed in magnitude except for transient effects. We now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the time-varying voltage that is commercially available in large quantities and is commonly called the *ac voltage.* (The letters *ac* are an abbreviation for *alternating current.*) To be absolutely rigorous, the terminology *ac voltage* or *ac current* is not sufficient to describe the type of signal we will be analyzing. Each waveform in Fig. 13.1 is an **alternating waveform** available from commercial supplies. The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence. To be absolutely correct, the term *sinusoidal, square-wave,* or *triangular* must also be applied.

The pattern of particular interest is the **sinusoidal ac voltage** in Fig. 13.1. Since this type of signal is encountered in the vast majority of instances, the abbreviated phrases *ac voltage* and *ac current* are commonly applied without confusion. For the other patterns in Fig. 13.1, the descriptive term is always present, but frequently the *ac* abbreviation is dropped, resulting in the designation *square-wave* or *triangular* waveforms.

One of the important reasons for concentrating on the sinusoidal ac voltage is that it is the voltage generated by utilities throughout the world. Other reasons include its application throughout electrical, electronic, communication, and industrial systems. In addition, the chapters to follow will reveal that the waveform itself has a number of characteristics that result in a unique response when it is applied to basic electrical elements. The wide range of theorems and methods introduced for dc networks will also be applied to sinusoidal ac systems. Although the application of sinusoidal signals raise the required math level, once the notation

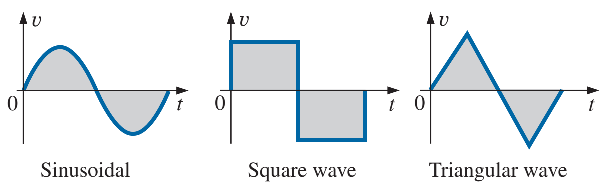


FIG. 13.1
Alternating waveforms.

given in Chapter 14 is understood, most of the concepts introduced in the dc chapters can be applied to ac networks with a minimum of added difficulty.

13.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

Generation

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant. Most power plants are fueled by water power, oil, gas, or nuclear fusion. In each case, an **ac generator** (also called an *alternator*), as shown in Fig. 13.2(a), is the primary component in the energy-conversion process. The power to the shaft developed by one of the energy sources listed turns a *rotor* (constructed of alternating magnetic poles) inside a set of windings housed in the *stator* (the stationary part of the dynamo) and induces a voltage across the windings of the stator, as defined by Faraday's law:

$$e = N\frac{d\phi}{dt}$$

Through proper design of the generator, a sinusoidal ac voltage is developed that can be transformed to higher levels for distribution through the power lines to the consumer. For isolated locations where power lines have not been installed, portable ac generators [Fig. 13.2(b)] are available that run on gasoline. As in the larger power plants, however, an ac generator is an integral part of the design.

In an effort to conserve our natural resources and reduce pollution, wind power, solar energy, and fuel cells are receiving increasing interest from various districts of the world that have such energy sources available in level and duration that make the conversion process viable. The turning propellers of the wind-power station [Fig. 13.2(c)] are connected directly to the shaft of an ac generator to provide the ac voltage described above. Through light energy absorbed in the form of *photons,* solar cells

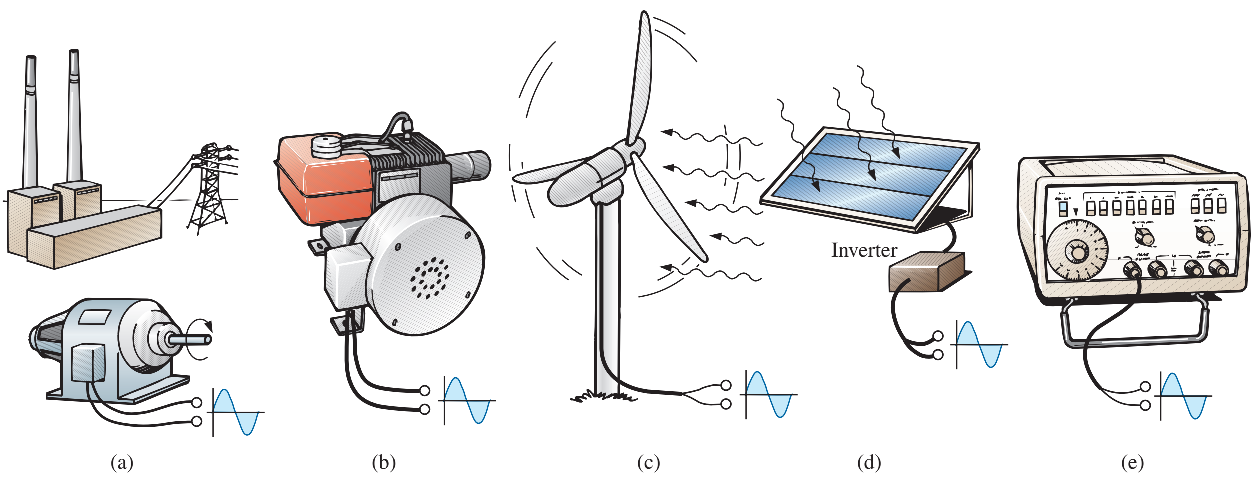


FIG. 13.2
Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

[Fig. 13.2(d)] can generate dc voltages. Through an electronic package called an *inverter,* the dc voltage can be converted to one of a sinusoidal nature. Boats, recreational vehicles (RVs), and so on, make frequent use of the inversion process in isolated areas.

Sinusoidal ac voltages with characteristics that can be controlled by the user are available from **function generators,** such as the one in Fig. 13.2(e). By setting the various switches and controlling the position of the knobs on the face of the instrument, you can make available sinusoidal voltages of different peak values and different repetition rates. The function generator plays an integral role in the investigation of the variety of theorems, methods of analysis, and topics to be introduced in the chapters that follow.

Definitions

The sinusoidal waveform in Fig. 13.3 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember, as you proceed through the various definitions, that the vertical scaling is in volts or amperes and the horizontal scaling is in units of time.

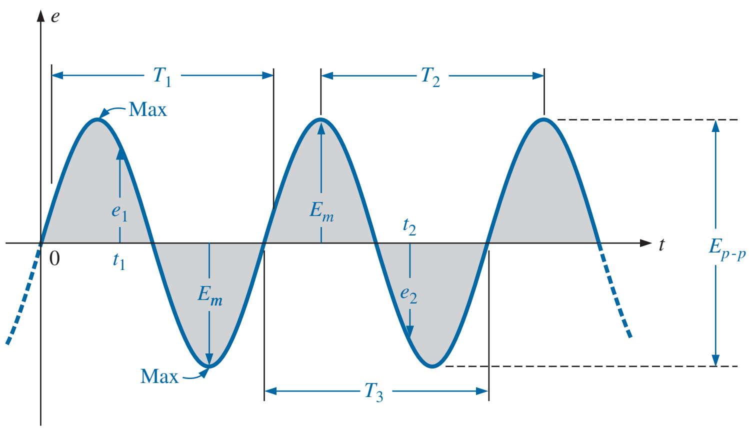


FIG. 13.3
Important parameters for a sinusoidal voltage.

Waveform: The path traced by a quantity, such as the voltage in Fig. 13.3, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2 in Fig. 13.3).

Peak amplitude: The maximum value of a waveform as measured from its *average,* or *mean,* value, denoted by uppercase letters [such as E_m (Fig. 13.3) for sources of voltage and V_m for the voltage drop across a load]. For the waveform in Fig. 13.3, the average value is zero volts, and E_m is as defined by the figure.

Peak value: The maximum instantaneous value of a function as measured from the zero volt level. For the waveform in Fig. 13.3, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by $E_{p\text{-}p}$ or $V_{p\text{-}p}$ (as shown in Fig. 13.3), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform in Fig. 13.3 is a periodic waveform.

Period (T): The time of a periodic waveform.

Cycle: The portion of a waveform contained in one period of time. The cycles within T_1, T_2, and T_3 in Fig. 13.3 may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

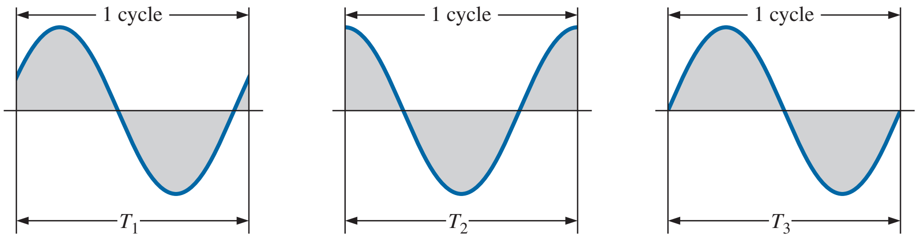


FIG. 13.4
Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform in Fig. 13.5(a) is 1 cycle per second, and for Fig. 13.5(b), 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13.5(c)], the frequency would be 2 cycles per second.

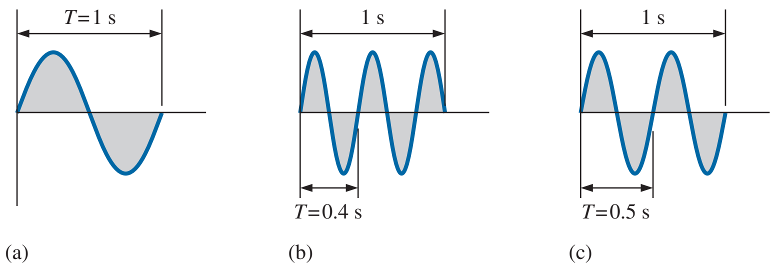


FIG. 13.5
Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

The unit of measure for frequency is the *hertz* (Hz), where

$$1 \text{ hertz (Hz)} = 1 \text{ cycle per second (cps)} \tag{13.1}$$

The unit hertz is derived from the surname of Heinrich Rudolph Hertz (Fig. 13.6), who did original research in the area of alternating currents and voltages and their effect on the basic R, L, and C elements. The frequency standard for North America is 60 Hz, whereas for Europe it is predominantly 50 Hz.

As with all standards, any variation from the norm will cause difficulties. In 1993, Berlin, Germany, received all its power from eastern plants, whose output frequency was varying between 50.03 Hz and 51 Hz. The result was that clocks were gaining as much as 4 minutes a day. Alarms went off too soon, VCRs clicked off before the end of the program, and so on, requiring that clocks be continually reset. In 1994, however, when power was linked with the rest of Europe, the precise standard of 50 Hz was reestablished and everyone was on time again.

FIG. 13.6
Heinrich Rudolph Hertz.
Courtesy of the Smithsonian Institution, Photo No. 66,606.

German (Hamburg, Berlin, Karlsruhe)
(1857–94)
Physicist
Professor of Physics, Karlsruhe Polytechnic and University of Bonn

Spurred on by the earlier predictions of the English physicist James Clerk Maxwell, Heinrich Hertz produced *electromagnetic waves* in his laboratory at the Karlsruhe Polytechnic while in his early 30s. The rudimentary *transmitter* and *receiver* were in essence the first to broadcast and receive radio waves. He was able to measure the *wavelength* of the electromagnetic waves and confirmed that the *velocity of propagation* is in the same order of magnitude as light. In addition, he demonstrated that the *reflective* and *refractive* properties of electromagnetic waves are the same as those for heat and light waves. It was indeed unfortunate that such an ingenious, industrious individual should pass away at the very early age of 37 due to a bone disease.

EXAMPLE 13.1 For the sinusoidal waveform in Fig. 13.7.

a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?

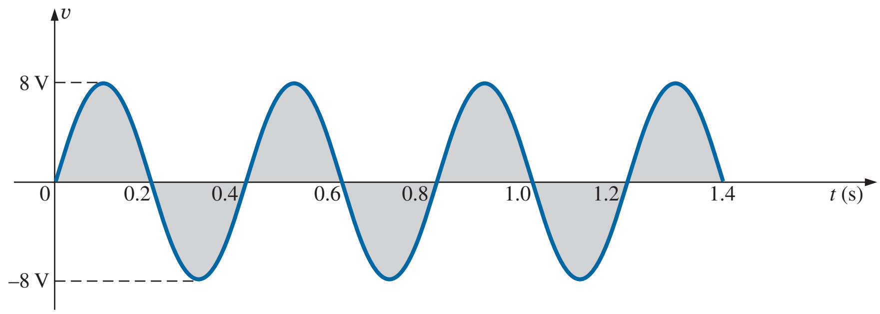


FIG. 13.7
Example 13.1.

Solutions:

a. **8 V.**
b. At 0.3 s, **−8 V;** at 0.6 s, **0 V.**
c. **16 V.**
d. **0.4 s.**
e. **3.5 cycles.**
f. **2.5 cps,** or **2.5 Hz.**

13.3 FREQUENCY SPECTRUM

Using a log scale (described in detail in Chapter 20), a frequency spectrum from 1 Hz to 1000 GHz can be scaled off on the same axis, as shown in Fig. 13.8. A number of terms in the various spectrums are probably familiar to you from everyday experiences. Note that the audio range (human ear) extends from only 15 Hz to 20 kHz, but the transmission of radio signals can occur between 3 kHz and 300 GHz. The uniform process of defining the intervals of the radio-frequency spectrum from VLF to EHF is quite evident from the length of the bars in the figure (although keep in mind that it is a log scale, so the frequencies encompassed within each segment are quite different). Other frequencies of particular interest (TV, CB, microwave, and so on) are also included for reference purposes. Although it is numerically easy to talk about frequencies in the megahertz and gigahertz range, keep in mind that a frequency of 100 MHz, for instance, represents a sinusoidal waveform that passes through 100,000,000 cycles in only 1 s—an incredible number when we compare it to the 60 Hz of our conventional power sources. The Intel® Pentium® 4 chip manufactured by Intel can run at speeds over 2 GHz. Imagine a product able to handle 2 billion instructions per second—an incredible achievement.

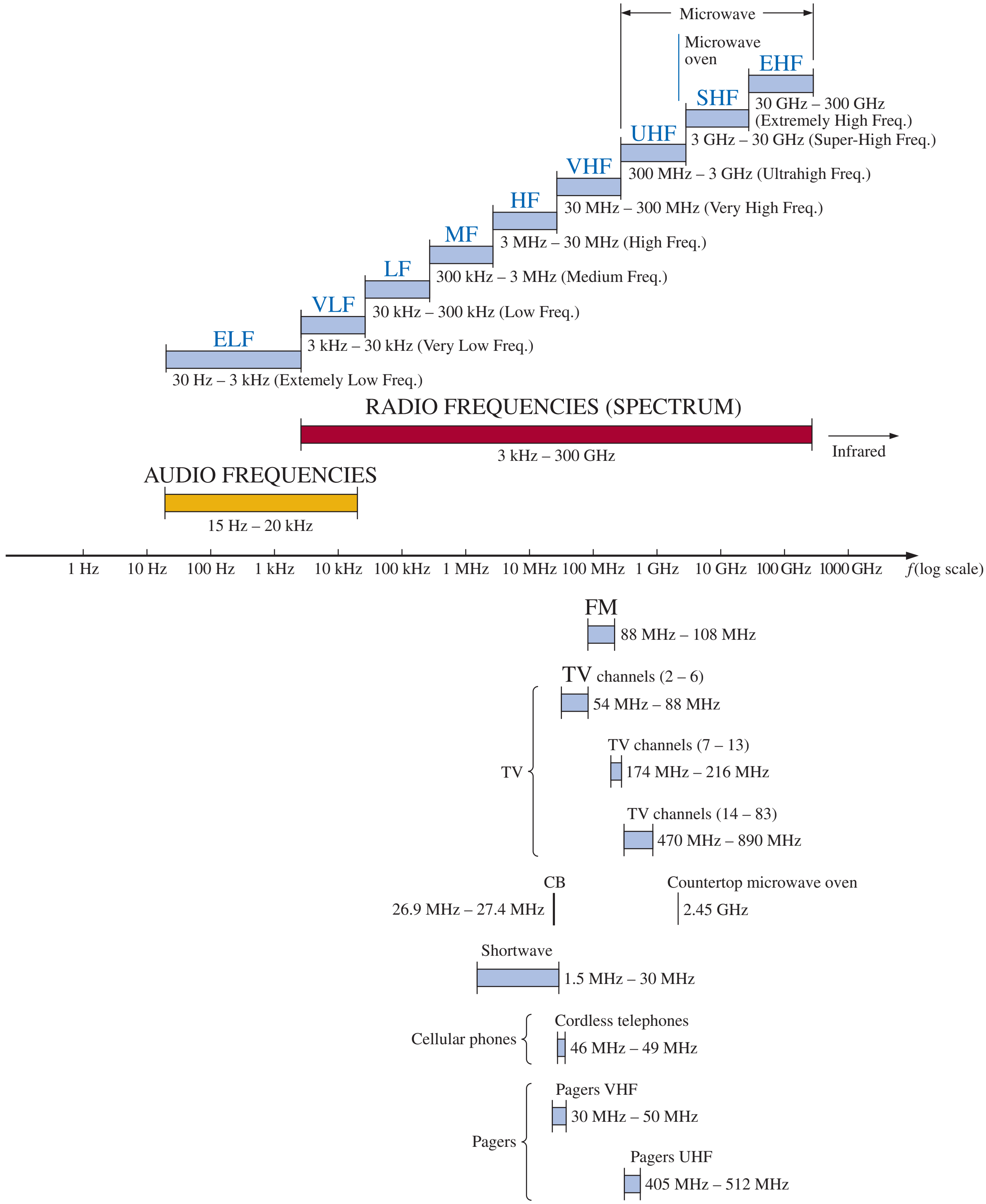


FIG. 13.8

Areas of application for specific frequency bands.

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$$f = \frac{1}{T} \qquad \begin{aligned} f &= \text{Hz} \\ T &= \text{seconds (s)} \end{aligned} \tag{13.2}$$

or

$$T = \frac{1}{f} \tag{13.3}$$

EXAMPLE 13.2 Find the period of periodic waveform with a frequency of

a. 60 Hz.
b. 1000 Hz.

Solutions:

a. $T = \dfrac{1}{f} = \dfrac{1}{60\text{ Hz}} \cong 0.01667\text{ s or } \mathbf{16.67\text{ ms}}$

(a recurring value since 60 Hz is so prevalent)

b. $T = \dfrac{1}{f} = \dfrac{1}{1000\text{ Hz}} = 10^{-3}\text{ s} = \mathbf{1\text{ ms}}$

EXAMPLE 13.3 Determine the frequency of the waveform in Fig. 13.9.

Solution: From the figure, $T = (25\text{ ms} - 5\text{ ms})$ or $(35\text{ ms} - 15\text{ ms}) = 20\text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\text{ s}} = \mathbf{50\text{ Hz}}$$

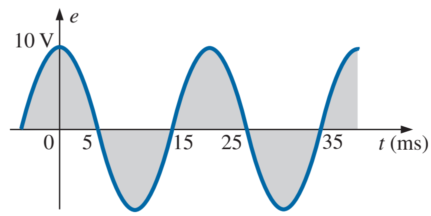


FIG. 13.9
Example 13.3.

In Fig. 13.10, the seismogram resulting from a seismometer near an earthquake is displayed. Prior to the disturbance, the waveform has a relatively steady level, but as the event is about to occur, the frequency begins

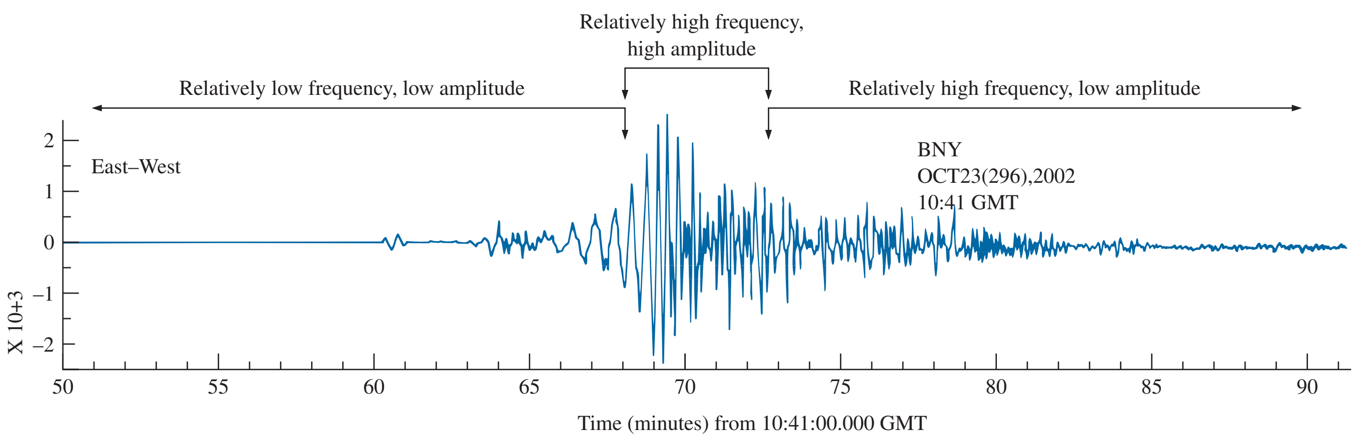


FIG. 13.10
Seismogram from station BNY (Binghamton University) in New York due to magnitude 6.7 earthquake in Central Alaska that occurred at 63.62°N, 148.04°W, with a depth of 10 km, on Wednesday, October 23, 2002.

to increase along with the amplitude. Finally, the earthquake occurs, and the frequency and the amplitude increase dramatically. In other words, the relative frequencies can be determined simply by looking at the tightness of the waveform and the associated period. The change in amplitude is immediately obvious from the resulting waveform. The fact that the earthquake lasts for only a few minutes is clear from the horizontal scale.

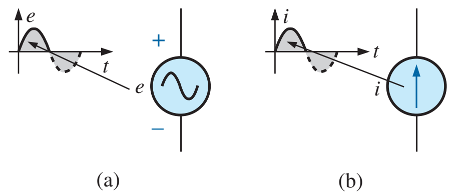


FIG. 13.11
(a) Sinusoidal ac voltage sources; (b) sinusoidal current sources.

Defined Polarities and Direction

You may be wondering how a polarity for a voltage or a direction for a current can be established if the waveform moves back and forth from the positive to the negative region. For a period of time, a voltage has one polarity, while for the next equal period it reverses. To take care of this problem, a positive sign is applied if the voltage is above the axis, as shown in Fig. 13.11(a). For a current source, the direction in the symbol corresponds with the positive region of the waveform, as shown in Fig. 13.11(b).

For any quantity that will not change with time, an uppercase letter such as *V* or *I* is used. For expressions that are time dependent or that represent a particular instant of time, a lowercase letter such as *e* or *i* is used.

The need for defining polarities and current direction becomes quite obvious when we consider multisource ac networks. Note in the last sentence the absence of the term *sinusoidal* before the phrase *ac networks.* This phrase will be used to an increasing degree as we progress; *sinusoidal* is to be understood unless otherwise indicated.

13.4 THE SINUSOIDAL WAVEFORM

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.

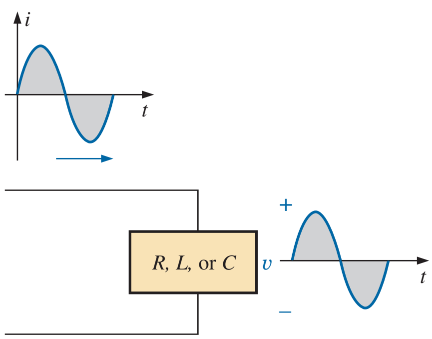


FIG. 13.12
The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, inductor, or capacitor.

In other words, if the voltage across (or current through) a resistor, inductor, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Fig. 13.12. If any other alternating waveform such as a square wave or a triangular wave were applied, such would not be the case.

The unit of measurement for the horizontal axis can be **time** (as appearing in the figures thus far), **degrees**, or **radians**. The term **radian** can be defined as follows: If we mark off a portion of the circumference of a circle by a length equal to the radius of the circle, as shown in Fig. 13.13, the angle resulting is called *1 radian.* The result is

$$\boxed{1 \text{ rad} = 57.296° \cong 57.3°} \qquad \textbf{(13.4)}$$

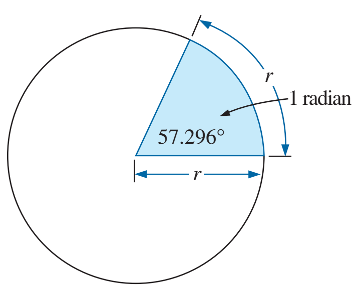


FIG. 13.13
Defining the radian.

where 57.3° is the usual approximation applied.

One full circle has 2π radians, as shown in Fig. 13.14. That is,

$$\boxed{2\pi \text{ rad} = 360°} \qquad \textbf{(13.5)}$$

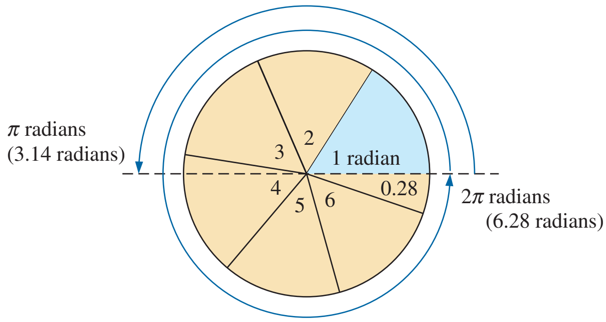


FIG. 13.14
There are 2π radians in one full circle of 360°.

so that $$2\pi = 2(3.142) = 6.28$$
and $$2\pi(57.3°) = 6.28(57.3°) = 359.84° \cong 360°$$

A number of electrical formulas contain a multiplier of π. For this reason, it is sometimes preferable to measure angles in radians rather than in degrees.

The quantity π is the ratio of the circumference of a circle to its diameter.

π has been determined to an extended number of places, primarily in an attempt to see if a repetitive sequence of numbers appears. It does not. A sampling of the effort appears below:

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ldots$$

Although the approximation $\pi \cong 3.14$ is often applied, all the calculations in the text use the π function as provided on all scientific calculators.

For 180° and 360°, the two units of measurement are related as shown in Fig. 13.14. The conversions equations between the two are the following:

$$\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees}) \tag{13.6}$$

$$\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians}) \tag{13.7}$$

Applying these equations, we find

$$\textbf{90°:}\ \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\pi}{2}\ \textbf{rad}$$

$$\textbf{30°:}\ \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\pi}{6}\ \textbf{rad}$$

$$\frac{\pi}{3}\ \textbf{rad}:\ \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\frac{3\pi}{2}\ \textbf{rad:}\ \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$

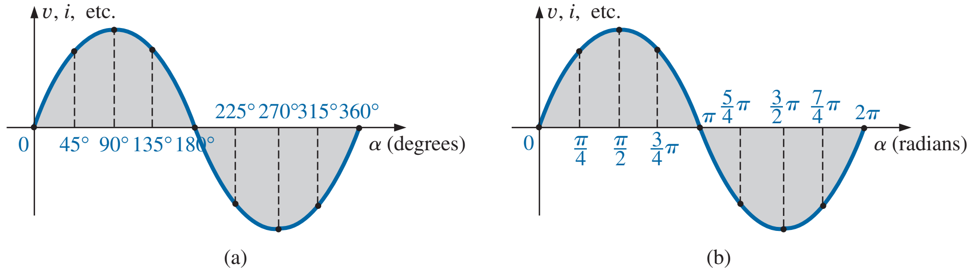


FIG. 13.15
Plotting a sine wave versus (a) degrees and (b) radians.

For comparison purposes, two sinusoidal voltages are plotted in Fig. 13.15 using degrees and radians as the units of measurement for the horizontal axis.

It is of particular interest that the sinusoidal waveform can be derived from the length of the *vertical projection* of a radius vector rotating in a uniform circular motion about a fixed point. Starting as shown in Fig. 13.16(a) and plotting the amplitude (above and below zero) on the coordinates drawn to the right [Figs. 13.16(b) through (i)], we will trace a complete sinusoidal waveform after the radius vector has completed a 360° rotation about the center.

The velocity with which the radius vector rotates about the center, called the **angular velocity,** can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}} \tag{13.8}$$

Substituting into Eq. (13.8) and assigning the lowercase Greek letter *omega* (ω) to the angular velocity, we have

$$\omega = \frac{\alpha}{t} \tag{13.9}$$

and

$$\alpha = \omega t \tag{13.10}$$

Since ω is typically provided in radians per second, the angle α obtained using Eq. (13.10) is usually in radians. If α is required in degrees, Eq. (13.7) must be applied. The importance of remembering the above will become obvious in the examples to follow.

In Fig. 13.16, the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform in Fig. 13.16(i). The radians subtended in this time interval are 2π. Substituting, we have

$$\omega = \frac{2\pi}{T} \quad \text{(rad/s)} \tag{13.11}$$

In words, this equation states that the smaller the period of the sinusoidal waveform of Fig. 13.16(i), or the smaller the time interval before one complete cycle is generated, the greater must be the angular velocity of the rotating radius vector. Certainly this statement agrees with what we have learned thus far. We can now go one step further and apply the fact

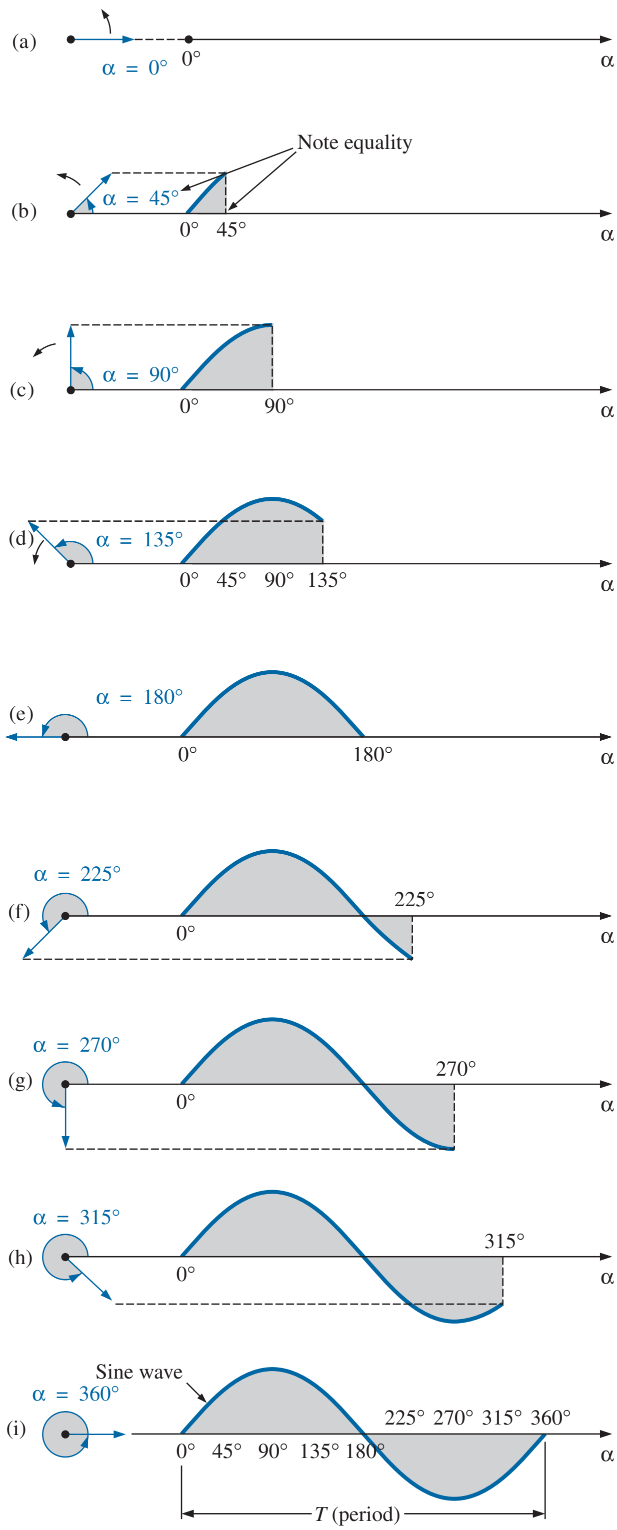


FIG. 13.16

Generating a sinusoidal waveform through the vertical projection of a rotating vector.

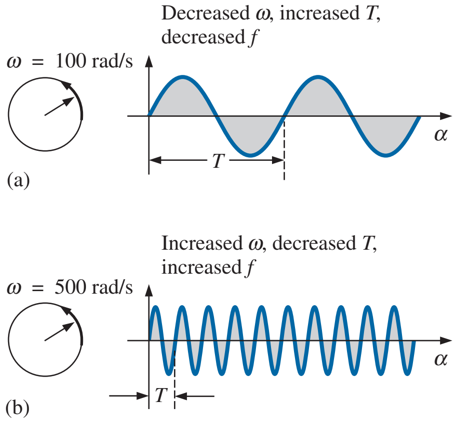


FIG. 13.17
Demonstrating the effect of ω on the frequency and period.

that the frequency of the generated waveform is inversely related to the period of the waveform; that is, $f = 1/T$. Thus,

$$\boxed{\omega = 2\pi f} \qquad \text{(rad/s)} \qquad \textbf{(13.12)}$$

This equation states that the higher the frequency of the generated sinusoidal waveform, the higher must be the angular velocity. Eqs. (13.11) and (13.12) are verified somewhat by Fig. 13.17, where for the same radius vector, $\omega = 100$ rad/s and 500 rad/s.

EXAMPLE 13.4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution:

$$\omega = 2\pi f = (2\pi)(60\text{ Hz}) \cong \mathbf{377\ rad/s}$$

(a recurring value due to 60 Hz predominance)

EXAMPLE 13.5 Determine the frequency and period of the sine wave in Fig. 13.17(b).

Solution: Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \mathbf{12.57\ ms}$$

and
$$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3}\text{ s}} = \mathbf{79.58\ Hz}$$

EXAMPLE 13.6 Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

Solution: Eq. (13.10): $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, α must be substituted as $\pi/2$ (= 90°) since ω is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2\text{ rad}}{200\text{ rad/s}} = \frac{\pi}{400}\text{ s} = \mathbf{7.85\ ms}$$

EXAMPLE 13.7 Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (13.11): $\alpha = \omega t$, or

$$\alpha = 2\pi f t = (2\pi)(60\text{ Hz})(5 \times 10^{-3}\text{ s}) = \mathbf{1.885\ rad}$$

If not careful, you might be tempted to interpret the answer as 1.885°. However,

$$\alpha\,(^\circ) = \frac{180^\circ}{\pi\ \cancel{\text{rad}}}(1.885\ \cancel{\text{rad}}) = \mathbf{108^\circ}$$

13.5 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is

$$A_m \sin \alpha \qquad (13.13)$$

where A_m is the peak value of the waveform and α is the unit of measure for the horizontal axis, as shown in Fig. 13.18.

The equation $\alpha = \omega t$ states that the angle α through which the rotating vector in Fig. 13.16 will pass is determined by the angular velocity of the rotating vector and the length of time the vector rotates. For example, for a particular angular velocity (fixed ω), the longer the radius vector is permitted to rotate (that is, the greater the value of t), the greater the number of degrees or radians through which the vector will pass. Relating this statement to the sinusoidal waveform, for a particular angular velocity, the longer the time, the greater the number of cycles shown. For a fixed time interval, the greater the angular velocity, the greater the number of cycles generated.

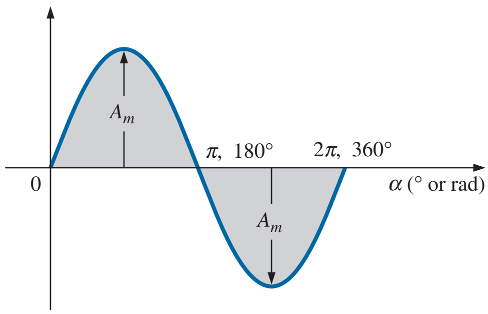


FIG. 13.18
Basic sinusoidal function.

Due to Eq. (13.10), the general format of a sine wave can also be written

$$A_m \sin \omega t \qquad (13.14)$$

with ωt as the horizontal unit of measure.

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$
$$e = E_m \sin \omega t = E_m \sin \alpha$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current and voltage, respectively, at any time t. This format is particularly important because it presents the sinusoidal voltage or current as a function of time, which is the horizontal scale for the oscilloscope. Recall that the horizontal sensitivity of a scope is in time per division, not degrees per centimeter.

EXAMPLE 13.8 Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.21\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha\,(°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and $$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.94\ V}$$

The angle at which a particular voltage level is attained can be determined by rearranging the equation

$$e = E_m \sin \alpha$$

in the following manner:

$$\sin \alpha = \frac{e}{E_m}$$

which can be written

$$\alpha = \sin^{-1} \frac{e}{E_m} \qquad \textbf{(13.15)}$$

Similarly, for a particular current level,

$$\alpha = \sin^{-1} \frac{i}{I_m} \qquad \textbf{(13.16)}$$

EXAMPLE 13.9

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
b. Determine the time at which the magnitude is attained.

Solutions:

a. Eq. (13.15):

$$\alpha_1 = \sin^{-1} \frac{v}{E_m} = \sin^{-1} \frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.58^\circ}$$

However, Fig.13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.42^\circ}$$

In general, therefore, keep in mind that Eqs. (13.15) and (13.16) will provide an angle with a magnitude between 0° and 90°.

b. Eq. (13.10): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha\,(\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412 \text{ rad}$$

and $$t_1 = \frac{\alpha}{\omega} = \frac{0.412 \text{ rad}}{377 \text{ rad/s}} = \mathbf{1.09\ ms}$$

For the second intersection,

$$\alpha\,(\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73 \text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73 \text{ rad}}{377 \text{ rad/s}} = \mathbf{7.24\ ms}$$

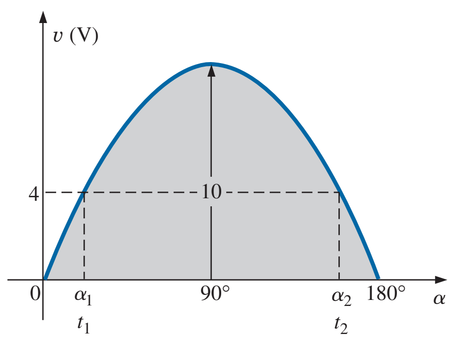


FIG. 13.19
Example 13.9.

Calculator Operations

Both sin and $\sin^{-1}$ are available on all scientific calculators. You can also use them to work with the angle in degrees or radians without having to convert from one form to the other. That is, if the angle is in radians and the mode setting is for radians, you can enter the radian measure directly.

To set the DEGREE mode, proceed as outlined in Fig. 13.20(a) using the TI-89 calculator. The magnitude of the voltage e at 40° can then be found using the sequence in Fig. 13.20(b).

HOME ENTER MODE ↓ Angle DEGREE ENTER ENTER

(a)

5 × 2ND SIN 4 0) ENTER **3.21**

(b)

FIG. 13.20

(a) Setting the DEGREE mode; (b) evaluating 5 sin 40°.

After establishing the RADIAN mode, the sequence in Fig. 13.21 determines the voltage at 0.8π.

5 × 2ND SIN 0 . 8 2ND π) ENTER **2.94**

FIG. 13.21

Finding $e = 5 \sin 0.8\pi$ using the calculator in the RADIAN mode.

Finally, the angle in degrees for α_1 in part (a) of Example 13.9 can be determined by the sequence in Fig. 13.22 with the mode set in degrees, whereas the angle in radians for part (a) of Example 13.9 can be determined by the sequence in Fig. 13.23 with the mode set in radians.

♦ SIN^{-1} 4 ÷ 1 0) ENTER **23.60**

FIG. 13.22

Finding $\alpha_1 = \sin^{-1}(4/10)$ using the calculator in the DEGREE mode.

♦ SIN^{-1} 4 ÷ 1 0) ENTER **0.41**

FIG. 13.23

Finding $\alpha_1 = \sin^{-1}(4/10)$ using the calculator in the RADIAN mode.

The sinusoidal waveform can also be plotted against *time* on the horizontal axis. The time period for each interval can be determined from $t = \alpha/\omega$, but the most direct route is simply to find the period T from $T = 1/f$ and break it up into the required intervals. This latter technique is demonstrated in Example 13.10.

Before reviewing the example, take special note of the relative simplicity of the mathematical equation that can represent a sinusoidal waveform. Any alternating waveform whose characteristics differ from those of the sine wave cannot be represented by a single term, but may require two, four, six, or perhaps an infinite number of terms to be represented accurately.

EXAMPLE 13.10 Sketch $e = 10 \sin 314t$ with the abscissa

a. angle (α) in degrees.
b. angle (α) in radians.
c. time (t) in seconds.

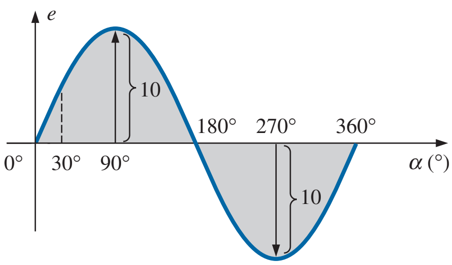


FIG. 13.24
Example 13.10, horizontal axis in degrees.

Solutions:

a. See Fig. 13.24. (Note that no calculations are required.)
b. See Fig. 13.25. (Once the relationship between degrees and radians is understood, no calculations are required.)
c. See Fig 13.26.

$$360°: \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{314} = 20 \text{ ms}$$
$$180°: \quad \frac{T}{2} = \frac{20 \text{ ms}}{2} = 10 \text{ ms}$$
$$90°: \quad \frac{T}{4} = \frac{20 \text{ ms}}{4} = 5 \text{ ms}$$
$$30°: \quad \frac{T}{12} = \frac{20 \text{ ms}}{12} = 1.67 \text{ ms}$$

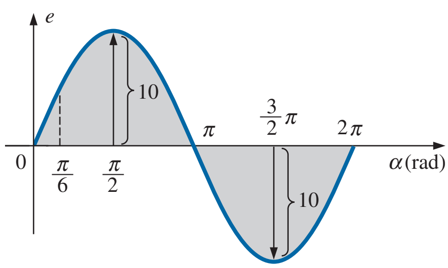


FIG. 13.25
Example 13.10, horizontal axis in radians.

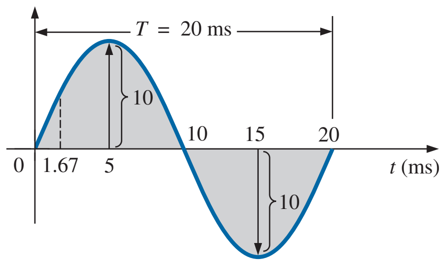


FIG. 13.26
Example 13.10, horizontal axis in milliseconds.

EXAMPLE 13.11 Given $i = 6 \times 10^{-3} \sin 1000t$, determine i at t = 2 ms.

Solution:

$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$
$$\alpha\,(°) = \frac{180°}{\pi \text{ rad}}(2 \text{ rad}) = 114.59°$$
$$i = (6 \times 10^{-3})(\sin 114.59°) = (6 \text{ mA})(0.9093) = \mathbf{5.46 \text{ mA}}$$

13.6 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π, as shown in Fig. 13.25. If the waveform is shifted to the right or left of 0°, the expression becomes

$$A_m \sin(\omega t \pm \theta) \qquad \textbf{(13.17)}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) slope *before* 0°, as shown in Fig. 13.27, the expression is

$$A_m \sin(\omega t + \theta) \qquad \textbf{(13.18)}$$

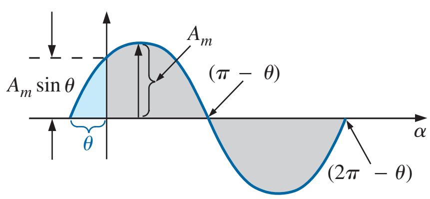


FIG. 13.27
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

At $\omega t = \alpha = 0°$, the magnitude is determined by $A_m \sin\theta$. If the waveform passes through the horizontal axis with a positive-going slope *after* 0°, as shown in Fig. 13.28, the expression is

$$\boxed{A_m \sin(\omega t - \theta)} \qquad \textbf{(13.19)}$$

Finally, at $\omega t = \alpha = 0°$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin\theta$.

If the waveform crosses the horizontal axis with a positive-going slope 90° ($\pi/2$) sooner, as shown in Fig. 13.29, it is called a *cosine wave;* that is,

$$\boxed{\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t} \qquad \textbf{(13.20)}$$

or

$$\boxed{\sin\omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)} \qquad \textbf{(13.21)}$$

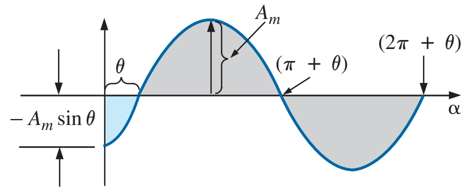


FIG. 13.28
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.

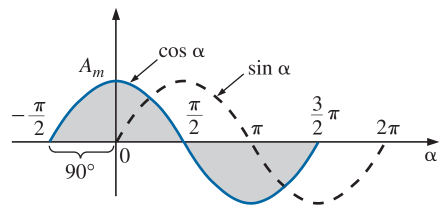


FIG. 13.29
Phase relationship between a sine wave and a cosine wave.

The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes. In Fig. 13.29, the cosine curve is said to *lead* the sine curve by 90°, and the sine curve is said to *lag* the cosine curve by 90°. The 90° is referred to as the phase angle between the two waveforms. In language commonly applied, the waveforms are *out of phase* by 90°. Note that the phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope.* If both waveforms cross the axis at the same point with the same slope, they are *in phase.*

The geometric relationship between various forms of the sine and cosine functions can be derived from Fig. 13.30. For instance, starting at the $+\sin\alpha$ position, we find that $+\cos\alpha$ is an additional 90° in the counterclockwise direction. Therefore, $\cos\alpha = \sin(\alpha + 90°)$. For $-\sin\alpha$ we must travel 180° in the counterclockwise (or clockwise) direction so that $-\sin\alpha = \sin(\alpha \pm 180°)$, and so on, as listed below:

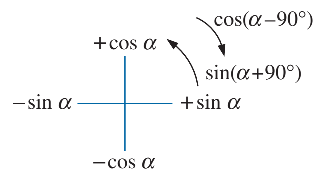


FIG. 13.30
Graphic tool for finding the relationship between specific sine and cosine functions.

$$\boxed{\begin{aligned} \cos\alpha &= \sin(\alpha + 90°) \\ \sin\alpha &= \cos(\alpha - 90°) \\ -\sin\alpha &= \sin(\alpha \pm 180°) \\ -\cos\alpha &= \sin(\alpha + 270°) = \sin(\alpha - 90°) \\ &\text{etc.} \end{aligned}} \qquad \textbf{(13.22)}$$

In addition, note that

$$\boxed{\begin{aligned} \sin(-\alpha) &= -\sin\alpha \\ \cos(-\alpha) &= \cos\alpha \end{aligned}} \qquad \textbf{(13.23)}$$

If a sinusoidal expression appears as

$$e = -E_m \sin\omega t$$

the negative sign is associated with the sine portion of the expression, not the peak value E_m. In other words, the expression, if not for convenience, would be written

$$e = E_m(-\sin\omega t)$$

Since $$-\sin \omega t = \sin(\omega t \pm 180°)$$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180°)$$

revealing that a negative sign can be replaced by a 180° change in phase angle (+ or −); that is,

$$e = -E_m \sin \omega t = E_m \sin(\omega t + 180°) = E_m \sin(\omega t - 180°)$$

A plot of each will clearly show their equivalence. There are, therefore, two correct mathematical representations for the functions.

The **phase relationship** between two waveforms indicates which one leads or lags the other, and by how many degrees or radians.

EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$
b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$
c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$
d. $i = -\sin(\omega t + 30°)$
   $v = 2 \sin(\omega t + 10°)$
e. $i = -2 \cos(\omega t - 60°)$
   $v = 3 \sin(\omega t - 150°)$

Solutions:

a. See Fig. 13.31.
   ***i* leads *v* by 40º, or *v* lags *i* by 40º.**

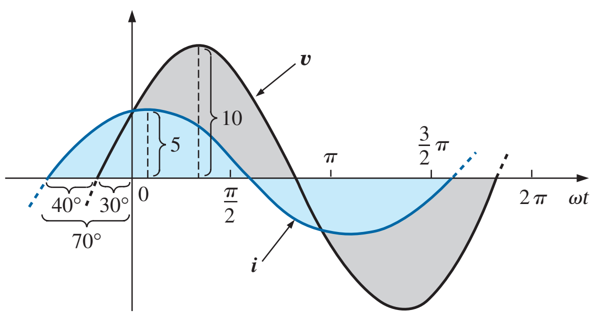


FIG. 13.31
Example 13.12(a): i leads v by 40°.

b. See Fig. 13.32.
   ***i* leads *v* by 80º, or *v* lags *i* by 80º.**

c. See Fig. 13.33.

$$\begin{aligned} i = 2 \cos(\omega t + 10°) &= 2 \sin(\omega t + 10° + 90°) \\ &= 2 \sin(\omega t + 100°) \end{aligned}$$

***i* leads *v* by 110º, or *v* lags *i* by 110º.**

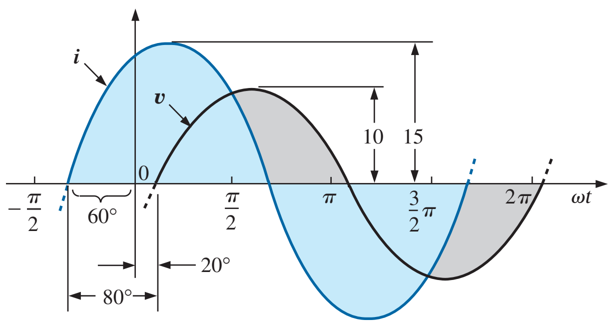


FIG. 13.32
Example 13.12(b): i leads v by 80°.

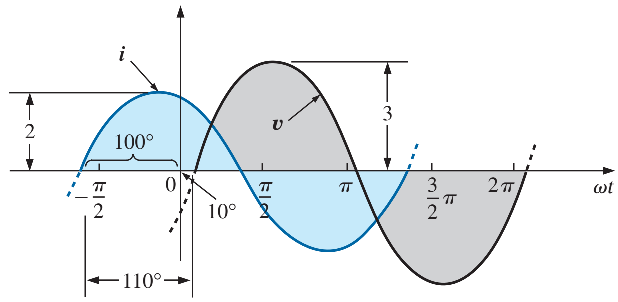


FIG. 13.33
Example 13.12(c): i leads v by 110°.

d. See Fig. 13.34.

Note

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° - 180°)$$
$$= \sin(\omega t - 150°)$$

***v* leads *i* by 160°, or *i* lags *v* by 160°.**

Or using

Note

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° + 180°)$$
$$= \sin(\omega t + 210°)$$

***i* leads *v* by 200°, or *v* lags *i* by 200°.**

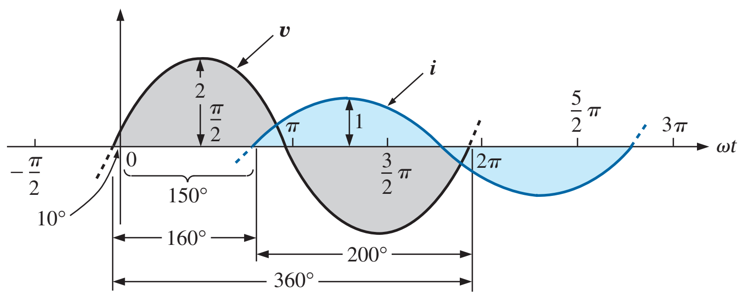


FIG. 13.34
Example 13.12(d): v leads i by 160°.

e. See Fig. 13.35.

By choice

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° - 180°)$$
$$= 2\cos(\omega t - 240°)$$

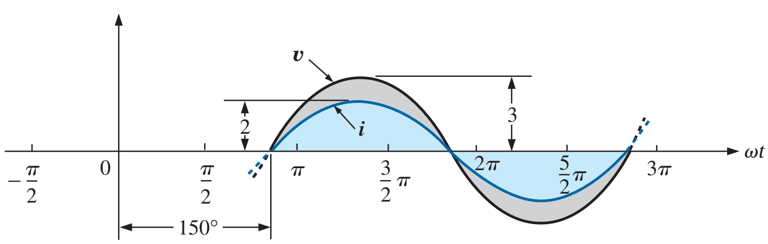


FIG. 13.35
Example 13.12(e): v and i are in phase.

However, $\cos \alpha = \sin(\alpha + 90^\circ)$

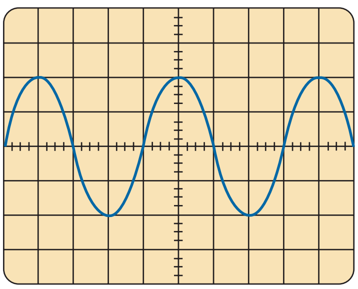
so that
$$2 \cos(\omega t - 240^\circ) = 2 \sin(\omega t - 240^\circ + 90^\circ) = 2 \sin(\omega t - 150^\circ)$$

***v* and *i* are in phase.**

The Oscilloscope

The **oscilloscope** is an instrument that will display the sinusoidal alternating waveform in a way that will permit the reviewing of all of the waveform's characteristics. In some ways, the screen and the dials give an oscilloscope the appearance of a small TV, but remember that *it can display only what you feed into it.* You can't turn it on and ask for a sine wave, a square wave, and so on; it must be connected to a source or an active circuit to pick up the desired waveform.

The screen has a standard appearance, with 10 horizontal divisions and 8 vertical divisions. The distance between divisions is 1 cm on the vertical and horizontal scales, providing you with an excellent opportunity to become aware of the length of 1 cm. *The vertical scale is set to display voltage levels, whereas the horizontal scale is always in units of time.* The vertical sensitivity control sets the voltage level for each division, whereas the horizontal sensitivity control sets the time associated with each division. In other words, if the vertical sensitivity is set at 1 V/div., each division displays a 1 V swing, so that a total vertical swing of 8 divisions represents 8 V peak-to-peak. If the horizontal control is set on 10 μs/div., 4 divisions equal a time period of 40 μs. Remember, the oscilloscope display presents a sinusoidal voltage versus time, not degrees or radians. Further, the vertical scale is always a voltage sensitivity, never units of amperes.



Vertical sensitivity = 0.1 V/div.
Horizontal sensitivity = 50 μs/div.

FIG. 13.36
Example 13.13.

EXAMPLE 13.13 Find the period, frequency, and peak value of the sinusoidal waveform appearing on the screen of the oscilloscope in Fig. 13.36. Note the sensitivities provided in the figure.

Solution: One cycle spans 4 divisions. Therefore, the period is

$$T = 4 \text{ div.} \left(\frac{50\ \mu\text{s}}{\text{div.}}\right) = \mathbf{200\ \mu s}$$

and the frequency is

$$f = \frac{1}{T} = \frac{1}{200 \times 10^{-6}\ \text{s}} = \mathbf{5\ kHz}$$

The vertical height above the horizontal axis encompasses 2 divisions. Therefore,

$$V_m = 2 \text{ div.} \left(\frac{0.1\ \text{V}}{\text{div.}}\right) = \mathbf{0.2\ V}$$

An oscilloscope can also be used to make phase measurements between two sinusoidal waveforms. Virtually all laboratory oscilloscopes today have the dual-trace option, that is, the ability to show two waveforms at the same time. It is important to remember, however, that both waveforms will and must have the same frequency. The hookup procedure for using an oscilloscope to measure phase angles is covered in detail in Section 15.13. However, the equation for determining the phase angle can be introduced using Fig. 13.37.

First, note that each sinusoidal function *has the same frequency,* permitting the use of either waveform to determine the period. For the waveform chosen in Fig. 13.37, the period encompasses 5 divisions at 0.2 ms/div. The phase shift between the waveforms (irrespective of which is leading or lagging) is 2 divisions. Since the full period represents a cycle of 360°, the following ratio [from which Eq. (13.24) can be derived] can be formed:

$$\frac{360°}{T\,(\text{no. of div.})} = \frac{\theta}{\text{phase shift (no. of div.)}}$$

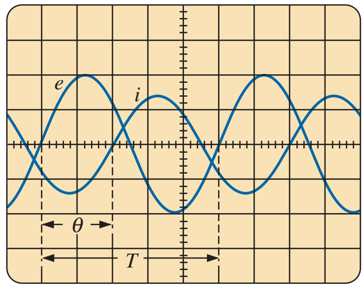
and

$$\theta = \frac{\text{phase shift (no. of div.)}}{T\,(\text{no. of div.})} \times 360° \qquad \textbf{(13.24)}$$

Substituting into Eq. (13.24) results in

$$\theta = \frac{(2\text{ div.})}{(5\text{ div.})} \times 360° = \mathbf{144°}$$

and e leads i by 144°.



Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 0.2 ms/div.

FIG. 13.37
Finding the phase angle between waveforms using a dual-trace oscilloscope.

13.7 AVERAGE VALUE

Even though the concept of the **average value** is an important one in most technical fields, its true meaning is often misunderstood. In Fig. 13.38(a), for example, the average height of the sand may be required to determine the volume of sand available. The average height of the sand is that height obtained if the distance from one end to the other is maintained while the sand is leveled off, as shown in Fig. 13.38(b). The area under the mound in Fig. 13.38(a) then equals the area under the rectangular shape in Fig. 13.38(b) as determined by $A = b \times h$. Of course, the depth (into the page) of the sand must be the same for Fig. 13.38(a) and (b) for the preceding conclusions to have any meaning.

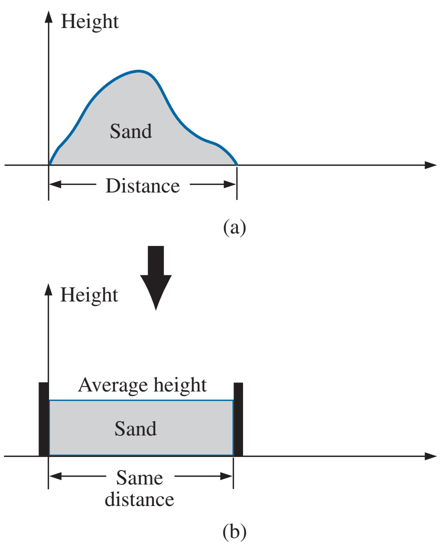


FIG. 13.38
Defining average value.

In Fig. 13.38, the distance was measured from one end to the other. In Fig. 13.39(a), the distance extends beyond the end of the original pile in Fig. 13.38. The situation could be one where a landscaper wants to know the average height of the sand if spread out over a distance such as defined in Fig. 13.39(a). The result of an increased distance is shown in Fig. 13.39(b). The average height has decreased compared to Fig. 13.38. Quite obviously, therefore, the longer the distance, the lower the average value.

If the distance parameter includes a depression, as shown in Fig. 13.40(a), some of the sand will be used to fill the depression, resulting in an even lower average value for the landscaper, as shown in Fig. 13.40(b). For a sinusoidal waveform, the depression would have the same shape as the mound of sand (over one full cycle), resulting in an average value at ground level (or zero volts for a sinusoidal voltage over one full period).

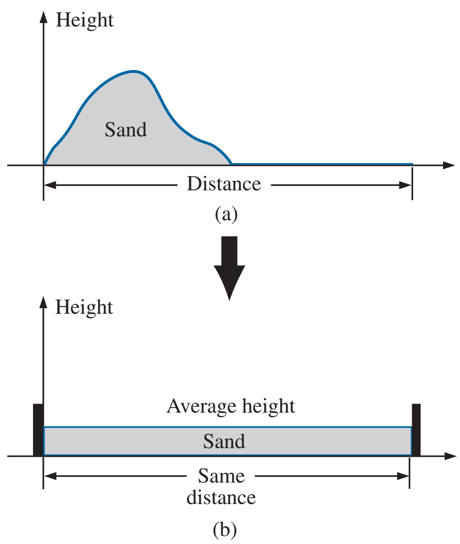


FIG. 13.39
Effect of distance (length) on average value.

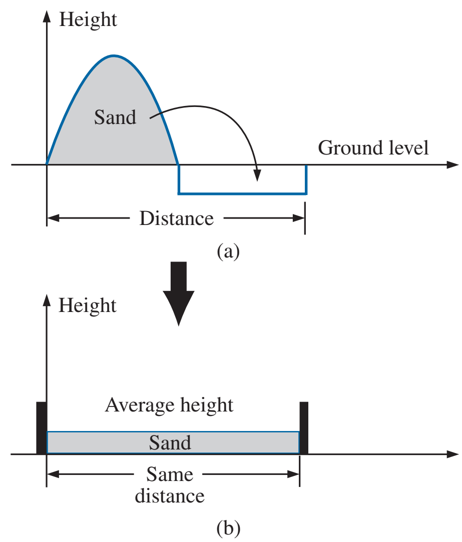


FIG. 13.40
Effect of depressions (negative excursions) on average value.

After traveling a considerable distance by car, some drivers like to calculate their average speed for the entire trip. This is usually done by dividing the miles traveled by the hours required to drive that distance. For example, if a person traveled 225 mi in 5 h, the average speed was 225 mi/5 h, or 45 mi/h. This same distance may have been traveled at various speeds for various intervals of time, as shown in Fig. 13.41.

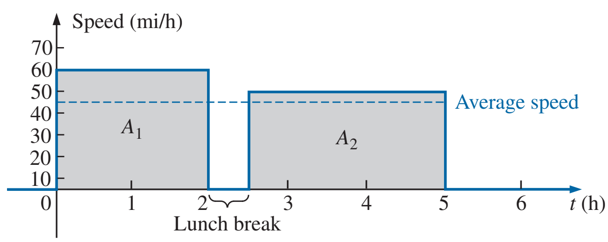


FIG. 13.41
Plotting speed versus time for an automobile excursion.

By finding the total area under the curve for the 5 h and then dividing the area by 5 h (the total time for the trip), we obtain the same result of 45 mi/h; that is,

$$\text{Average speed} = \frac{\text{area under curve}}{\text{length of curve}} \tag{13.25}$$

$$\text{Average speed} = \frac{A_1 + A_2}{5\text{ h}} = \frac{(60\text{ mi/h})(2\text{ h}) + (50\text{ mi/h})(2.5\text{ h})}{5\text{ h}}$$

$$= \frac{225}{5}\text{ mi/h} = \mathbf{45\ mi/h}$$

Eq. (13.25) can be extended to include any variable quantity, such as current or voltage, if we let G denote the average value, as follows:

$$G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}} \tag{13.26}$$

The *algebraic* sum of the areas must be determined, since some area contributions are from below the horizontal axis. Areas above the axis are assigned a positive sign, and those below, a negative sign. A positive average value is then above the axis, and a negative value, below.

The average value of *any* current or voltage is the value indicated on a dc meter. In other words, over a complete cycle, the average value is the equivalent dc value. In the analysis of electronic circuits to be considered in a later course, both dc and ac sources of voltage will be applied to the same network. You will then need to know or determine the dc (or average value) and ac components of the voltage or current in various parts of the system.

EXAMPLE 13.14 Determine the average value of the waveforms in Fig. 13.42.

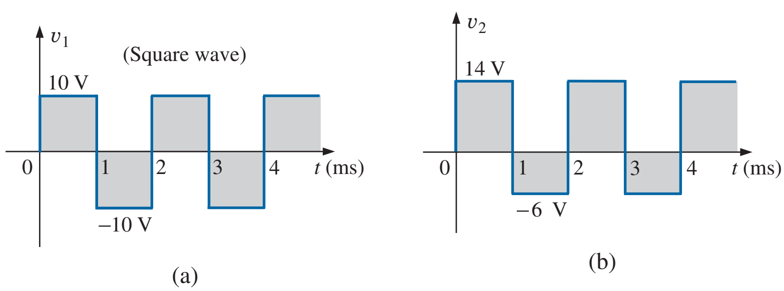


FIG. 13.42
Example 13.14.

Solutions:

a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using Eq. (13.26):

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

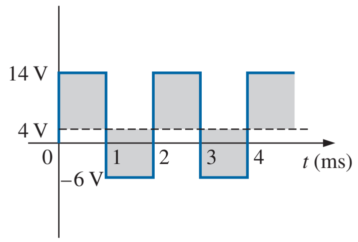


FIG. 13.43
Defining the average value for the waveform in Fig. 13.42(b).

b. Using Eq. (13.26):

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\text{ V}}$$

as shown in Fig. 13.43.

In reality, the waveform in Fig. 13.42(b) is simply the square wave in Fig. 13.42(a) with a dc shift of 4 V; that is,

$$v_2 = v_1 + 4\text{ V}$$

EXAMPLE 13.15 Find the average values of the following waveforms over one full cycle:

a. Fig. 13.44.
b. Fig. 13.45.

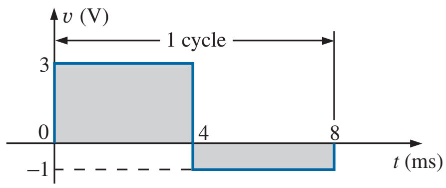


FIG. 13.44
Example 13.15(a).

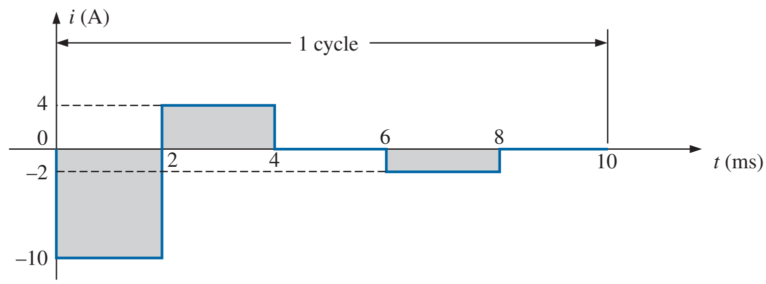


FIG. 13.45
Example 13.15(b).

Solutions:

a. $G = \frac{+(3\text{ V})(4\text{ ms}) - (1\text{ V})(4\text{ ms})}{8\text{ ms}} = \frac{12\text{ V} - 4\text{ V}}{8} = \mathbf{1\text{ V}}$

Note Fig. 13.46.

b. $G = \frac{-(10\text{ V})(2\text{ ms}) + (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}}$

$$= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = -\frac{16\text{ V}}{10} = \mathbf{-1.6\text{ V}}$$

Note Fig. 13.47.

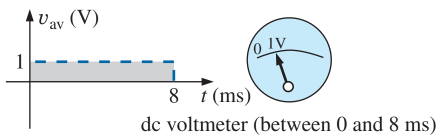


FIG. 13.46
The response of a dc meter to the waveform in Fig. 13.44.

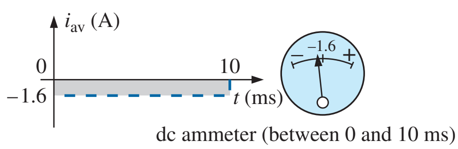


FIG. 13.47
The response of a dc meter to the waveform in Fig. 13.45.

We found the areas under the curves in Example 13.15 by using a simple geometric formula. If we should encounter a sine wave or any other unusual shape, however, we must find the area by some other means. We can obtain a good approximation of the area by attempting to reproduce the original wave shape using a number of small rectangles or other familiar shapes, the area of which we already know through simple geometric formulas. For example,

the area of the positive (or negative) pulse of a sine wave is $2A_m$.

Approximating this waveform by two triangles (Fig. 13.48), we obtain (using *area = 1/2 base × height* for the area of a triangle) a rough idea of the actual area:

$$\text{Area shaded} = 2\left(\frac{1}{2}bh\right) = 2\left[\left(\frac{1}{2}\right)\overbrace{\left(\frac{\pi}{2}\right)}^{b}\overbrace{(A_m)}^{h}\right] = \frac{\pi}{2}A_m \cong 1.58A_m$$

A closer approximation may be a rectangle with two similar triangles (Fig. 13.49):

$$\text{Area} = A_m\frac{\pi}{3} + 2\left(\frac{1}{2}bh\right) = A_m\frac{\pi}{3} + \frac{\pi}{3}A_m = \frac{2}{3}\pi A_m = 2.094A_m$$

which is certainly close to the actual area. If an infinite number of forms is used, an exact answer of $2A_m$ can be obtained. For irregular waveforms, this method can be especially useful if data such as the average value are desired.

The procedure of calculus that gives the exact solution $2A_m$ is known as *integration*. Integration is presented here only to make the method recognizable to you; it is not necessary to be proficient in its use to continue with this text. It is a useful mathematical tool, however, and should be learned. Finding the area under the positive pulse of a sine wave using integration, we have

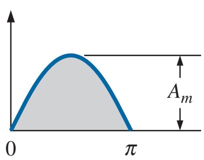
$$\text{Area} = \int_0^{\pi} A_m \sin\alpha \, d\alpha$$

where $\int$ is the sign of integration, 0 and π are the limits of integration, $A_m \sin\alpha$ is the function to be integrated, and $d\alpha$ indicates that we are integrating with respect to α.

Integrating, we obtain

$$\begin{aligned}\text{Area} &= A_m[-\cos\alpha]_0^{\pi}\\ &= -A_m(\cos\pi - \cos 0°)\\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$$\boxed{\text{Area} = 2A_m} \qquad \textbf{(13.27)}$$

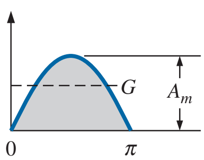


Since we know the area under the positive (or negative) pulse, we can easily determine the average value of the positive (or negative) region of a sine wave pulse by applying Eq. (13.26):

$$G = \frac{2A_m}{\pi}$$

and

$$\boxed{G = \frac{2A_m}{\pi} = 0.637A_m} \qquad \textbf{(13.28)}$$



For the waveform in Fig. 13.50,

$$G = \frac{(2A_m/2)}{\pi/2} = \frac{2A_m}{\pi} \qquad \text{(The average is the same as for a full pulse.)}$$

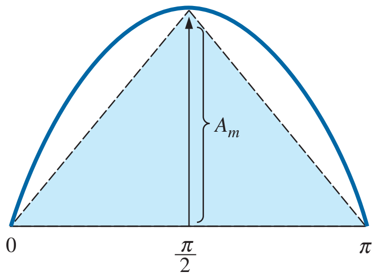


FIG. 13.48
Approximating the shape of the positive pulse of a sinusoidal waveform with two right triangles.

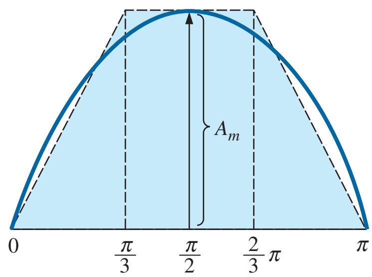


FIG. 13.49
A better approximation for the shape of the positive pulse of a sinusoidal waveform.

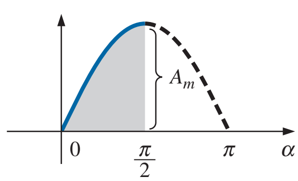


FIG. 13.50
Finding the average value of one-half the positive pulse of a sinusoidal waveform.

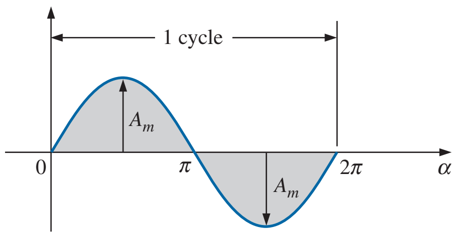


FIG. 13.51
Example 13.16.

EXAMPLE 13.16 Determine the average value of the sinusoidal waveform in Fig. 13.51.

Solution: By inspection it is fairly obvious that

the average value of a pure sinusoidal waveform over one full cycle is zero.

Eq. (13.26):

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

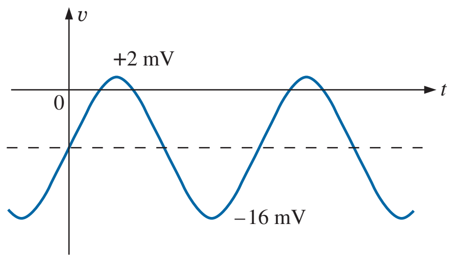


FIG. 13.52
Example 13.17.

EXAMPLE 13.17 Determine the average value of the waveform in Fig. 13.52.

Solution: The peak-to-peak value of the sinusoidal function is 16 mV + 2 mV = 18 mV. The peak amplitude of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV. Counting down 9 mV from 2 mV (or 9 mV up from −16 mV) results in an average or dc level of **−7 mV,** as noted by the dashed line in Fig. 13.52.

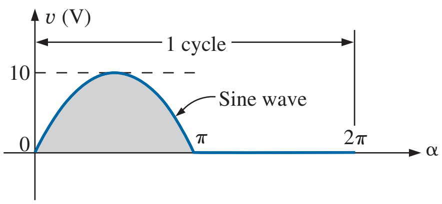


FIG. 13.53
Example 13.18.

EXAMPLE 13.18 Determine the average value of the waveform in Fig. 13.53.

Solution:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\ \text{V})}{2\pi} \cong \mathbf{3.18\ V}$$

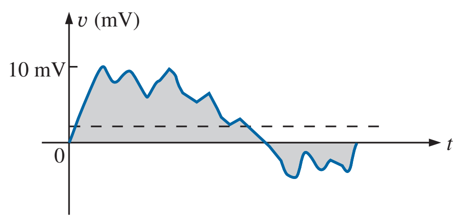


FIG. 13.54
Example 13.19.

EXAMPLE 13.19 For the waveform in Fig. 13.54, determine whether the average value is positive or negative, and determine its approximate value.

Solution: From the appearance of the waveform, the average value is positive and in the vicinity of 2 mV. Occasionally, judgments of this type will have to be made.

Instrumentation

The dc level or average value of any waveform can be found using a digital multimeter (DMM) or an **oscilloscope.** For purely dc circuits, set the DMM on dc, and read the voltage or current levels. Oscilloscopes are limited to voltage levels using the sequence of steps listed below:

1. First choose GND from the DC-GND-AC option list associated with each vertical channel. The GND option blocks any signal to which the oscilloscope probe may be connected from entering the oscilloscope and responds with just a horizontal line. Set the resulting line in the middle of the vertical axis on the horizontal axis, as shown in Fig. 13.55(a).
2. Apply the oscilloscope probe to the voltage to be measured (if not already connected), and switch to the DC option. If a dc voltage is present, the horizontal line shifts up or down, as demonstrated in Fig. 13.55(b). Multiplying the shift by the vertical sensitivity

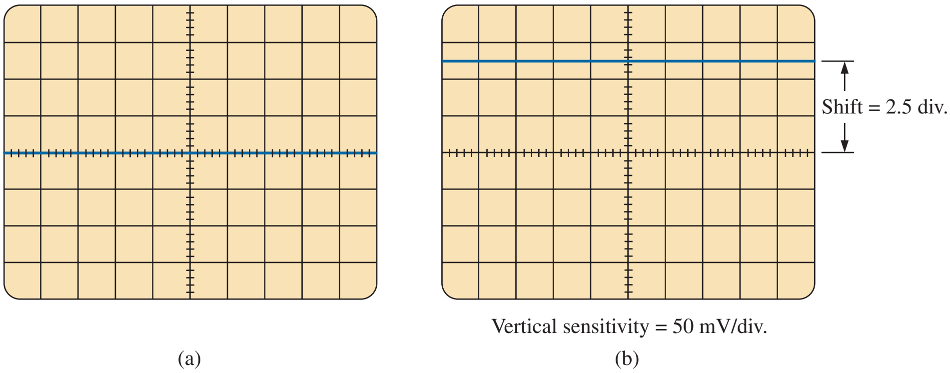


FIG. 13.55
Using the oscilloscope to measure dc voltages; (a) setting the GND condition; (b) the vertical shift resulting from a dc voltage when shifted to the DC option.

results in the dc voltage. An upward shift is a positive voltage (higher potential at the red or positive lead of the oscilloscope), while a downward shift is a negative voltage (lower potential at the red or positive lead of the oscilloscope).

In general,

$$V_{dc} = (\text{vertical shift in div.}) \times (\text{vertical sensitivity in V/div.}) \quad \textbf{(13.29)}$$

For the waveform in Fig. 13.55(b),

$$V_{dc} = (2.5 \text{ div.})(50 \text{ mV/div.}) = \mathbf{125 \text{ mV}}$$

The oscilloscope can also be used to measure the dc or average level of any waveform using the following sequence:

1. Using the GND option, reset the horizontal line to the middle of the screen.
2. Switch to AC (all dc components of the signal to which the probe is connected will be blocked from entering the oscilloscope—only the alternating, or changing, components are displayed). Note the location of some definitive point on the waveform, such as the bottom of the half-wave rectified waveform of Fig. 13.56(a); that is,

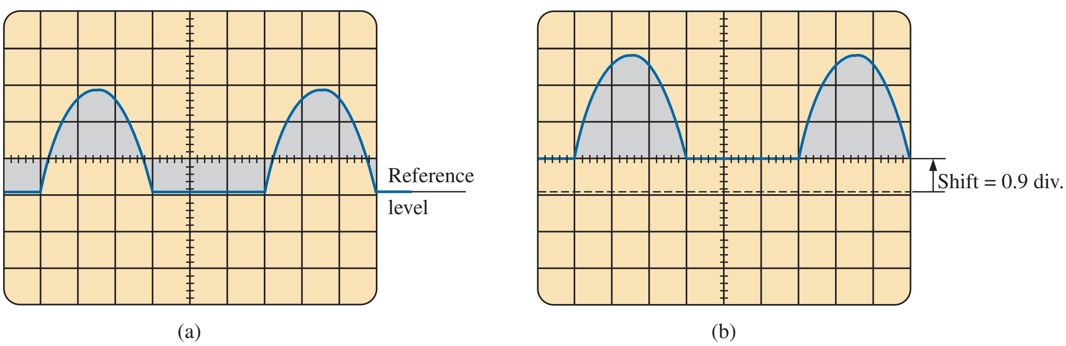


FIG. 13.56
Determining the average value of a nonsinusoidal waveform using the oscilloscope: (a) vertical channel on the ac mode; (b) vertical channel on the dc mode.

note its position on the vertical scale. For the future, *whenever you use the AC option, keep in mind that the computer will distribute the waveform above and below the horizontal axis such that the average value is zero;* that is, the area above the axis will equal the area below.

3. Then switch to DC (to permit both the dc and the ac components of the waveform to enter the oscilloscope), and note the shift in the chosen level of part 2, as shown in Fig. 13.56(b). Eq. (13.29) can then be used to determine the dc or average value of the waveform. For the waveform in Fig. 13.56(b), the average value is about

$$V_{av} = V_{dc} = (0.9 \text{ div.})(5 \text{ V/div.}) = \mathbf{4.5\ V}$$

The procedure outlined above can be applied to any alternating waveform such as the one in Fig. 13.54. In some cases the average value may require moving the starting position of the waveform under the AC option to a different region of the screen or choosing a higher voltage scale. By choosing the appropriate scale, you can enable DMMs to read the average or dc level of any waveform.

13.8 EFFECTIVE (rms) VALUES

This section begins to relate dc and ac quantities with respect to the power delivered to a load. It will help us determine the amplitude of a sinusoidal ac current required to deliver the same power as a particular dc current. The question frequently arises, How is it possible for a sinusoidal ac quantity to deliver a net power if, over a full cycle, the net current in any one direction is zero (average value = 0)? It would almost appear that the power delivered during the positive portion of the sinusoidal waveform is withdrawn during the negative portion, and since the two are equal in magnitude, the net power delivered is zero. However, understand that regardless of *direction,* current of any magnitude through a resistor delivers power *to that resistor.* In other words, during the positive or negative portions of a sinusoidal ac current, power is being delivered at *each instant of time* to the resistor. The power delivered at each instant, of course, varies with the magnitude of the sinusoidal ac current, but there will be a net flow during either the positive or the negative pulses with a net flow over the full cycle. The net power flow equals twice that delivered by either the positive or the negative regions of sinusoidal quantity.

A fixed relationship between ac and dc voltages and currents can be derived from the experimental setup shown in Fig. 13.57. A resistor in a

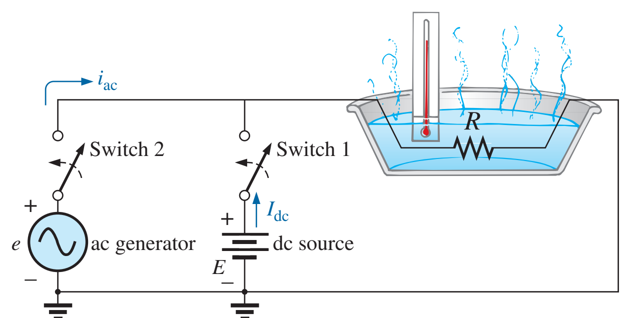


FIG. 13.57
An experimental setup to establish a relationship between dc and ac quantities.

water bath is connected by switches to a dc and an ac supply. If switch 1 is closed, a dc current *I,* determined by the resistance *R* and battery voltage *E,* is established through the resistor *R.* The temperature reached by the water is determined by the dc power dissipated in the form of heat by the resistor.

If switch 2 is closed and switch 1 left open, the ac current through the resistor has a peak value of I_m. The temperature reached by the water is now determined by the ac power dissipated in the form of heat by the resistor. The ac input is varied until the temperature is the same as that reached with the dc input. When this is accomplished, the average electrical power delivered to the resistor *R* by the ac source is the same as that delivered by the dc source.

The power delivered by the ac supply at any instant of time is

$$P_{ac} = (i_{ac})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 \sin^2 \omega t) R$$

However,

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t) \qquad \text{(trigonometric identity)}$$

Therefore,

$$P_{ac} = I_m^2 \left[\frac{1}{2}(1 - \cos 2\omega t)\right] R$$

and

$$P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t \qquad \textbf{(13.30)}$$

The *average power* delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform. Equating the average power delivered by the ac generator to that delivered by the dc source,

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2 R}{2} = I_{dc}^2 R$$

and

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

which, in words, states that

the equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its peak value.

The equivalent dc value is called the **rms** or **effective value** of the sinusoidal quantity.

As a simple numerical example, it requires an ac current with a peak value of $\sqrt{2}(10) = 14.14$ A to deliver the same power to the resistor in Fig. 13.57 as a dc current of 10 A. The effective value of any quantity plotted as a function of time can be found by using the following equation derived from the experiment just described.

Calculus format:

$$I_{rms} = \sqrt{\frac{\int_0^T i^2(t)\, dt}{T}} \qquad \textbf{(13.31)}$$

which means:

$$I_{\text{rms}} = \sqrt{\frac{\text{area } (i^2(t))}{T}} \tag{13.32}$$

In words, Eqs. (13.31) and (13.32) state that to find the rms value, the function $i(t)$ must first be squared. After $i(t)$ is squared, the area under the curve is found by integration. It is then divided by T, the length of the cycle or the period of the waveform, to obtain the average or *mean* value of the squared waveform. The final step is to take the *square root* of the mean value. This procedure is the source for the other designation for the effective value, the **root-mean-square (rms) value.** In fact, since *rms* is the most commonly used term in the educational and industrial communities, it is used throughout this text.

The relationship between the peak value and the rms value is the same for voltages, resulting in the following set of relationships for the examples and text material to follow:

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$$
$$E_{\text{rms}} = \frac{1}{\sqrt{2}} E_m = 0.707 E_m \tag{13.33}$$

Similarly,

$$I_m = \sqrt{2} I_{\text{rms}} = 1.414 I_{\text{rms}}$$
$$E_m = \sqrt{2} E_{\text{rms}} = 1.414 E_{\text{rms}} \tag{13.34}$$

EXAMPLE 13.20 Find the rms values of the sinusoidal waveform in each part in Fig. 13.58.

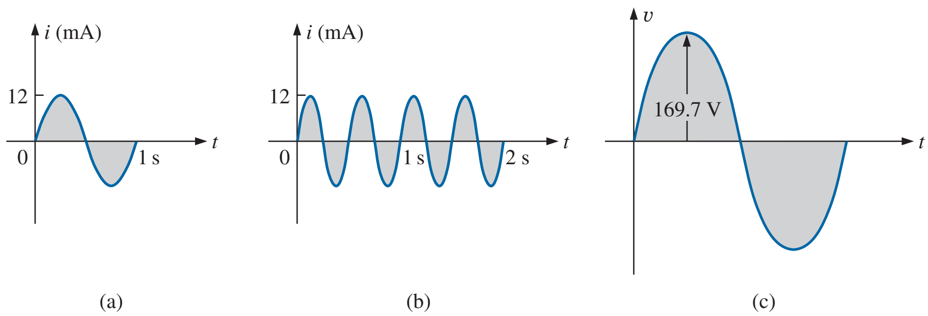


FIG. 13.58
Example 13.20.

Solution: For part (a), $I_{\text{rms}} = 0.707(12 \times 10^{-3}\ \text{A}) =$ **8.48 mA.** For part (b), again $I_{\text{rms}} =$ **8.48 mA.** Note that frequency did not change the effective value in (b) compared to (a). For part (c), $V_{\text{rms}} = 0.707(169.73\ \text{V}) \cong$ **120 V,** the same as available from a home outlet.

EXAMPLE 13.21 The 120 V dc source in Fig. 13.59(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13.59(b)] is to deliver the same power to the load.

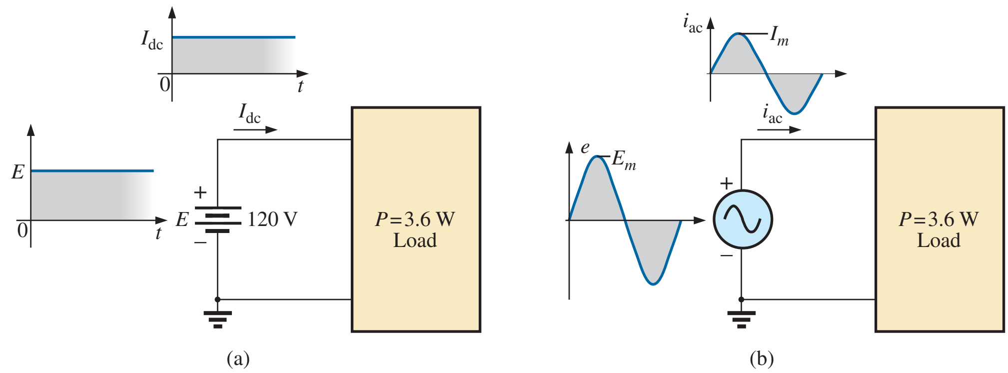


FIG. 13.59
Example 13.21.

Solution:

$$P_{dc} = V_{dc}I_{dc}$$

and $$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2}I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$

$$E_m = \sqrt{2}E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

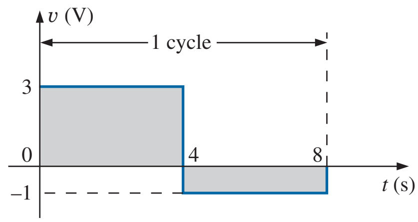


FIG. 13.60
Example 13.22.

EXAMPLE 13.22 Find the rms value of the waveform in Fig. 13.60.

Solution: v^2 (Fig. 13.61):

$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\text{ V}}$$

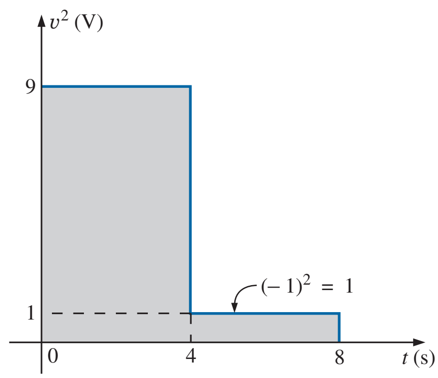


FIG. 13.61
The squared waveform of Fig. 13.60.

EXAMPLE 13.23 Calculate the rms value of the voltage in Fig. 13.62.

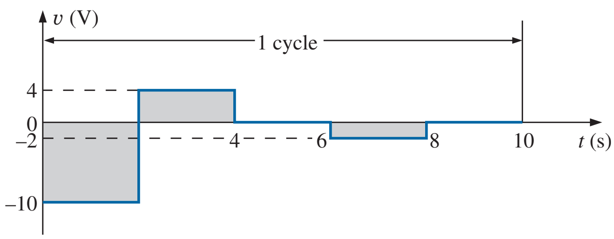


FIG. 13.62
Example 13.23.

Solution: v^2 (Fig. 13.63):

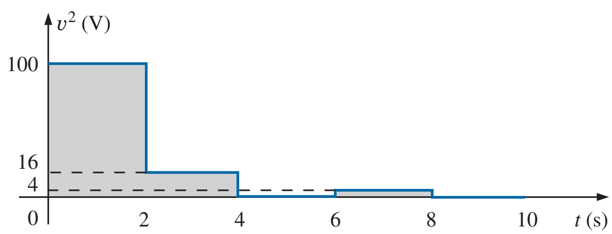


FIG. 13.63
The squared waveform of Fig. 13.62.

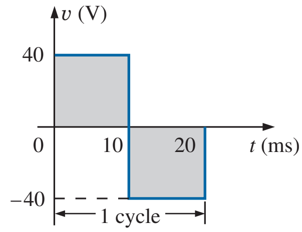


FIG. 13.64
Example 13.24.

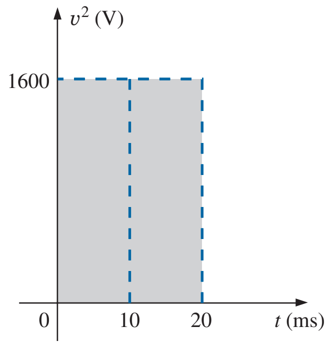


FIG. 13.65
The squared waveform of Fig. 13.64.

EXAMPLE 13.24 Determine the average and rms values of the square wave in Fig. 13.64.

Solution: By inspection, the average value is zero.

v^2 (Fig. 13.65):

$$V_{\text{rms}} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{(32{,}000 \times 10^{-3})}{20 \times 10^{-3}}} = \sqrt{1600} = \mathbf{40\ V}$$

(the maximum value of the waveform in Fig. 13.64).

The waveforms appearing in these examples are the same as those used in the examples on the average value. It may prove interesting to compare the rms and average values of these waveforms.

The rms values of sinusoidal quantities such as voltage or current are represented by E and I. These symbols are the same as those used for dc voltages and currents. To avoid confusion, the peak value of a waveform always has a subscript m associated with it: $I_m \sin \omega t$. *Caution:* When finding the rms value of the positive pulse of a sine wave, note that the squared area is *not* simply $(2A_m)^2 = 4A_m^2$; it must be found by a completely new integration. This is always true for any waveform that is not rectangular.

A unique situation arises if a waveform has both a dc and an ac component that may be due to a source such as the one in Fig. 13.66. The combination appears frequently in the analysis of electronic networks where both dc and ac levels are present in the same system.

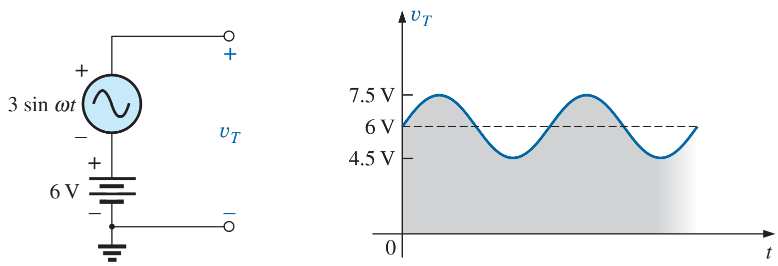


FIG. 13.66
Generation and display of a waveform having a dc and an ac component.

The question arises, What is the rms value of the voltage v_T? You may be tempted to assume that it is the sum of the rms values of each component of the waveform; that is, $V_{T_{\text{rms}}} = 0.7071(1.5\text{ V}) + 6\text{ V} = 1.06\text{ V} + 6\text{ V} = 7.06\text{ V}$. However, the rms value is actually determined by

$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^2 + V_{\text{ac(rms)}}^2} \tag{13.35}$$

which for the waveform in Fig. 13.66 is

$$V_{\text{rms}} = \sqrt{(6\text{ V})^2 + (1.06\text{ V})^2} = \sqrt{37.124}\text{ V} \cong \mathbf{6.1\ V}$$

This result is noticeably less than the solution of 7.06 V.

13.9 ac METERS AND INSTRUMENTS

It is important to note whether the DMM in use is a *true rms* meter or simply a meter where the average value is calibrated (as described in the next section) to indicate the rms level. *A true rms meter reads the effective value of any waveform (such as Figs. 13.54 and 13.66) and is not limited to only sinusoidal waveforms.* Since the label *true rms* is normally not placed on the face of the meter, it is prudent to check the manual if waveforms other than purely sinusoidal are to be encountered. For any type of rms meter, be sure to check the manual for its frequency range of application. For most, it is less than 1 kHz.

If an average reading movement such as the d'Arsonval movement used in the **VOM** of Fig. 2.29 is used to measure an ac current or voltage, the level indicated by the movement must be multiplied by a **calibration factor.** In other words, if the movement of any voltmeter or ammeter is reading the average value, that level must be multiplied by a specific constant, or calibration factor, to indicate the rms level. For ac waveforms, the signal must first be converted to one having an average value over the time period. Recall that it is zero over a full period for a sinusoidal waveform. This is usually accomplished for sinusoidal waveforms using a bridge rectifier such as in Fig. 13.67. The conversion process, involving four diodes in a bridge configuration, is well documented in most electronic texts.

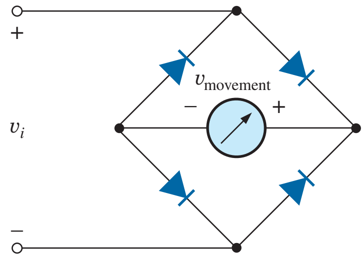


FIG. 13.67
Full-wave bridge rectifier.

Fundamentally, conduction is permitted through the diodes in such a manner as to convert the sinusoidal input of Fig. 13.68(a) to one having the appearance of Fig. 13.68(b). The negative portion of the input has been effectively "flipped over" by the bridge configuration. The resulting waveform in Fig. 13.68(b) is called a *full-wave rectified waveform.*

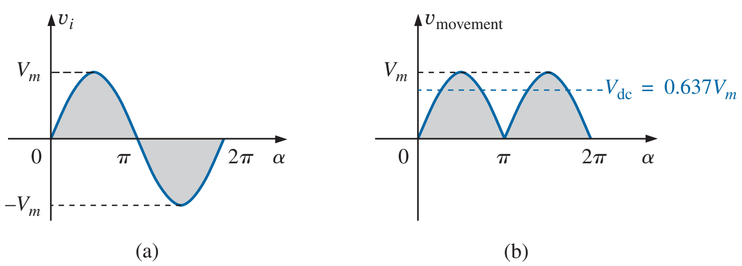


FIG. 13.68
(a) Sinusoidal input; (b) full-wave rectified signal.

The zero average value in Fig. 13.68(a) has been replaced by a pattern having an average value determined by

$$G = \frac{2V_m + 2V_m}{2\pi} = \frac{4V_m}{2\pi} = \frac{2V_m}{\pi} = 0.637V_m$$

The movement of the pointer is therefore directly related to the peak value of the signal by the factor 0.637.

Forming the ratio between the rms and dc levels results in

$$\frac{V_{rms}}{V_{dc}} = \frac{0.707V_m}{0.637V_m} \cong 1.11$$

revealing that the scale indication is 1.11 times the dc level measured by the movement; that is,

$$\boxed{\text{Meter indication} = 1.11\ (\text{dc or average value})} \quad \text{full-wave} \qquad \textbf{(13.36)}$$

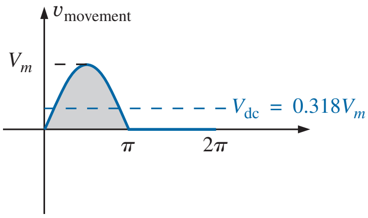


FIG. 13.69
Half-wave rectified signal.

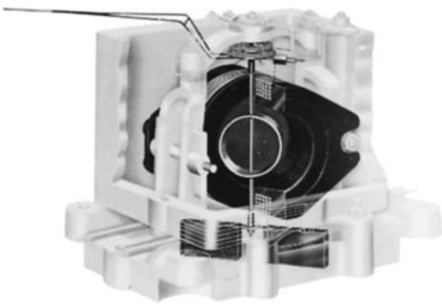


FIG. 13.70
Electrodynamometer movement.
(Courtesy of Schlumberger Technology Corp.)

Some ac meters use a half-wave rectifier arrangement that results in the waveform in Fig. 13.69, which has half the average value in Fig. 13.68(b) over one full cycle. The result is

$$\boxed{\text{Meter indication} = 2.22\,(\text{dc or average value})} \quad \text{half-wave} \qquad \textbf{(13.37)}$$

A second movement, called the **electrodynamometer movement** (Fig. 13.70), can measure both ac and dc quantities without a change in internal circuitry. The movement can, in fact, read the effective value of any periodic or nonperiodic waveform because a reversal in current direction reverses the fields of both the stationary and the movable coils, so the deflection of the pointer is always up-scale.

EXAMPLE 13.25 Determine the reading of each meter for each situation in Fig. 13.71(a) and (b).

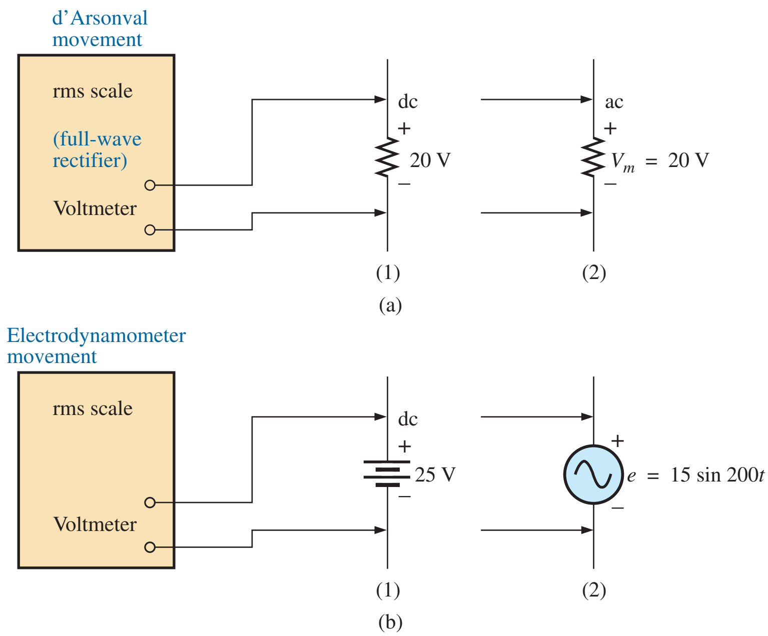


FIG. 13.71
Example 13.25.

Solution: For Fig. 13.71(a), situation (1): By Eq. (13.36),

$$\text{Meter indication} = 1.11(20\text{ V}) = \mathbf{22.2\text{ V}}$$

For Fig. 13.71(a), situation (2):

$$V_{\text{rms}} = 0.707V_m = 0.707(20\text{ V}) = \mathbf{14.14\text{ V}}$$

For Fig. 13.71(b), situation (1):

$$V_{\text{rms}} = V_{\text{dc}} = \mathbf{25\text{ V}}$$

For Fig. 13.71(b), situation (2):

$$V_{\text{rms}} = 0.707V_m = 0.707(15\text{ V}) \cong \mathbf{10.6\text{ V}}$$

Most DMMs employ a full-wave rectification system to convert the input ac signal to one with an average value. In fact, for the VOM in Fig. 2.29, the same scale factor of Eq. (13.36) is employed; that is, the average value is scaled up by a factor of 1.11 to obtain the rms value. In digital meters, however, there are no moving parts such as in the d'Arsonval movement to display the signal level. Rather, the average value is sensed by a multiprocessor integrated circuit (IC), which in turn determines which digits should appear on the digital display.

Digital meters can also be used to measure nonsinusoidal signals, but the scale factor of each input waveform must first be known (normally provided by the manufacturer in the operator's manual.) For instance, the scale factor for an average responding DMM on the ac rms scale produces an indication for a square-wave input that is 1.11 times the peak value. For a triangular input, the response is 0.555 times the peak value. Obviously, for a sine wave input, the response is 0.707 times the peak value.

For any instrument, it is always good practice to read the operator's manual if you will use the instrument on a regular basis.

For frequency measurements, the **frequency counter** in Fig. 13.72 provides a digital readout of sine, square, and triangular waves from 1 Hz to 1.3 GHz. Note the relative simplicity of the panel and the high degree of accuracy available. The temperature-compensated, crystal-controlled time base is stable to ±1 part per million per year.

FIG. 13.72
Frequency counter. Tektronix CMC251 1.3 GHz multifunction counter.
(Photo courtesy of Tektronix, Inc.)

The AEMC® **Clamp Meter** in Fig. 13.73 is an instrument that can measure alternating current in the ampere range without having to open the circuit. The loop is opened by squeezing the "trigger"; then it is placed around the current-carrying conductor. Through transformer action, the level of current in rms units appears on the appropriate scale. The Model 501 is auto-ranging (that is, each scale changes automatically) and can measure dc or ac currents up to 400 mA. Through the use of additional leads, it can also be used as a voltmeter (up to 400 V, dc or ac) and an ohmmeter (from zero to 400 Ω).

One of the most versatile and important instruments in the electronics industry is the **oscilloscope,** which has already been introduced in this chapter. It provides a display of the waveform on a cathode-ray tube to permit the detection of irregularities and the determination of quantities such as magnitude, frequency, period, dc component, and so on. The

FIG. 13.73
Clamp-on ammeter and voltmeter.
(Courtesy of AEMC® Instruments, Foxborough, MA.)

FIG. 13.74
Four-channel digital phosphor oscilloscope. Tektronix TDS3000B series oscilloscope.
(Photo courtesy of Tektronix, Inc.)

digital oscilloscope in Fig. 13.74 can display four waveforms at the same time. You use menu buttons to set the vertical and horizontal scales by choosing from selections appearing on the screen. The TDS model in Fig. 13.74 can display, store, and analyze the amplitude, time, and distribution of amplitude over time. It is also completely portable due to its battery-capable design.

A student accustomed to watching TV may be confused when first introduced to an oscilloscope. There is, at least initially, an assumption that the oscilloscope is generating the waveform on the screen—much like a TV broadcast. However, it is important to clearly understand that

an oscilloscope displays only those signals generated elsewhere and connected to the input terminals of the oscilloscope. The absence of an external signal will simply result in a horizontal line on the screen of the scope.

On most oscilloscopes today, there is a switch or knob with the choice DC/GND/AC, as shown in Fig. 13.75(a), that is often ignored or treated too lightly in the early stages of scope utilization. The effect of each position is fundamentally as shown in Fig. 13.75(b). In the DC mode, the dc and ac components of the input signal can pass directly to the display. In the AC mode, the dc input is blocked by the capacitor, but the ac portion of the signal can pass through to the screen. In the GND position, the input signal is prevented from reaching the scope display by a direct ground connection, which reduces the scope display to a single horizontal line.

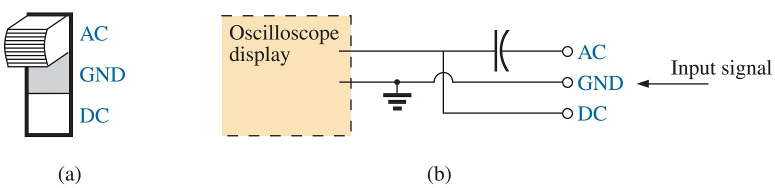


FIG. 13.75
AC-GND-DC switch for the vertical channel of an oscilloscope.

Before we leave the subject of ac meters and instrumentation, you should understand that

an ohmmeter cannot be used to measure the ac reactance or impedance of an element or system even though reactance and impedance are measured in ohms.

Recall that ohmmeters cannot be used on energized networks—the power must be shut off or disconnected. For an inductor, if the ac power is removed, the reactance of the coil is simply the dc resistance of the windings because the applicable frequency will be 0 Hz. For a capacitor, if the ac power is removed, the reactance of the capacitor is simply the leakage resistance of the capacitor. In general, therefore, always keep in mind that *ohmmeters can read only the dc resistance of an element or network, and only after the applied power has been removed.*

13.10 APPLICATIONS

(120 V at 60 Hz) versus (220 V at 50 Hz)

In North and South America, the most common available ac supply is 120 V at 60 Hz; in Europe and the Eastern countries, it is 220 V at 50 Hz. The choices of rms value and frequency were obviously made carefully because they have such an important impact on the design and operation of so many systems.

The fact that the frequency difference is only 10 Hz reveals that there was agreement on the general frequency range that should be used for power generation and distribution. History suggests that the question of frequency selection originally focused on the frequency that would not exhibit *flicker in the incandescent lamps* available in those days. Technically, however, there really wouldn't be a noticeable difference between 50 and 60 cycles per second based on this criterion. Another important factor in the early design stages was the effect of frequency on the size of transformers, which play a major role in power generation and distribution. Working through the fundamental equations for transformer design, you will find that *the size of a transformer is inversely proportional to frequency.* The result is that transformers operating at 50 Hz must be larger (on a purely mathematical basis about 17% larger) than those operating at 60 Hz. You will therefore find that transformers designed for the international market where they can operate on 50 Hz or 60 Hz are designed around the 50 Hz frequency. On the other side of the coin, however, higher frequencies result in increased concerns about arcing, increased losses in the transformer core due to eddy current and hysteresis losses, and skin effect phenomena. Somewhere in the discussion we may wonder about the fact that 60 Hz is an exact multiple of 60 seconds in a minute and 60 minutes in an hour. On the other side of the coin, however, a 60 Hz signal has a period of 16.67 ms (an awkward number), but the period of a 50 Hz signal is exactly 20 ms. Since accurate timing is such a critical part of our technological design, was this a significant motive in the final choice? There is also the question about whether the 50 Hz is a result of the close affinity of this value to the metric system. Keep in mind that powers of ten are all powerful in the metric system, with 100 cm in a meter, 100°C the boiling point of water, and so on. Note that 50 Hz is exactly half of this special number. All in all, it would seem that both sides have an argument that is worth defending. However, in the final analysis, we must also wonder whether the difference is simply political in nature.

The difference in voltage between the Americas and Europe is a different matter entirely in the sense that the difference is close to 100%. Again, however, there are valid arguments for both sides. There is no question that larger voltages such as 220 V *raise safety issues* beyond those raised by voltages of 120 V. However, when higher voltages are supplied, there is less current in the wire for the same power demand, permitting the use of smaller conductors—a real money saver. In addition, motors and some appliances *can be smaller in size.* Higher voltages, however, also bring back the concern about arcing effects, insulation requirements, and, due to real safety concerns, higher installation costs. In general, however, international travelers are prepared for most situations if they have a transformer that can convert from their home level to that of the country they plan to visit. Most equipment (not clocks, of course) can run quite well on 50 Hz or 60 Hz for most travel periods. For any unit not operating at its design frequency, it simply has to "work a little harder" to perform the given task. The major problem for the traveler is not the transformer itself but the wide variety of plugs used from one country to another. Each country has its own design for the "female" plug in the wall. For a three-week tour, this could mean as many as 6 to 10 different plugs of the type shown in Fig. 13.76. For a 120 V, 60 Hz supply, the plug is quite standard in appearance with its two spade leads (and possible ground connection).

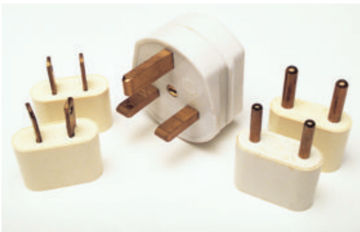


FIG. 13.76
Variety of plugs for a 220 V, 50 Hz connection.

In any event, both the 120 V at 60 Hz and the 220 V at 50 Hz are obviously meeting the needs of the consumer. It is a debate that could go on at length without an ultimate victor.

Safety Concerns (High Voltages and dc versus ac)

Be aware that any "live" network should be treated with a calculated level of respect. Electricity in its various forms is not to be feared but used with some awareness of its potentially dangerous side effects. It is common knowledge that electricity and water do not mix (never use extension cords or plug in TVs or radios in the bathroom) because a full 120 V in a layer of water of any height (from a shallow puddle to a full bath) can be *lethal.* However, other effects of dc and ac voltages are less known. In general, as the voltage and current increase, your concern about safety should increase exponentially. For instance, under dry conditions, most human beings can survive a 120 V ac shock such as obtained when changing a light bulb, turning on a switch, and so on. Most electricians have experienced such a jolt many times in their careers. However, ask an electrician to relate how it feels to hit 220 V, and the response (if he or she has been unfortunate to have had such an experience) will be totally different. How often have you heard of a back-hoe operator hitting a 220 V line and having a fatal heart attack? Remember, the operator is sitting in a metal container on a damp ground which provides an excellent path for the resulting current to flow from the line to ground. If only for a short period of time, with the best environment (rubber-sole shoes, and so on), in a situation where you can quickly escape the situation, most human beings can also survive a 220 V shock. However, as mentioned above, it is one you will not quickly forget. For voltages beyond 220 V rms, the chances of survival go down exponentially with increase in voltage. It takes only about 10 mA of steady current through the heart to put it in defibrillation. In general, therefore, always be sure that the power is disconnected when working on the repair of electrical equipment. Don't assume that throwing a wall switch will disconnect the power. Throw the main circuit breaker and test the lines with a voltmeter before working on the system. Since voltage is a two-point phenomenon, be sure to work with only one line at at time—accidents happen!

You should also be aware that the reaction to dc voltages is quite different from that to ac voltages. You have probably seen in movies or comic strips that people are often unable to let go of a *hot* wire. This is evidence of the most important difference between the two types of voltages. As mentioned above, if you happen to touch a "hot" 120 V ac line, you will probably get a good sting, but *you can let go.* If it happens to be a "hot" 120 V dc line, you will probably not be able to let go, and you could die. Time plays an important role when this happens, because the longer you are subjected to the dc voltage, the more the resistance in the body decreases until a fatal current can be established. The reason that we can let go of an ac line is best demonstrated by carefully examining the 120 V rms, 60 Hz voltage in Fig. 13.77. Since the voltage is oscillating, there is a period when the voltage is near zero or less than, say, 20 V, and is reversing in direction. Although this time interval is very short, it appears every 8.3 ms and provides a window for you to *let go.*

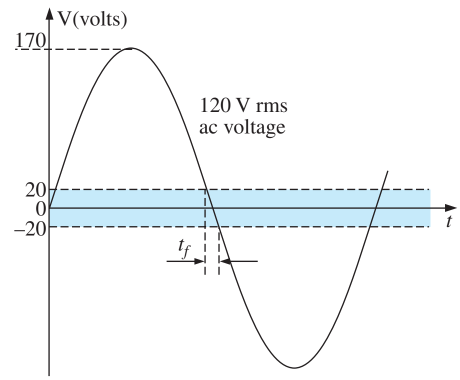


FIG. 13.77
Interval of time when sinusoidal voltage is near zero volts.

Now that we are aware of the additional dangers of dc voltages, it is important to mention that under the wrong conditions, dc voltages as low as 12 V such as from a car battery can be quite dangerous. If you happen to be working on a car under wet conditions, or if you are sweating badly for some reason or, worse yet, wearing a wedding ring that may have moisture and body salt underneath, touching the positive terminal may initiate the process whereby the body resistance begins to drop and serious injury could take place. It is one of the reasons you seldom see a professional electrician wearing any rings or jewelry—it is just not worth the risk.

Before leaving this topic of safety concerns, you should also be aware of the dangers of high-frequency supplies. We are all aware of what 2.45 GHz at 120 V can do to a meat product in a microwave oven, and it is therefore very important that the seal around the oven be as tight as possible. However, don't ever assume that anything is absolutely perfect in design—so don't make it a habit to view the cooking process in the microwave 6 in. from the door on a continuing basis. Find something else to do, and check the food only when the cooking process is complete. If you ever visit the Empire State Building, you will notice that you are unable to get close to the antenna on the dome due to the high-frequency signals being emitted with a great deal of power. Also note the large KEEP OUT signs near radio transmission towers for local radio stations. Standing within 10 ft of an AM transmitter working at 540 kHz would bring on disaster. Simply holding (do not try!) a fluorescent bulb near the tower could make it light up due to the excitation of the molecules inside the bulb.

In total, therefore, treat any situation with high ac voltages or currents, high-energy dc levels, and high frequencies with added care.

13.11 COMPUTER ANALYSIS

PSpice

OrCAD Capture offers a variety of ac voltage and current sources. However, for the purposes of this text, the voltage source **VSIN** and the current source **ISIN** are the most appropriate because they have a list of attributes that cover current areas of interest. Under the library **SOURCE,** a number of others are listed, but they don't have the full range of the above, or they are dedicated to only one type of analysis. On occasion, **ISRC** is used because it has an arrow symbol like that appearing in the text, and it can be used for dc, ac, and some transient analyses. The symbol for **ISIN** is a sine wave that utilizes the plus-and-minus sign (±) to indicate direction. The sources **VAC, IAC, VSRC,** and **ISRC** are fine if the magnitude and the phase of a specific quantity are desired or if a transient plot against frequency is desired. However, they will not provide a transient response against time even if the frequency and the transient information are provided for the simulation.

For all of the sinusoidal sources, the magnitude (**VAMPL**) is the peak value of the waveform, not the rms value. This becomes clear when a plot of a quantity is desired and the magnitude calculated by PSpice is the peak value of the transient response. However, for a purely steady-state ac response, the magnitude provided can be the rms value, and the output read as the rms value. Only when a plot is desired will it be clear that PSpice is accepting every ac magnitude as the peak value of the waveform. Of course, the phase angle is the same whether the magnitude is the peak or the rms value.

Before examining the mechanics of getting the various sources, remember that

Transient Analysis provides an ac or a dc output versus time, while AC Sweep is used to obtain a plot versus frequency.

To obtain any of the sources listed above, apply the following sequence: **Place part** key-**Place Part** dialog box-**Source-**(enter type of source). Once you select the source, the ac source **VSIN** appears on the schematic with **OFF, VAMPL,** and **FREQ.** Always specify **VOFF** as 0 V (unless a specific value is part of the analysis), and provide a value for the amplitude and frequency. Enter the remaining quantities of **PHASE, AC, DC, DF,** and **TD** by double-clicking on the source symbol to obtain the **Property Editor,** although **PHASE, DF** (damping factor), and **TD** (time delay) do have a default of 0 s. To add a phase angle, click on **PHASE,** enter the phase angle in the box below, and then select **Apply.** If you want to display a factor such as a phase angle of 60°, click on **PHASE** followed by **Display** to obtain the **Display Properties** dialog box. Then choose **Name and Value** followed by **OK** and **Apply,** and leave the **Properties Editor** dialog box (**X**) to see **PHASE=60** next to the **VSIN** source. The next chapter includes the use of the ac source in a simple circuit.

Multisim

For Multisim, the ac voltage source is available from two sources—the **Sources** parts bin and the **Function Generator.** The major difference between the two is that the phase angle can be set when using the **Sources** parts bin, whereas it cannot be set using the **Function Generator.**

Under **Sources,** select **SIGNAL_VOLTAGE_SOURCES** group under the **Family** heading. When selected and placed, it displays the default values for the amplitude, frequency, and phase. All the parameters of the source can be changed by double-clicking on the source symbol to obtain the dialog box. The listing clearly indicates that the set voltage is the peak value. Note that the unit of measurement is controlled by the scrolls to the right of the default label and cannot be set by typing in the desired unit of measurement. The label can be changed by switching the **Label** heading and inserting the desired label. After all the changes have been made in the dialog box, click **OK,** and all the changes appear next to the

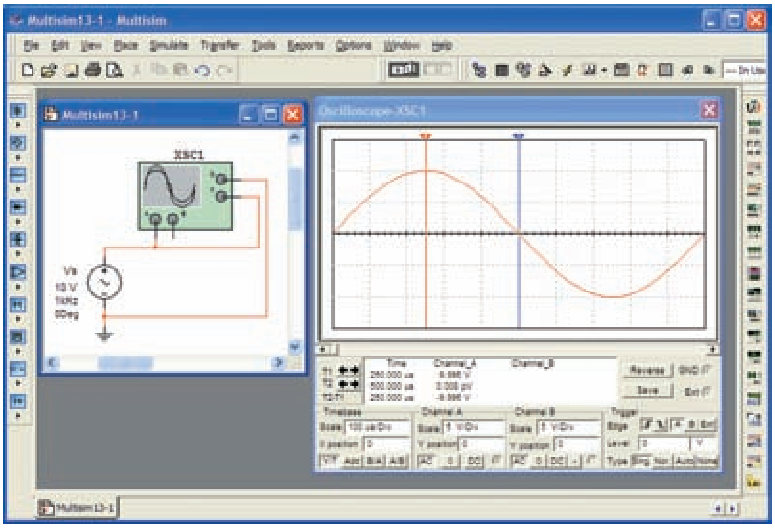


FIG. 13.78

*Using the oscilloscope to display the sinusoidal ac voltage source available in the Multisim **Sources** tool bin.*

ac voltage source symbol. In Fig. 13.78, the label was changed to **Vs** and the amplitude to 10 V while the freqency and phase angle were left with their default values. It is particularly important to realize that

for any frequency analysis (that is, where the frequency will change), the AC Magnitude of the ac source must be set under Analysis Setup in the SIGNAL_VOLTAGE_SOURCES dialog box. Failure to do so will create results linked to the default values rather than the value set under the Value heading.

To view the sinusoidal voltage set in Fig. 13.78, select an oscilloscope from the **Instrument** toolbar at the right of the screen. It is the fourth option down and has the appearance shown in Fig. 13.78 when selected. Note that it is a dual-channel oscilloscope with an **A** channel and a **B** channel. It has a ground (**G**) connection and a trigger (**T**) connection. The connections for viewing the ac voltage source on the **A** channel are provided in Fig. 13.78. Note that the trigger control is also connected to the **A** channel for sync control. The screen appearing in Fig. 13.78 can be displayed by double-clicking on the oscilloscope symbol on the screen. It has all the major controls of a typical laboratory oscilloscope. When you select **Simulate-Run** or select **1** on the **Simulate Switch,** the ac voltage appears on the screen. Changing the **Time base** to 100 μs/div. results in the display of Fig. 13.78 since there are 10 divisions across the screen and 10(100 μs) = 1 ms (the period of the applied signal). Changes in the **Time base** are made by clicking on the default value to obtain the scrolls in the same box. It is important to remember, however, that

changes in the oscilloscope setting or any network should not be made until the simulation is ended by disabling the Simulate-Run option or placing the Simulate switch in the 0 mode.

The options within the time base are set by the scroll bars and cannot be changed—again they match those typically available on a laboratory oscilloscope. The vertical sensitivity of the **A** channel was automatically set by the program at 5 V/div. to result in two vertical boxes for the peak value as shown in Fig. 13.78. Note the **AC** and **DC** keypads below Channel **A.** Since there is no dc component in the applied signal, either one results in the same display. The **Trigger** control is set on the positive transition at a level of 0 V. The **T1** and **T2** refer to the cursor positions on the horizontal time axis. By clicking on the small red triangle at the top of the red line at the far left edge of the screen and dragging the triangle, you can move the vertical red line to any position along the axis. In Fig. 13.78, it was moved to the peak value of the waveform at one-quarter of the total period or 0.25 ms $=$ 250 μs. Note the value of **T1** (250 μs) and the corresponding value of **VA1** (9.995 V $\cong$ 10.0 V). By moving the other cursor with a blue triangle at the top to one-half the total period or 0.5 ms $=$ 500 μs, we find that the value at **T2** (500 μs) is 0.008 pV (**VA2**), which is essentially 0 V for a waveform with a peak value of 10 V. The accuracy is controlled by the number of data points called for in the simulation setup. The more data points, the higher the likelihood of a higher degree of accuracy for the desired quantity. However, an increased number of data points also extends the running time of the simulation. The third line provides the difference between **T2** and **T1** as 250 μs and difference between their magnitudes (**VA2–VA1**) as $-$9.995 V, with the negative sign appearing because **VA1** is greater than **VA2.**

As mentioned above, you can also obtain an ac voltage from the **Function Generator** appearing as the second option down on the **Instrument** toolbar. Its symbol appears in Fig. 13.79 with positive, negative, and ground connections. Double-click on the generator graphic symbol, and the **Function Generator** dialog box appears in which selections can be made. For this example, the sinusoidal waveform is chosen. The **Frequency** is set at 1 kHz, the **Amplitude** is set at 10 V, and the **Offset** is left at 0 V. Note that there is no option to set the phase angle as was possible for the source above. Double-clicking on the oscilloscope generates the **Oscilloscope-XSCI** dialog box in which a **Timebase** of 100 μs/div. can be set again with a vertical sensitivity of 5 V/div. Select **1** on the **Simulate** switch, and the waveform of Fig. 13.79 appears. Choosing **Singular** under **Trigger** results in a fixed display. Set the **Simulate** switch on **0** to end the simulation. Placing the cursors in the same position shows that the waveforms for Figs. 13.78 and 13.79 are the same.

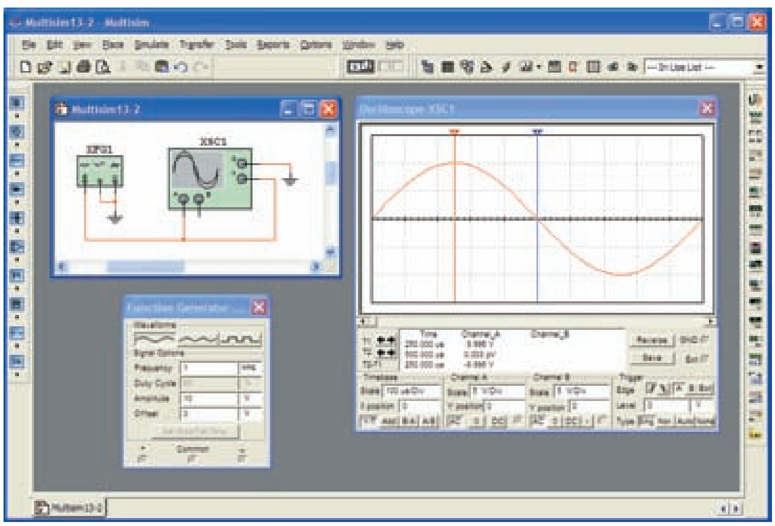


FIG. 13.79

Using the function generator to place a sinusoidal ac voltage waveform on the screen of the oscilloscope.

For most of the Multisim analyses to appear in this text, the **AC_VOLTAGE_SOURCE** under **Sources** will be employed. However, with such a limited introduction to Multisim, it seemed appropriate to introduce the use of the **Function Generator** because of its close linkage to the laboratory experience.

PROBLEMS

SECTION 13.2 Sinusoidal ac Voltage Characteristics and Definitions

1. For the sinusoidal waveform in Fig. 13.80:
  - **a.** What is the peak value?
  - **b.** What is the instantaneous value at 15 ms and at 20 ms?
  - **c.** What is the peak-to-peak value of the waveform?
  - **d.** What is the period of the waveform?
  - **e.** How many cycles are shown?

2. For the square-wave signal in Fig. 13.81:
  - **a.** What is the peak value?
  - **b.** What is the instantaneous value at 5 μs and at 11 μs?
  - **c.** What is the peak-to-peak value of the waveform?
  - **d.** What is the period of the waveform?
  - **e.** How many cycles are shown?

3. For the periodic waveform in Fig. 13.82:
  - **a.** What is the peak value?
  - **b.** What is the instantaneous value at 3 μs and at 9 μs?
  - **c.** What is the peak-to-peak value of the waveform?
  - **d.** What is the period of the waveform?
  - **e.** How many cycles are shown?

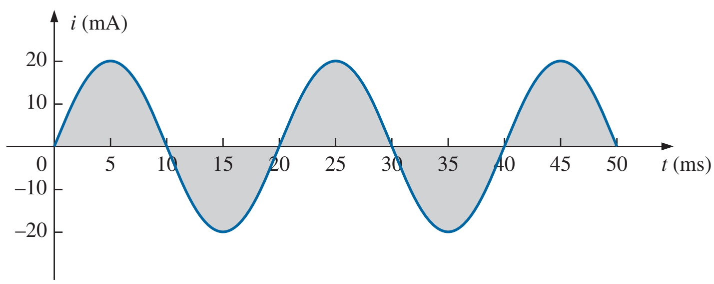


FIG. 13.80
Problem 1.

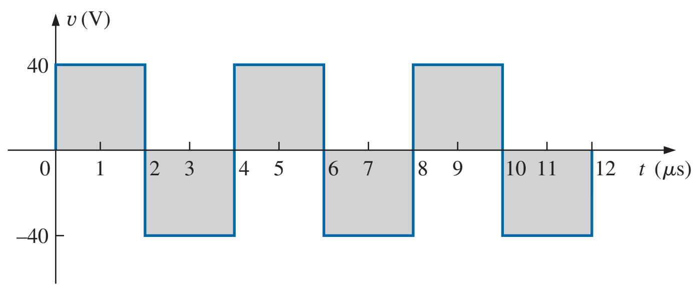


FIG. 13.81
Problem 2.

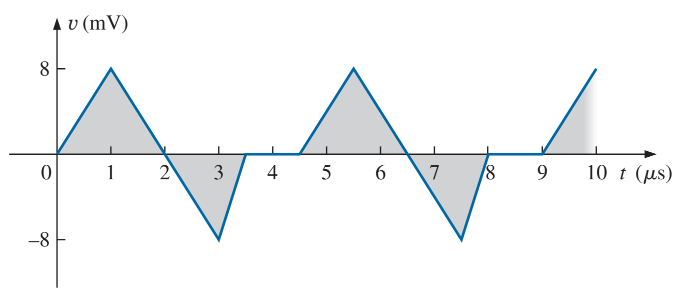


FIG. 13.82
Problem 3.

SECTION 13.3 Frequency Spectrum

4. Find the period of a periodic waveform whose frequency is
a. 25 Hz. **b.** 40 MHz.
c. 25 kHz. **d.** 1 Hz.

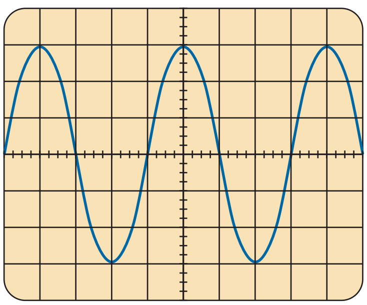
5. Find the frequency of a repeating waveform whose period is
a. 1/60 s. **b.** 0.01 s.
c. 40 ms. **d.** 25 μs.

6. If a periodic waveform has a frequency of 20 Hz, how long (in seconds) will it take to complete five cycles?

7. Find the period of a sinusoidal waveform that completes 80 cycles in 24 ms.

8. What is the frequency of a periodic waveform that completes 42 cycles in 6 s?

9. For the oscilloscope pattern of Fig. 13.83:
a. Determine the peak amplitude.
b. Find the period.
c. Calculate the frequency.
Redraw the oscilloscope pattern if a +25 mV dc level were added to the input waveform.



Vertical sensitivity = 50 mV/div.
Horizontal sensitivity = 10 μs/div.

FIG. 13.83
Problem 9.

SECTION 13.4 The Sinusoidal Waveform

10. Convert the following degrees to radians:
a. 45° **b.** 60° **c.** 270° **d.** 170°

11. Convert the following radians to degrees:
a. $\pi/4$ **b.** $\pi/6$ **c.** $\frac{1}{10}\pi$ **d.** 0.6π

12. Find the angular velocity of a waveform with a period of
a. 2 s. **b.** 0.3 ms. **c.** 4 μs. **d.** $\frac{1}{26}$ s.

13. Find the angular velocity of a waveform with a frequency of
a. 50 Hz. **b.** 600 Hz.
c. 2 kHz. **d.** 0.004 MHz.

14. Find the frequency and period of sine waves having an angular velocity of
a. 754 rad/s. **b.** 8.4 rad/s.
c. 6000 rad/s. **d.** $\frac{1}{16}$ rad/s.

***15.** Given f = 60 Hz, determine how long it will take the sinusoidal waveform to pass through an angle of 45°.

***16.** If a sinusoidal waveform passes through an angle of 30° in 5 ms, determine the angular velocity of the waveform.

SECTION 13.5 General Format for the Sinusoidal Voltage or Current

17. Find the amplitude and frequency of the following waves:
a. $20 \sin 377t$ **b.** $5 \sin 754t$
c. $10^6 \sin 10{,}000t$ **d.** $-6.4 \sin 942t$

18. Sketch $5 \sin 754t$ with the abscissa
a. angle in degrees. **b.** angle in radians.
c. time in seconds.

***19.** Sketch $-7.6 \sin 43.6t$ with the abscissa
a. angle in degrees. **b.** angle in radians.
c. time in seconds.

20. If $e = 300 \sin 157t$, how long (in seconds) does it take this waveform to complete 1/2 cycle?

21. Given $i = 0.5 \sin \alpha$, determine i at $\alpha = 72°$.

22. Given $\upsilon = 20 \sin \alpha$, determine υ at $\alpha = 1.2\pi$.

***23.** Given $\upsilon = 30 \times 10^{-3} \sin \alpha$, determine the angles at which υ will be 6 mV.

***24.** If υ = 40 V at α = 30 and t = 1 ms, determine the mathematical expression for the sinusoidal voltage.

SECTION 13.6 Phase Relations

25. Sketch $\sin(377t + 60°)$ with the abscissa
a. angle in degrees. **b.** angle in radians.
c. time in seconds.

26. Sketch the following waveforms:
a. $50 \sin(\omega t + 0°)$ **b.** $5 \sin(\omega t + 60°)$
c. $2 \cos(\omega t + 10°)$ **d.** $-20 \sin(\omega t + 2°)$

27. Write the analytical expression for the waveforms in Fig. 13.84 with the phase angle in degrees.

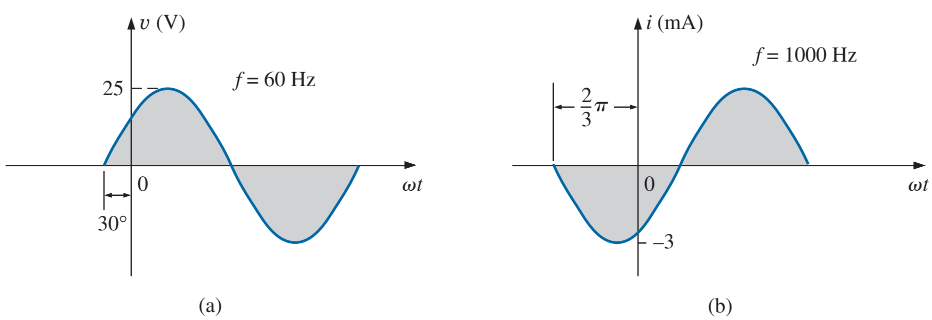


FIG. 13.84
Problem 27.

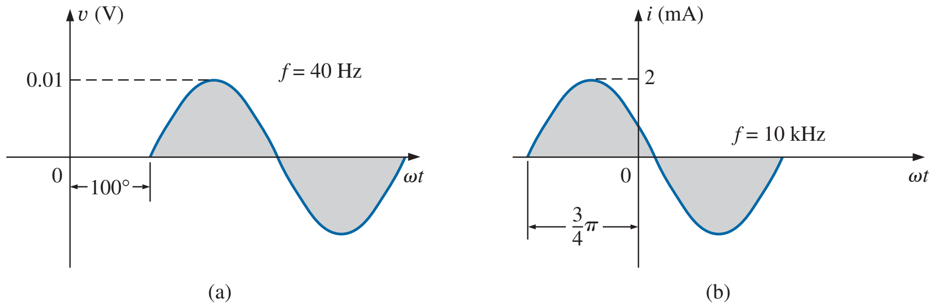


FIG. 13.85
Problem 28.

28. Write the analytical expression for the waveforms in Fig. 13.85 with the phase angle in degrees.

29. Find the phase relationship between the following waveforms:

$$v = 4\sin(\omega t + 50°)$$
$$i = 6\sin(\omega t + 40°)$$

30. Find the phase relationship between the following waveforms:

$$v = 25\sin(\omega t - 80°)$$
$$i = 5 \times 10^{-3}\sin(\omega t - 10°)$$

31. Find the phase relationship between the following waveforms:

$$v = 0.2\sin(\omega t - 60°)$$
$$i = 0.1\sin(\omega t + 20°)$$

***32.** Find the phase relationship between the following waveforms:

$$v = 2\cos(\omega t - 30°)$$
$$i = 5\sin(\omega t + 60°)$$

***33.** Find the phase relationship between the following waveforms:

$$v = -4\cos(\omega t + 90°)$$
$$i = -2\sin(\omega t + 10°)$$

***34.** The sinusoidal voltage $v = 200\sin(2\pi\,1000t + 60°)$ is plotted in Fig. 13.86. Determine the time t_1 when the waveform crosses the axis.

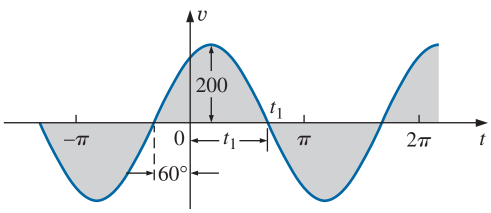


FIG. 13.86
Problem 34.

***35.** The sinusoidal current $i = 4\sin(50{,}000t - 40°)$ is plotted in Fig. 13.87. Determine the time t_1 when the waveform crosses the axis.

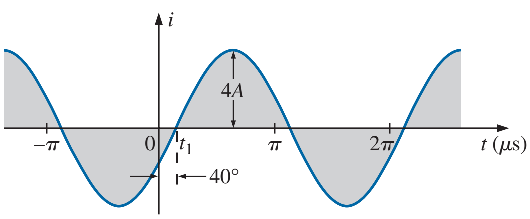
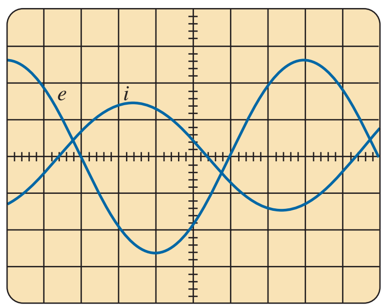


FIG. 13.87
Problem 35.

36. For the oscilloscope display in Fig. 13.88:

a. Determine the period of the waveform.
b. Determine the frequency of each waveform.
c. Find the rms value of each waveform.
d. Determine the phase shift between the two waveforms and determine which leads and which lags.



Vertical sensitivity = 0.5 V/div.
Horizontal sensitivity = 1 ms/div.

FIG. 13.88
Problem 36.

SECTION 13.7 Average Value

37. Find the average value of the periodic waveform in Fig. 13.89.

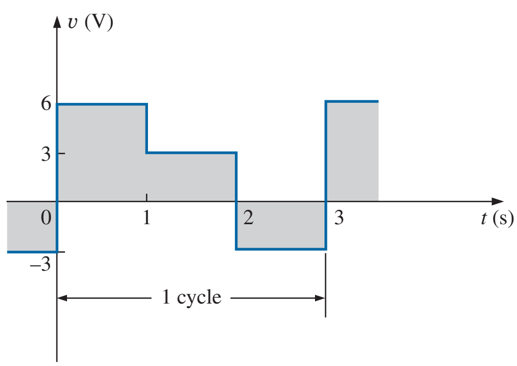


FIG. 13.89
Problem 37.

38. Find the average value of the periodic waveform in Fig. 13.90.

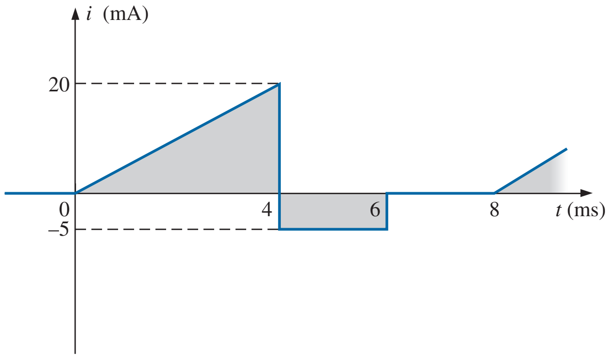


FIG. 13.90
Problem 38.

39. Find the average value of the periodic waveform in Fig. 13.91.

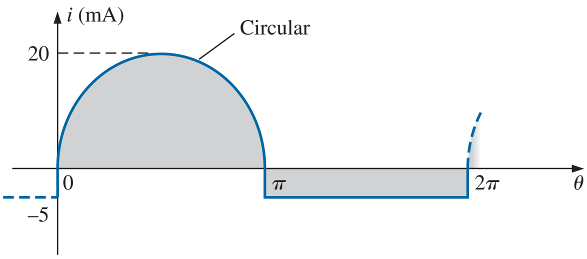
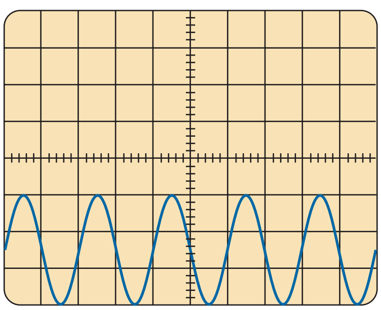


FIG. 13.91
Problem 39.

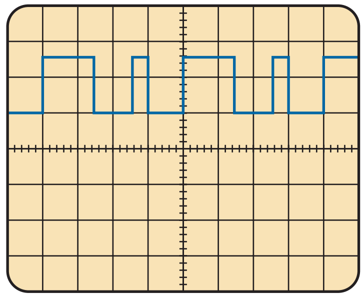
40. For the waveform in Fig. 13.92:
- **a.** Determine the period.
- **b.** Find the frequency.
- **c.** Determine the average value.
- **d.** Sketch the resulting oscilloscope display if the vertical channel is switched from DC to AC.



Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 0.2 ms/div.

FIG. 13.92
Problem 40.

***41.** For the waveform in Fig. 13.93:
- **a.** Determine the period.
- **b.** Find the frequency.
- **c.** Determine the average value.
- **d.** Sketch the resulting oscilloscope display if the vertical channel is switched from DC to AC.



Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 10 μs/div.

FIG. 13.93
Problem 41.

SECTION 13.8 Effective (rms) Values

42. Find the rms values of the following sinusoidal waveforms:
- **a.** $\upsilon = 140 \sin(377t + 60°)$
- **b.** $i = 6 \times 10^{-3} \sin(2\pi\, 1000t)$
- **c.** $\upsilon = 40 \times 10^{-6} (\sin(2\pi\, 5000t + 30°)$

43. Write the sinusoidal expressions for voltages and currents having the following rms values at a frequency of 60 Hz with zero phase shift:
- **a.** 10 V
- **b.** 50 mA
- **c.** 2 kV

44. Find the rms value of the periodic waveform in Fig. 13.94 over one full cycle.

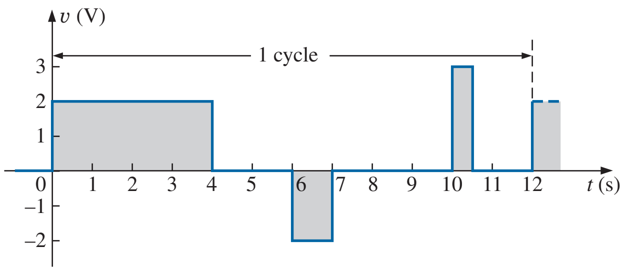


FIG. 13.94
Problem 44.

45. Find the rms value of the periodic waveform in Fig. 13.95 over one full cycle.

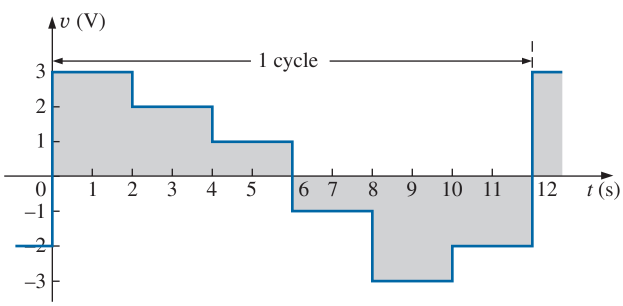


FIG. 13.95
Problem 45.

46. What are the average and rms values of the square wave in Fig. 13.96?

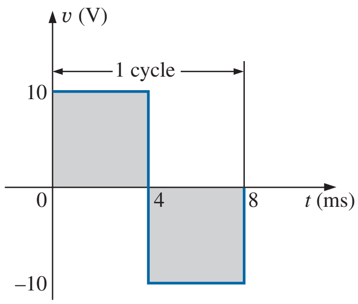


FIG. 13.96
Problem 46.

*__47.__ For each waveform in Fig. 13.97, determine the period, frequency, average value, and rms value.

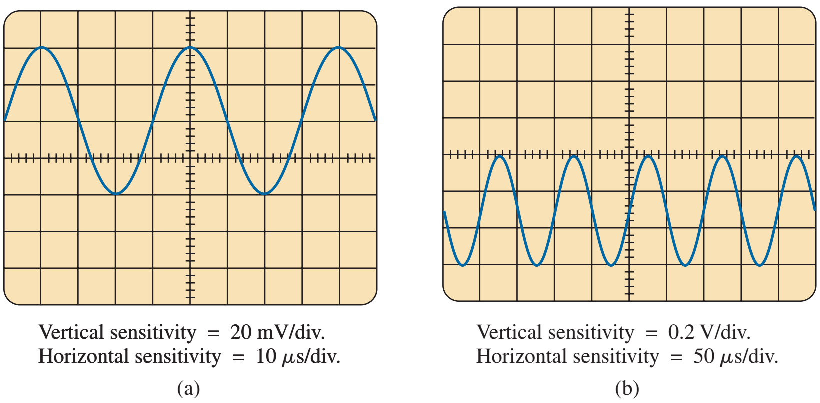


FIG. 13.97
Problem 47.

SECTION 13.9 ac Meters and Instruments

48. Determine the reading of the meter for each situation in Fig. 13.98.

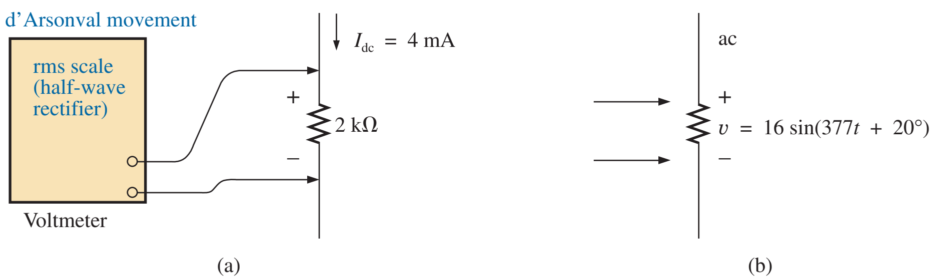


FIG. 13.98
Problem 48.

GLOSSARY

Alternating waveform A waveform that oscillates above and below a defined reference level.

Angular velocity The velocity with which a radius vector projecting a sinusoidal function rotates about its center.

Average value The level of a waveform defined by the condition that the area enclosed by the curve above this level is exactly equal to the area enclosed by the curve below this level.

Calibration factor A multiplying factor used to convert from one meter indication to another.

Clamp Meter® A clamp-type instrument that will permit noninvasive current measurements and that can be used as a conventional voltmeter or ohmmeter.

Cycle A portion of a waveform contained in one period of time.

Effective value The equivalent dc value of any alternating voltage or current.

Electrodynamometer meters Instruments that can measure both ac and dc quantities without a change in internal circuitry.

Frequency (*f*) The number of cycles of a periodic waveform that occur in 1 second.

Frequency counter An instrument that will provide a digital display of the frequency or period of a periodic time-varying signal.

Instantaneous value The magnitude of a waveform at any instant of time, denoted by lowercase letters.

Lagging waveform A waveform that crosses the time axis at a point in time later than another waveform of the same frequency.

Leading waveform A waveform that crosses the time axis at a point in time ahead of another waveform of the same frequency.

Oscilloscope An instrument that will display, through the use of a cathode-ray tube, the characteristics of a time-varying signal.

Peak amplitude The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters.

Peak-to-peak value The magnitude of the total swing of a signal from positive to negative peaks. The sum of the absolute values of the positive and negative peak values.

Peak value The maximum value of a waveform, denoted by uppercase letters.

Period (*T*) The time interval necessary for one cycle of a periodic waveform.

Periodic waveform A waveform that continually repeats itself after a defined time interval.

Phase relationship An indication of which of two waveforms leads or lags the other, and by how many degrees or radians.

Radian (rad) A unit of measure used to define a particular segment of a circle. One radian is approximately equal to 57.3°; 2π rad are equal to 360°.

Root-mean-square (rms) value The root-mean-square or effective value of a waveform.

Sinusoidal ac waveform An alternating waveform of unique characteristics that oscillates with equal amplitude above and below a given axis.

VOM A multimeter with the capability to measure resistance and both ac and dc levels of current and voltage.

Waveform The path traced by a quantity, plotted as a function of some variable such as position, time, degrees, temperature, and so on.