

# INTRODUCTION

# 1

## OBJECTIVES

- *Become aware of the rapid growth of the electrical/electronics industry over the past century.*
- *Understand the importance of applying a unit of measurement to a result or measurement and to ensuring that the numerical values substituted into an equation are consistent with the unit of measurement of the various quantities.*
- *Become familiar with the SI system of units used throughout the electrical/electronics industry.*
- *Understand the importance of powers of ten and how to work with them in any numerical calculation.*
- *Be able to convert any quantity, in any system of units, to another system with confidence.*

## 1.1 THE ELECTRICAL/ELECTRONICS INDUSTRY

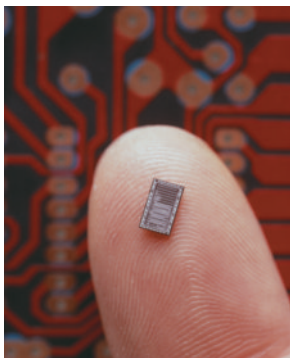
Over the past few decades, technology has been changing at an ever-increasing rate. The pressure to develop new products, improve the performance of existing systems, and create new markets will only accelerate that rate. This pressure, however, is also what makes the field so exciting. New ways of storing information, constructing integrated circuits, or hardware that contains software components that can “think” on their own based on data input are only a few possibilities.

Change has always been part of the human experience, but it used to be gradual. This is no longer true. Just think, for example, that it was only a few years ago that TVs with wide, flat screens were introduced. Already, these have been eclipsed by high-definition TVs with images so crystal clear that they seem almost three-dimensional.

Miniaturization has also made possible huge advances in electronic systems. Cell phones that originally were the size of notebooks are now about the size of a deck of playing cards. Plus, these new versions record short videos, transmit photos, send text messages, and have calendars, reminders, calculators, games, and lists of frequently called numbers. Boom boxes playing audio cassettes have been replaced by pocket-sized iPods® that can store 15,000 songs or 25,000 photos. Hearing aids with higher power levels that are almost invisible in the ear, TVs with 1-inch screens—the list of new or improved products continues to expand because significantly smaller electronic systems have been developed.

This reduction in size of electronic systems is due primarily to an important innovation introduced less than 50 years ago (1958)—the **integrated circuit (IC)**. An integrated circuit can now contain features less than 100 nanometers across. The fact that measurements are now being made in nanometers has resulted in the terminology **nanotechnology** producing integrated circuits called *nanochips*. To understand nanometers, consider drawing one hundred lines


$$\sum_{S} I$$



**FIG. 1.1**  
*Computer chip.*  
(© Roger Du Buisson/Corbis)

within the boundaries of one inch. Then attempt drawing 1000 lines within the same length. Cutting 100 nanometer features would require drawing over 250,000 lines in one inch. The integrated circuit shown in Fig. 1.1 is the Intel® Pentium® 4 processor, which has 42 million transistors in an area measuring 0.34 square inches. Intel Corporation recently presented a technical paper describing 20 nanometer (0.02 micrometers) transistors, developed in its silicon research laboratory. These very small, ultrafast transistors will permit placing nearly 1 billion transistors on a sliver of silicon smaller than a stamp—an incredible accomplishment.

However, before a decision is made on such dramatic reductions in size, the system must be designed and tested to determine if it is worth constructing as an integrated circuit. That design process requires engineers who know the characteristics of each device used in the system, including undesirable characteristics that are part of any electronic element. In other words, there are *no ideal (perfect) elements* in an electronic design. Considering the limitations of each component is necessary to ensure a reliable response under all conditions of temperature, vibration, and effects of the surrounding environment. To develop this awareness requires time and must begin with understanding the basic characteristics of the device, as covered in this text. One of the objectives of this text is to explain how ideal components work and their function in a network. Another is to explain conditions in which components may not be ideal.

One of the very positive aspects of the learning process associated with electric and electronic circuits is that once a concept or procedure is clearly and correctly understood, it will be useful throughout the career of the individual at any level in the industry. Once a law or equation is understood, it will not be replaced by another equation as the material becomes more advanced and complicated. For instance, one of the first laws to be introduced is Ohm's law. This law provides a relationship between forces and components that will always be true, no matter how complicated the system becomes. In fact, it is an equation that will be applied in various forms throughout the design of the entire system. The use of the basic laws may change, but the laws will not change and will always be applicable.

It is vitally important to understand that the learning process for circuit analysis is sequential. That is, the first few chapters establish the foundation for the remaining chapters. Failure to properly understand the opening chapters will only lead to difficulties understanding the material in the chapters to follow. This first chapter provides a brief history of the field followed by a review of mathematical concepts necessary to understand the rest of the material.

## 1.2 A BRIEF HISTORY

In the sciences, once a hypothesis is proven and accepted, it becomes one of the building blocks of that area of study, permitting additional investigation and development. Naturally, the more pieces of a puzzle available, the more obvious the avenue toward a possible solution. In fact, history demonstrates that a single development may provide the key that will result in a mushroom effect that brings the science to a new plateau of understanding and impact.

If the opportunity presents itself, read one of the many publications reviewing the history of this field. Space requirements are such that only a brief review can be provided here. There are many more contributors than

could be listed, and their efforts have often provided important keys to the solution of some very important concepts.

Throughout history, some periods were characterized by what appeared to be an explosion of interest and development in particular areas. As you will see from the discussion of the late 1700s and the early 1800s, inventions, discoveries, and theories came fast and furiously. Each new concept has broadened the possible areas of application until it becomes almost impossible to trace developments without picking a particular area of interest and following it through. In the review, as you read about the development of the radio, television, and computer, keep in mind that similar progressive steps were occurring in the areas of the telegraph, the telephone, power generation, the phonograph, appliances, and so on.

There is a tendency when reading about the great scientists, inventors, and innovators to believe that their contribution was a totally individual effort. In many instances, this was not the case. In fact, many of the great contributors were friends or associates who provided support and encouragement in their efforts to investigate various theories. At the very least, they were aware of one another's efforts to the degree possible in the days when a letter was often the best form of communication. In particular, note the closeness of the dates during periods of rapid development. One contributor seemed to spur on the efforts of the others or possibly provided the key needed to continue with the area of interest.

In the early stages, the contributors were not electrical, electronic, or computer engineers as we know them today. In most cases, they were physicists, chemists, mathematicians, or even philosophers. In addition, they were not from one or two communities of the Old World. The home country of many of the major contributors introduced in the paragraphs to follow is provided to show that almost every established community had some impact on the development of the fundamental laws of electrical circuits.

As you proceed through the remaining chapters of the text, you will find that a number of the units of measurement bear the name of major contributors in those areas—*volt* after Count Alessandro Volta, *ampere* after André Ampère, *ohm* after Georg Ohm, and so forth—fitting recognition for their important contributions to the birth of a major field of study.

Time charts indicating a limited number of major developments are provided in Fig. 1.2, primarily to identify specific periods of rapid development and to reveal how far we have come in the last few decades. In essence, the current state of the art is a result of efforts that began in earnest some 250 years ago, with progress in the last 100 years almost exponential.

As you read through the following brief review, try to sense the growing interest in the field and the enthusiasm and excitement that must have accompanied each new revelation. Although you may find some of the terms used in the review new and essentially meaningless, the remaining chapters will explain them thoroughly.

## The Beginning

The phenomenon of **static electricity** has intrigued scholars throughout history. The Greeks called the fossil resin substance so often used to demonstrate the effects of static electricity *elektron*, but no extensive study was made of the subject until William Gilbert researched the event in 1600. In the years to follow, there was a continuing investigation of electrostatic charge by many individuals such as Otto von Guericke, who

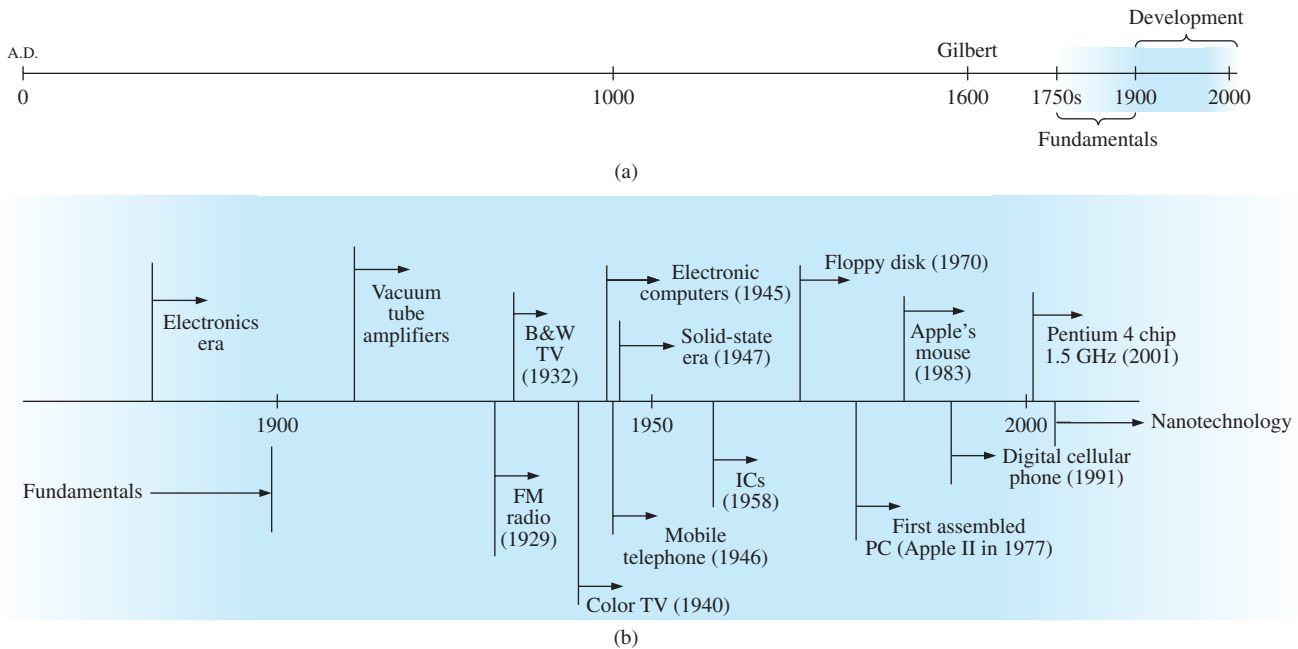


FIG. 1.2

Time charts: (a) long-range; (b) expanded.

developed the first machine to generate large amounts of charge, and Stephen Gray, who was able to transmit electrical charge over long distances on silk threads. Charles DuFay demonstrated that charges either attract or repel each other, leading him to believe that there were two types of charge—a theory we subscribe to today with our defined positive and negative charges.

There are many who believe that the true beginnings of the electrical era lie with the efforts of Pieter van Musschenbroek and Benjamin Franklin. In 1745, van Musschenbroek introduced the **Leyden jar** for the storage of electrical charge (the first capacitor) and demonstrated electrical shock (and therefore the power of this new form of energy). Franklin used the Leyden jar some seven years later to establish that lightning is simply an electrical discharge, and he expanded on a number of other important theories including the definition of the two types of charge as *positive* and *negative*. From this point on, new discoveries and theories seemed to occur at an increasing rate as the number of individuals performing research in the area grew.

In 1784, Charles Coulomb demonstrated in Paris that the force between charges is inversely related to the square of the distance between the charges. In 1791, Luigi Galvani, professor of anatomy at the University of Bologna, Italy, performed experiments on the effects of electricity on animal nerves and muscles. The first **voltaic cell**, with its ability to produce electricity through the chemical action of a metal dissolving in an acid, was developed by another Italian, Alessandro Volta, in 1799.

The fever pitch continued into the early 1800s with Hans Christian Oersted, a Danish professor of physics, announcing in 1820 a relationship between magnetism and electricity that serves as the foundation for the theory of **electromagnetism** as we know it today. In the same year, a French physicist, André Ampère, demonstrated that there are magnetic

effects around every current-carrying conductor and that current-carrying conductors can attract and repel each other just like magnets. In the period 1826 to 1827, a German physicist, Georg Ohm, introduced an important relationship between potential, current, and resistance which we now refer to as *Ohm's law*. In 1831, an English physicist, Michael Faraday, demonstrated his theory of *electromagnetic induction*, whereby a changing current in one coil can induce a changing current in another coil, even though the two coils are not directly connected. Professor Faraday also did extensive work on a storage device he called the condenser, which we refer to today as a *capacitor*. He introduced the idea of adding a dielectric between the plates of a capacitor to increase the storage capacity (Chapter 10). James Clerk Maxwell, a Scottish professor of natural philosophy, performed extensive mathematical analyses to develop what are currently called *Maxwell's equations*, which support the efforts of Faraday linking electric and magnetic effects. Maxwell also developed the *electromagnetic theory of light* in 1862, which, among other things, revealed that electromagnetic waves travel through air at the velocity of light (186,000 miles per second or  $3 \times 10^8$  meters per second). In 1888, a German physicist, Heinrich Rudolph Hertz, through experimentation with lower-frequency electromagnetic waves (microwaves), substantiated Maxwell's predictions and equations. In the mid-1800s, Professor Gustav Robert Kirchhoff introduced a series of laws of voltages and currents that find application at every level and area of this field (Chapters 5 and 6). In 1895, another German physicist, Wilhelm Röntgen, discovered electromagnetic waves of high frequency, commonly called *X-rays* today.

By the end of the 1800s, a significant number of the fundamental equations, laws, and relationships had been established, and various fields of study, including electricity, electronics, power generation and distribution, and communication systems, started to develop in earnest.

## The Age of Electronics

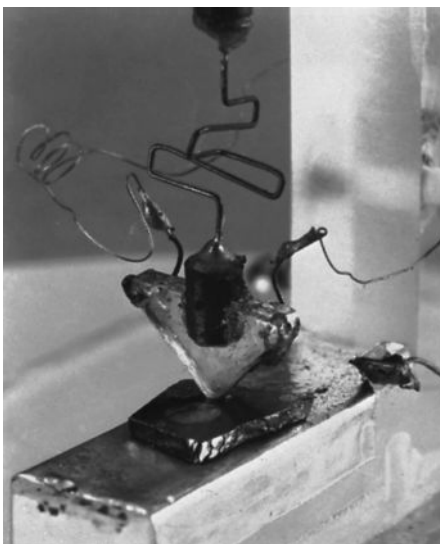
**Radio** The true beginning of the electronics era is open to debate and is sometimes attributed to efforts by early scientists in applying potentials across evacuated glass envelopes. However, many trace the beginning to Thomas Edison, who added a metallic electrode to the vacuum of the tube and discovered that a current was established between the metal electrode and the filament when a positive voltage was applied to the metal electrode. The phenomenon, demonstrated in 1883, was referred to as the **Edison effect**. In the period to follow, the transmission of radio waves and the development of the radio received widespread attention. In 1887, Heinrich Hertz, in his efforts to verify Maxwell's equations, transmitted radio waves for the first time in his laboratory. In 1896, an Italian scientist, Guglielmo Marconi (often called the father of the radio), demonstrated that telegraph signals could be sent through the air over long distances (2.5 kilometers) using a grounded antenna. In the same year, Aleksandr Popov sent what might have been the first radio message some 300 yards. The message was the name "*Heinrich Hertz*" in respect for Hertz's earlier contributions. In 1901, Marconi established radio communication across the Atlantic.

In 1904, John Ambrose Fleming expanded on the efforts of Edison to develop the first diode, commonly called **Fleming's valve**—actually the first of the *electronic devices*. The device had a profound impact on the

design of detectors in the receiving section of radios. In 1906, Lee De Forest added a third element to the vacuum structure and created the first amplifier, the triode. Shortly thereafter, in 1912, Edwin Armstrong built the first regenerative circuit to improve receiver capabilities and then used the same contribution to develop the first nonmechanical oscillator. By 1915, radio signals were being transmitted across the United States, and in 1918 Armstrong applied for a patent for the superheterodyne circuit employed in virtually every television and radio to permit amplification at one frequency rather than at the full range of incoming signals. The major components of the modern-day radio were now in place, and sales in radios grew from a few million dollars in the early 1920s to over \$1 billion by the 1930s. The 1930s were truly the golden years of radio, with a wide range of productions for the listening audience.

**Television** The 1930s were also the true beginnings of the television era, although development on the picture tube began in earlier years with Paul Nipkow and his *electrical telescope* in 1884 and John Baird and his long list of successes, including the transmission of television pictures over telephone lines in 1927 and over radio waves in 1928, and simultaneous transmission of pictures and sound in 1930. In 1932, NBC installed the first commercial television antenna on top of the Empire State Building in New York City, and RCA began regular broadcasting in 1939. The war slowed development and sales, but in the mid-1940s the number of sets grew from a few thousand to a few million. Color television became popular in the early 1960s.

**Computers** The earliest computer system can be traced back to Blaise Pascal in 1642 with his mechanical machine for adding and subtracting numbers. In 1673, Gottfried Wilhelm von Leibniz used the *Leibniz wheel* to add multiplication and division to the range of operations, and in 1823 Charles Babbage developed the **difference engine** to add the mathematical operations of sine, cosine, logs, and several others. In the years to follow, improvements were made, but the system remained primarily mechanical until the 1930s when electromechanical systems using components such as relays were introduced. It was not until the 1940s that totally electronic systems became the new wave. It is interesting to note that, even though IBM was formed in 1924, it did not enter the computer industry until 1937. An entirely electronic system known as **ENIAC** was dedicated at the University of Pennsylvania in 1946. It contained 18,000 tubes and weighed 30 tons but was several times faster than most electromechanical systems. Although other vacuum tube systems were built, it was not until the birth of the solid-state era that computer systems experienced a major change in size, speed, and capability.



**FIG. 1.3**

*The first transistor.*

(Used with permission of Lucent Technologies Inc./Bell Labs.)

## The Solid-State Era

In 1947, physicists William Shockley, John Bardeen, and Walter H. Brattain of Bell Telephone Laboratories demonstrated the point-contact **transistor** (Fig. 1.3), an amplifier constructed entirely of solid-state materials with no requirement for a vacuum, glass envelope, or heater voltage for the filament. Although reluctant at first due to the vast amount of material available on the design, analysis, and synthesis of tube networks, the industry eventually accepted this new technology as the wave of the future. In 1958, the first **integrated circuit (IC)** was developed at Texas

Instruments, and in 1961 the first commercial integrated circuit was manufactured by the Fairchild Corporation.

It is impossible to review properly the entire history of the electrical/electronics field in a few pages. The effort here, both through the discussion and the time graphs in Fig. 1.2, was to reveal the amazing progress of this field in the last 50 years. The growth appears to be truly exponential since the early 1900s, raising the interesting question, Where do we go from here? The time chart suggests that the next few decades will probably contain many important innovative contributions that may cause an even faster growth curve than we are now experiencing.

### 1.3 UNITS OF MEASUREMENT

One of the most important rules to remember and apply when working in any field of technology is to use the correct units when substituting numbers into an equation. Too often we are so intent on obtaining a numerical solution that we overlook checking the units associated with the numbers being substituted into an equation. Results obtained, therefore, are often meaningless. Consider, for example, the following very fundamental physics equation:

$$\boxed{v = \frac{d}{t}} \quad \begin{array}{l} v = \text{velocity} \\ d = \text{distance} \\ t = \text{time} \end{array} \quad (1.1)$$

Assume, for the moment, that the following data are obtained for a moving object:

$$\begin{array}{l} d = 4000 \text{ ft} \\ t = 1 \text{ min} \end{array}$$

and  $v$  is desired in miles per hour. Often, without a second thought or consideration, the numerical values are simply substituted into the equation, with the result here that

$$v = \frac{d}{t} = \frac{4000 \text{ ft}}{1 \text{ min}} = \cancel{4000 \text{ mph}}$$

As indicated above, the solution is totally incorrect. If the result is desired in *miles per hour*, the unit of measurement for distance must be *miles*, and that for time, *hours*. In a moment, when the problem is analyzed properly, the extent of the error will demonstrate the importance of ensuring that

***the numerical value substituted into an equation must have the unit of measurement specified by the equation.***

The next question is normally, How do I convert the distance and time to the proper unit of measurement? A method is presented in Section 1.9 of this chapter, but for now it is given that

$$\begin{array}{l} 1 \text{ mi} = 5280 \text{ ft} \\ 4000 \text{ ft} = 0.76 \text{ mi} \\ 1 \text{ min} = \frac{1}{60} \text{ h} = 0.017 \text{ h} \end{array}$$

Substituting into Eq. (1.1), we have

$$v = \frac{d}{t} = \frac{0.76 \text{ mi}}{0.017 \text{ h}} = \mathbf{44.71 \text{ mph}}$$

which is significantly different from the result obtained before.

To complicate the matter further, suppose the distance is given in kilometers, as is now the case on many road signs. First, we must realize that the prefix *kilo* stands for a multiplier of 1000 (to be introduced in Section 1.5), and then we must find the conversion factor between kilometers and miles. If this conversion factor is not readily available, we must be able to make the conversion between units using the conversion factors between meters and feet or inches, as described in Section 1.9.

Before substituting numerical values into an equation, try to mentally establish a reasonable range of solutions for comparison purposes. For instance, if a car travels 4000 ft in 1 min, does it seem reasonable that the speed would be 4000 mph? Obviously not! This self-checking procedure is particularly important in this day of the hand-held calculator, when ridiculous results may be accepted simply because they appear on the digital display of the instrument.

Finally,

*if a unit of measurement is applicable to a result or piece of data, then it must be applied to the numerical value.*

To state that  $v = 44.71$  without including the unit of measurement *mph* is meaningless.

Eq. (1.1) is not a difficult one. A simple algebraic manipulation will result in the solution for any one of the three variables. However, in light of the number of questions arising from this equation, the reader may wonder if the difficulty associated with an equation will increase at the same rate as the number of terms in the equation. In the broad sense, this will not be the case. There is, of course, more room for a mathematical error with a more complex equation, but once the proper system of units is chosen and each term properly found in that system, there should be very little added difficulty associated with an equation requiring an increased number of mathematical calculations.

In review, before substituting numerical values into an equation, be absolutely sure of the following:

1. *Each quantity has the proper unit of measurement as defined by the equation.*
2. *The proper magnitude of each quantity as determined by the defining equation is substituted.*
3. *Each quantity is in the same system of units (or as defined by the equation).*
4. *The magnitude of the result is of a reasonable nature when compared to the level of the substituted quantities.*
5. *The proper unit of measurement is applied to the result.*

## 1.4 SYSTEMS OF UNITS

In the past, the *systems of units* most commonly used were the English and metric, as outlined in Table 1.1. Note that while the English system is based on a single standard, the metric is subdivided into two interrelated standards: the **MKS** and the **CGS**. Fundamental quantities of these systems are compared in Table 1.1 along with their abbreviations. The MKS and CGS systems draw their names from the units of measurement used with each system; the MKS system uses *Meters*, *Kilograms*, and *Seconds*, while the CGS system uses *Centimeters*, *Grams*, and *Seconds*.

Understandably, the use of more than one system of units in a world that finds itself continually shrinking in size, due to advanced technical



**TABLE 1.1**  
*Comparison of the English and metric systems of units.*

English	Metric		SI
	MKS	CGS	
<i>Length:</i> Yard (yd) (0.914 m)	Meter (m) (39.37 in.) (100 cm)	Centimeter (cm) (2.54 cm = 1 in.)	<b>Meter (m)</b>
<i>Mass:</i> <b>Slug</b> (14.6 kg)	Kilogram (kg) (1000 g)	Gram (g)	<b>Kilogram (kg)</b>
<i>Force:</i> <b>Pound (lb)</b> (4.45 N)	Newton (N) (100,000 dynes)	Dyne	<b>Newton (N)</b>
<i>Temperature:</i> Fahrenheit (°F) $\left( = \frac{9}{5}^{\circ}\text{C} + 32 \right)$	Celsius or Centigrade (°C) $\left( = \frac{5}{9} (^{\circ}\text{F} - 32) \right)$	Centigrade (°C)	<b>Kelvin (K)</b> $\text{K} = 273.15 + ^{\circ}\text{C}$
<i>Energy:</i> Foot-pound (ft-lb) (1.356 joules)	Newton-meter (N·m) or joule (J) (0.7376 ft-lb)	Dyne-centimeter or erg (1 joule = $10^7$ ergs)	<b>Joule (J)</b>
<i>Time:</i> Second (s)	Second (s)	Second (s)	<b>Second (s)</b>

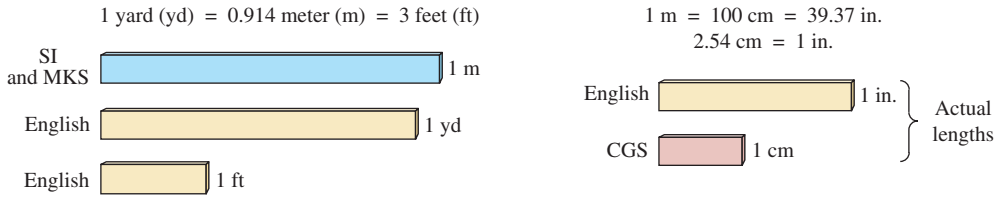
developments in communications and transportation, would introduce unnecessary complications to the basic understanding of any technical data. The need for a standard set of units to be adopted by all nations has become increasingly obvious. The International Bureau of Weights and Measures located at Sèvres, France, has been the host for the General Conference of Weights and Measures, attended by representatives from all nations of the world. In 1960, the General Conference adopted a system called Le Système International d'Unités (International System of Units), which has the international abbreviation **SI**. Since then, it has been adopted by the Institute of Electrical and Electronic Engineers, Inc. (IEEE) in 1965 and by the United States of America Standards Institute in 1967 as a standard for all scientific and engineering literature.

For comparison, the SI units of measurement and their abbreviations appear in Table 1.1. These abbreviations are those usually applied to each unit of measurement, and they were carefully chosen to be the most effective. Therefore, it is important that they be used whenever applicable to ensure universal understanding. Note the similarities of the SI system to the MKS system. This text uses, whenever possible and practical, all of the major units and abbreviations of the SI system in an effort to support the need for a universal system. Those readers requiring additional information on the SI system should contact the information office of the American Society for Engineering Education (ASEE).\*

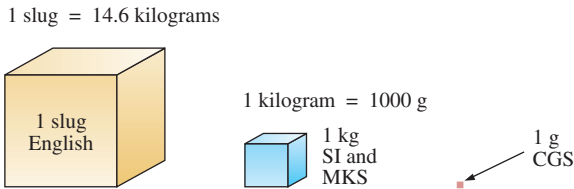
Figure 1.4 should help you develop some feeling for the relative magnitudes of the units of measurement of each system of units. Note

\*American Society for Engineering Education (ASEE), 1818 N Street N.W., Suite 600, Washington, D.C. 20036-2479; (202) 331-3500; <http://www.asee.org/>.

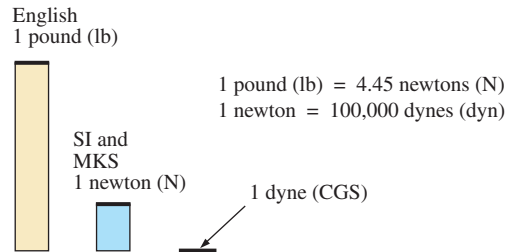
**Length:**



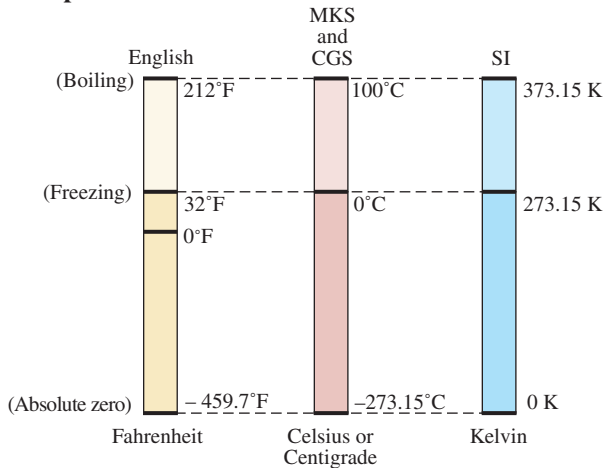
**Mass:**



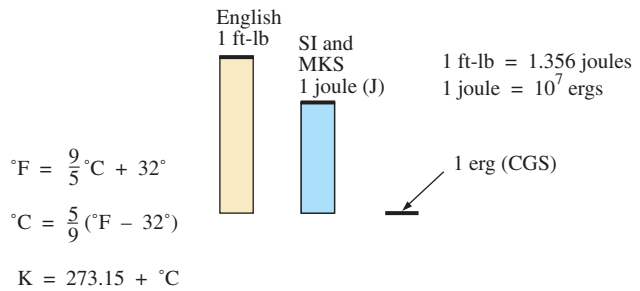
**Force:**



**Temperature:**



**Energy:**



**FIG. 1.4**

Comparison of units of the various systems of units.

in the figure the relatively small magnitude of the units of measurement for the CGS system.

A standard exists for each unit of measurement of each system. The standards of some units are quite interesting.

The **meter** was originally defined in 1790 to be 1/10,000,000 the distance between the equator and either pole at sea level, a length preserved on a platinum-iridium bar at the International Bureau of Weights and Measures at Sèvres, France.

*The meter is now defined with reference to the speed of light in a vacuum, which is 299,792,458 m/s.*

*The kilogram is defined as a mass equal to 1000 times the mass of one cubic centimeter of pure water at 4°C.*

This standard is preserved in the form of a platinum-iridium cylinder in Sèvres.

The **second** was originally defined as 1/86,400 of the mean solar day. However, since Earth's rotation is slowing down by almost 1 second every 10 years,

*the second was redefined in 1967 as 9,192,631,770 periods of the electromagnetic radiation emitted by a particular transition of cesium atom.*

## 1.5 SIGNIFICANT FIGURES, ACCURACY, AND ROUNDING OFF

This section emphasizes the importance of knowing the source of a piece of data, how a number appears, and how it should be treated. Too often we write numbers in various forms with little concern for the format used, the number of digits that should be included, and the unit of measurement to be applied.

For instance, measurements of 22.1" and 22.10" imply different levels of accuracy. The first suggests that the measurement was made by an instrument accurate only to the tenths place; the latter was obtained with instrumentation capable of reading to the hundredths place. The use of zeros in a number, therefore, must be treated with care and the implications must be understood.

In general, there are two types of numbers: *exact* and *approximate*. Exact numbers are precise to the exact number of digits presented, just as we know that there are 12 apples in a dozen and not 12.1. Throughout the text, the numbers that appear in the descriptions, diagrams, and examples are considered *exact*, so that a battery of 100 V can be written as 100.0 V, 100.00 V, and so on, since it is 100 V at any level of precision. The additional zeros were not included for purposes of clarity. However, in the laboratory environment, where measurements are continually being taken and the level of accuracy can vary from one instrument to another, it is important to understand how to work with the results. Any reading obtained in the laboratory should be considered *approximate*. The analog scales with their pointers may be difficult to read, and even though the digital meter provides only specific digits on its display, it is limited to the number of digits it can provide, leaving us to wonder about the less significant digits not appearing on the display.

The precision of a reading can be determined by the number of *significant figures (digits)* present. Significant digits are those integers (0 to 9) that can be assumed to be accurate for the measurement being made. The result is that all nonzero numbers are considered significant, with zeros being significant in only some cases. For instance, the zeros in 1005 are considered significant because they define the size of the number and are surrounded by nonzero digits. For the number 0.4020, the zero to the left of the decimal point is not significant, but the other two zeros are because they define the magnitude of the number and the fourth-place accuracy of the reading.

When adding approximate numbers, it is important to be sure that the accuracy of the readings is consistent throughout. To add a quantity accurate only to the tenths place to a number accurate to the thousandths place will result in a total having accuracy only to the tenths place. One cannot expect the reading with the higher level of accuracy to improve the reading with only tenths-place accuracy.

*In the addition or subtraction of approximate numbers, the entry with the lowest level of accuracy determines the format of the solution.*

*For the multiplication and division of approximate numbers, the result has the same number of significant figures as the number with the least number of significant figures.*

For approximate numbers (and exact, for that matter), there is often a need to *round off* the result; that is, you must decide on the appropriate level of accuracy and alter the result accordingly. The accepted procedure is simply to note the digit following the last to appear in the rounded-off form, and add a 1 to the last digit if it is greater than or equal to 5, and leave it alone if it is less than 5. For example,  $3.186 \cong 3.19 \cong 3.2$ , depending on the level of precision desired. The symbol  $\cong$  appearing means *approximately equal to*.

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**EXAMPLE 1.1** Perform the indicated operations with the following approximate numbers and round off to the appropriate level of accuracy.

- a.  $532.6 + 4.02 + 0.036 = 536.656 \cong \mathbf{536.7}$  (as determined by 532.6)
- b.  $0.04 + 0.003 + 0.0064 = 0.0494 \cong \mathbf{0.05}$  (as determined by 0.04)

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**EXAMPLE 1.2** Round off the following numbers to the hundredths place.

- a.  $32.419 = \mathbf{32.42}$
- b.  $0.05328 = \mathbf{0.05}$

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**EXAMPLE 1.3** Round off the following numbers to the thousandths place.

- a.  $0.00738 = \mathbf{0.007}$
  - b.  $46.23550 = \mathbf{46.236}$
- 

## 1.6 POWERS OF TEN

It should be apparent from the relative magnitude of the various units of measurement that very large and very small numbers are frequently encountered in the sciences. To ease the difficulty of mathematical operations with numbers of such varying size, *powers of ten* are usually employed. This notation takes full advantage of the mathematical properties of powers of ten. The notation used to represent numbers that are integer powers of ten is as follows:

$$\begin{array}{lll} 1 = 10^0 & 1/10 = & 0.1 = 10^{-1} \\ 10 = 10^1 & 1/100 = & 0.01 = 10^{-2} \\ 100 = 10^2 & 1/1000 = & 0.001 = 10^{-3} \\ 1000 = 10^3 & 1/10,000 = & 0.0001 = 10^{-4} \end{array}$$

In particular, note that  $10^0 = 1$ , and, in fact, any quantity to the zero power is 1 ( $x^0 = 1$ ,  $1000^0 = 1$ , and so on). Numbers in the list *greater than 1 are associated with positive powers of ten*, and numbers in the list *less than 1 are associated with negative powers of ten*.

A quick method of determining the proper power of ten is to place a caret mark to the right of the numeral 1 wherever it may occur; then count from this point to the number of places to the right or left before arriving at the decimal point. Moving to the right indicates a positive power of ten, whereas moving to the left indicates a negative power. For example,

$$10,000.0 = 1 \underbrace{0,000}_{1 \ 2 \ 3 \ 4} = 10^{+4}$$

$$0.00001 = 0 \underbrace{,00001}_{5 \ 4 \ 3 \ 2 \ 1} = 10^{-5}$$

Some important mathematical equations and relationships pertaining to powers of ten are listed below, along with a few examples. In each case,  $n$  and  $m$  can be any positive or negative real number.

$$\boxed{\frac{1}{10^n} = 10^{-n} \quad \frac{1}{10^{-n}} = 10^n} \quad (1.2)$$

Eq. (1.2) clearly reveals that shifting a power of ten from the denominator to the numerator, or the reverse, requires simply changing the sign of the power.

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#### EXAMPLE 1.4

a.  $\frac{1}{1000} = \frac{1}{10^{+3}} = 10^{-3}$

b.  $\frac{1}{0.00001} = \frac{1}{10^{-5}} = 10^{+5}$

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The product of powers of ten:

$$\boxed{(10^n)(10^m) = (10)^{(n+m)}} \quad (1.3)$$

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#### EXAMPLE 1.5

a.  $(1000)(10,000) = (10^3)(10^4) = 10^{(3+4)} = 10^7$

b.  $(0.00001)(100) = (10^{-5})(10^2) = 10^{(-5+2)} = 10^{-3}$

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The division of powers of ten:

$$\boxed{\frac{10^n}{10^m} = 10^{(n-m)}} \quad (1.4)$$

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#### EXAMPLE 1.6

a.  $\frac{100,000}{100} = \frac{10^5}{10^2} = 10^{(5-2)} = 10^3$

b.  $\frac{1000}{0.0001} = \frac{10^3}{10^{-4}} = 10^{(3-(-4))} = 10^{(3+4)} = 10^7$

Note the use of parentheses in part (b) to ensure that the proper sign is established between operators.

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The power of powers of ten:

$$\boxed{(10^n)^m = 10^{nm}} \quad (1.5)$$

**EXAMPLE 1.7**

- a.  $(100)^4 = (10^2)^4 = 10^{(2)(4)} = \mathbf{10^8}$   
 b.  $(1000)^{-2} = (10^3)^{-2} = 10^{(3)(-2)} = \mathbf{10^{-6}}$   
 c.  $(0.01)^{-3} = (10^{-2})^{-3} = 10^{(-2)(-3)} = \mathbf{10^6}$

**Basic Arithmetic Operations**

Let us now examine the use of powers of ten to perform some basic arithmetic operations using numbers that are not just powers of ten. The number 5000 can be written as  $5 \times 1000 = 5 \times 10^3$ , and the number 0.0004 can be written as  $4 \times 0.0001 = 4 \times 10^{-4}$ . Of course,  $10^5$  can also be written as  $1 \times 10^5$  if it clarifies the operation to be performed.

**Addition and Subtraction** To perform addition or subtraction using powers of ten, the power of ten *must be the same for each term*; that is,

$$A \times 10^n \pm B \times 10^n = (A \pm B) \times 10^n \quad (1.6)$$

Eq. (1.6) covers all possibilities, but students often prefer to remember a verbal description of how to perform the operation.

Eq. (1.6) states

*when adding or subtracting numbers in a power-of-ten format, be sure that the power of ten is the same for each number. Then separate the multipliers, perform the required operation, and apply the same power of ten to the result.*

**EXAMPLE 1.8**

- a.  $6300 + 75,000 = (6.3)(1000) + (75)(1000)$   
 $= 6.3 \times 10^3 + 75 \times 10^3$   
 $= (6.3 + 75) \times 10^3$   
 $= \mathbf{81.3 \times 10^3}$
- b.  $0.00096 - 0.000086 = (96)(0.00001) - (8.6)(0.00001)$   
 $= 96 \times 10^{-5} - 8.6 \times 10^{-5}$   
 $= (96 - 8.6) \times 10^{-5}$   
 $= \mathbf{87.4 \times 10^{-5}}$

**Multiplication** In general,

$$(A \times 10^n)(B \times 10^m) = (A)(B) \times 10^{n+m} \quad (1.7)$$

revealing that the *operations with the power of ten can be separated from the operation with the multipliers*.

Eq. (1.7) states

*when multiplying numbers in the power-of-ten format, first find the product of the multipliers and then determine the power of ten for the result by adding the power-of-ten exponents.*

**EXAMPLE 1.9**

- a.  $(0.0002)(0.000007) = [(2)(0.0001)][(7)(0.000001)]$   
 $= (2 \times 10^{-4})(7 \times 10^{-6})$   
 $= (2)(7) \times (10^{-4})(10^{-6})$   
 $= \mathbf{14 \times 10^{-10}}$
- b.  $(340,000)(0.00061) = (3.4 \times 10^5)(61 \times 10^{-5})$   
 $= (3.4)(61) \times (10^5)(10^{-5})$   
 $= 207.4 \times 10^0$   
 $= \mathbf{207.4}$

**Division** In general,

$$\frac{A \times 10^n}{B \times 10^m} = \frac{A}{B} \times 10^{n-m} \quad (1.8)$$

revealing again that the *operations with the power of ten can be separated from the same operation with the multipliers.*

Eq. (1.8) states

*when dividing numbers in the power-of-ten format, first find the result of dividing the multipliers. Then determine the associated power for the result by subtracting the power of ten of the denominator from the power of ten of the numerator.*

**EXAMPLE 1.10**

- a.  $\frac{0.00047}{0.002} = \frac{47 \times 10^{-5}}{2 \times 10^{-3}} = \left(\frac{47}{2}\right) \times \left(\frac{10^{-5}}{10^{-3}}\right) = \mathbf{23.5 \times 10^{-2}}$
- b.  $\frac{690,000}{0.00000013} = \frac{69 \times 10^4}{13 \times 10^{-8}} = \left(\frac{69}{13}\right) \times \left(\frac{10^4}{10^{-8}}\right) = \mathbf{5.31 \times 10^{12}}$

**Powers** In general,

$$(A \times 10^n)^m = A^m \times 10^{nm} \quad (1.9)$$

which again permits the separation of the *operation with the power of ten from the multiplier.*

Eq. (1.9) states

*when finding the power of a number in the power-of-ten format, first separate the multiplier from the power of ten and determine each separately. Determine the power-of-ten component by multiplying the power of ten by the power to be determined.*

**EXAMPLE 1.11**

- a.  $(0.00003)^3 = (3 \times 10^{-5})^3 = (3)^3 \times (10^{-5})^3$   
 $= 27 \times 10^{-15}$
- b.  $(90,800,000)^2 = (9.08 \times 10^7)^2 = (9.08)^2 \times (10^7)^2$   
 $= 82.45 \times 10^{14}$

In particular, remember that the following operations are not the same. One is the product of two numbers in the power-of-ten format, while the other is a number in the power-of-ten format taken to a power. As noted below, the results of each are quite different:

$$\begin{aligned} (10^3)(10^3) &\neq (10^3)^3 \\ (10^3)(10^3) &= 10^6 = 1,000,000 \\ (10^3)^3 &= (10^3)(10^3)(10^3) = 10^9 = 1,000,000,000 \end{aligned}$$

**1.7 FIXED-POINT, FLOATING-POINT, SCIENTIFIC, AND ENGINEERING NOTATION**

When you are using a computer or a calculator, numbers generally appear in one of four ways. If powers of ten are not employed, numbers are written in the **fixed-point** or **floating-point notation**.

*The fixed-point format requires that the decimal point appear in the same place each time. In the floating-point format, the decimal point appears in a location defined by the number to be displayed.*

Most computers and calculators permit a choice of fixed- or floating-point notation. In the fixed format, the user can choose the level of accuracy for the output as tenths place, hundredths place, thousandths place, and so on. Every output will then fix the decimal point to one location, such as the following examples using thousandths-place accuracy:

$$\frac{1}{3} = \mathbf{0.333} \quad \frac{1}{16} = \mathbf{0.063} \quad \frac{2300}{2} = \mathbf{1150.000}$$

If left in the floating-point format, the results will appear as follows for the above operations:

$$\frac{1}{3} = \mathbf{0.333333333333} \quad \frac{1}{16} = \mathbf{0.0625} \quad \frac{2300}{2} = \mathbf{1150}$$

Powers of ten will creep into the fixed- or floating-point notation if the number is too small or too large to be displayed properly.

**Scientific** (also called *standard*) **notation** and **engineering notation** make use of powers of ten, with restrictions on the mantissa (multiplier) or scale factor (power of ten).

*Scientific notation requires that the decimal point appear directly after the first digit greater than or equal to 1 but less than 10.*

A power of ten will then appear with the number (usually following the power notation E), even if it has to be to the zero power. A few examples:

$$\frac{1}{3} = \mathbf{3.33333333333E-1} \quad \frac{1}{16} = \mathbf{6.25E-2} \quad \frac{2300}{2} = \mathbf{1.15E3}$$

Within scientific notation, the fixed- or floating-point format can be chosen. In the above examples, floating was employed. If fixed is chosen



and set at the hundredths-point accuracy, the following will result for the above operations:

$$\frac{1}{3} = 3.33\text{E}-1 \quad \frac{1}{16} = 6.25\text{E}-2 \quad \frac{2300}{2} = 1.15\text{E}3$$

**Engineering notation** specifies that

*all powers of ten must be multiples of 3, and the mantissa must be greater than or equal to 1 but less than 1000.*

This restriction on the powers of ten is because specific powers of ten have been assigned prefixes that are introduced in the next few paragraphs. Using scientific notation in the floating-point mode results in the following for the above operations:

$$\frac{1}{3} = 333.33333333\text{E}-3 \quad \frac{1}{16} = 62.5\text{E}-3 \quad \frac{2300}{2} = 1.15\text{E}3$$

Using engineering notation with two-place accuracy will result in the following:

$$\frac{1}{3} = 333.33\text{E}-3 \quad \frac{1}{16} = 62.50\text{E}-3 \quad \frac{2300}{2} = 1.15\text{E}3$$

## Prefixes

Specific powers of ten in engineering notation have been assigned prefixes and symbols, as appearing in Table 1.2. They permit easy recognition of the power of ten and an improved channel of communication between technologists.

**TABLE 1.2**

Multiplication Factors	SI Prefix	SI Symbol
1 000 000 000 000 000 000 = $10^{18}$	exa	E
1 000 000 000 000 000 = $10^{15}$	peta	P
1 000 000 000 000 = $10^{12}$	tera	T
1 000 000 000 = $10^9$	giga	G
1 000 000 = $10^6$	mega	M
1 000 = $10^3$	kilo	k
0.001 = $10^{-3}$	milli	m
0.000 001 = $10^{-6}$	micro	$\mu$
0.000 000 001 = $10^{-9}$	nano	n
0.000 000 000 001 = $10^{-12}$	pico	p
0.000 000 000 000 001 = $10^{-15}$	femto	f
0.000 000 000 000 000 001 = $10^{-18}$	atto	a

### EXAMPLE 1.12

- 1,000,000 ohms =  $1 \times 10^6$  ohms  
= **1 megohm** = **1 M $\Omega$**
- 100,000 meters =  $100 \times 10^3$  meters  
= **100 kilometers** = **100 km**
- 0.0001 second =  $0.1 \times 10^{-3}$  second  
= **0.1 millisecond** = **0.1 ms**

$$\begin{aligned} \text{d. } 0.000001 \text{ farad} &= 1 \times 10^{-6} \text{ farad} \\ &= \mathbf{1 \text{ microfarad} = 1 \mu\text{F}} \end{aligned}$$

Here are a few examples with numbers that are not strictly powers of ten.

### EXAMPLE 1.13

- 41,200 m is equivalent to  $41.2 \times 10^3 \text{ m} = 41.2 \text{ kilometers} = \mathbf{41.2 \text{ km}}$ .
- 0.00956 J is equivalent to  $9.56 \times 10^{-3} \text{ J} = 9.56 \text{ millijoules} = \mathbf{9.56 \text{ mJ}}$ .
- 0.000768 s is equivalent to  $768 \times 10^{-6} \text{ s} = 768 \text{ microseconds} = \mathbf{768 \mu\text{s}}$ .
- $\frac{8400 \text{ m}}{0.06} = \frac{8.4 \times 10^3 \text{ m}}{6 \times 10^{-2}} = \left(\frac{8.4}{6}\right) \times \left(\frac{10^3}{10^{-2}}\right) \text{ m}$   
 $= 1.4 \times 10^5 \text{ m} = 140 \times 10^3 \text{ m} = 140 \text{ kilometers} = \mathbf{140 \text{ km}}$
- $(0.0003)^4 \text{ s} = (3 \times 10^{-4})^4 \text{ s} = 81 \times 10^{-16} \text{ s}$   
 $= 0.0081 \times 10^{-12} \text{ s} = 0.0081 \text{ picosecond} = \mathbf{0.0081 \text{ ps}}$

## 1.8 CONVERSION BETWEEN LEVELS OF POWERS OF TEN

It is often necessary to convert from one power of ten to another. For instance, if a meter measures kilohertz (kHz—a unit of measurement for the frequency of an ac waveform), it may be necessary to find the corresponding level in megahertz (MHz). If time is measured in milliseconds (ms), it may be necessary to find the corresponding time in microseconds ( $\mu\text{s}$ ) for a graphical plot. The process is not difficult if we simply keep in mind that an increase or a decrease in the power of ten must be associated with the opposite effect on the multiplying factor. The procedure is best described by the following steps:

- Replace the prefix by its corresponding power of ten.*
- Rewrite the expression, and set it equal to an unknown multiplier and the new power of ten.*
- Note the change in power of ten from the original to the new format. If it is an increase, move the decimal point of the original multiplier to the left (smaller value) by the same number. If it is a decrease, move the decimal point of the original multiplier to the right (larger value) by the same number.*

**EXAMPLE 1.14** Convert 20 kHz to megahertz.

**Solution:** In the power-of-ten format:

$$20 \text{ kHz} = 20 \times 10^3 \text{ Hz}$$

The conversion requires that we find the multiplying factor to appear in the space below:

$$20 \times 10^3 \text{ Hz} \Rightarrow \underline{\hspace{1cm}} \times 10^6 \text{ Hz}$$

↑ Increase by 3  
↓ Decrease by 3

Since the power of ten will be *increased* by a factor of *three*, the multiplying factor must be *decreased* by moving the decimal point *three* places to the left, as shown below:

$$\underbrace{020.}_{3} = 0.02$$

and  $20 \times 10^3 \text{ Hz} = 0.02 \times 10^6 \text{ Hz} = \mathbf{0.02 \text{ MHz}}$

**EXAMPLE 1.15** Convert 0.01 ms to microseconds.

**Solution:** In the power-of-ten format:

$$0.01 \text{ ms} = 0.01 \times 10^{-3} \text{ s}$$

and  $0.01 \times 10^{-3} \text{ s} \Rightarrow \underline{\quad} \times 10^{-6} \text{ s}$

Decrease by 3  
Increase by 3

Since the power of ten will be *reduced* by a factor of three, the multiplying factor must be *increased* by moving the decimal point three places to the right, as follows:

$$\underbrace{0.010}_{3} = 10$$

and  $0.01 \times 10^{-3} \text{ s} = 10 \times 10^{-6} \text{ s} = \mathbf{10 \mu\text{s}}$

There is a tendency when comparing  $-3$  to  $-6$  to think that the power of ten has increased, but keep in mind when making your judgment about increasing or decreasing the magnitude of the multiplier that  $10^{-6}$  is a great deal smaller than  $10^{-3}$ .

**EXAMPLE 1.16** Convert 0.002 km to millimeters.

**Solution:**

$$0.002 \times 10^3 \text{ m} \Rightarrow \underline{\quad} \times 10^{-3} \text{ m}$$

Decrease by 6  
Increase by 6

In this example we have to be very careful because the difference between  $+3$  and  $-3$  is a factor of 6, requiring that the multiplying factor be modified as follows:

$$\underbrace{0.002000}_{6} = 2000$$

and  $0.002 \times 10^3 \text{ m} = 2000 \times 10^{-3} \text{ m} = \mathbf{2000 \text{ mm}}$

## 1.9 CONVERSION WITHIN AND BETWEEN SYSTEMS OF UNITS

The conversion within and between systems of units is a process that cannot be avoided in the study of any technical field. It is an operation, however, that is performed incorrectly so often that this section was included to provide one approach that, if applied properly, will lead to the correct result.

There is more than one method of performing the conversion process. In fact, some people prefer to determine mentally whether the conversion factor is multiplied or divided. This approach is acceptable for some elementary conversions, but it is risky with more complex operations.

The procedure to be described here is best introduced by examining a relatively simple problem such as converting inches to meters. Specifically, let us convert 48 in. (4 ft) to meters.

If we multiply the 48 in. by a factor of **1**, the magnitude of the quantity remains the same:

$$48 \text{ in.} = 48 \text{ in.}(\mathbf{1}) \quad (1.10)$$

Let us now look at the conversion factor for this example:

$$1 \text{ m} = 39.37 \text{ in.}$$

Dividing both sides of the conversion factor by 39.37 in. results in the following format:

$$\frac{1 \text{ m}}{39.37 \text{ in.}} = (\mathbf{1})$$

Note that the end result is that the ratio 1 m/39.37 in. equals 1, as it should since they are equal quantities. If we now substitute this factor (**1**) into Eq. (1.10), we obtain

$$48 \text{ in.}(\mathbf{1}) = 48 \text{ in.} \left( \frac{1 \text{ m}}{39.37 \text{ in.}} \right)$$

which results in the cancellation of inches as a unit of measure and leaves meters as the unit of measure. In addition, since the 39.37 is in the denominator, it must be divided into the 48 to complete the operation:

$$\frac{48}{39.37} \text{ m} = \mathbf{1.219 \text{ m}}$$

Let us now review the method:

1. *Set up the conversion factor to form a numerical value of (1) with the unit of measurement to be removed from the original quantity in the denominator.*
2. *Perform the required mathematics to obtain the proper magnitude for the remaining unit of measurement.*

**EXAMPLE 1.17** Convert 6.8 min to seconds.

**Solution:** The conversion factor is

$$1 \text{ min} = 60 \text{ s}$$

Since the minute is to be removed as the unit of measurement, it must appear in the denominator of the (**1**) factor, as follows:

$$\text{Step 1.} \quad \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = (\mathbf{1})$$

$$\begin{aligned} \text{Step 2.} \quad 6.8 \text{ min}(\mathbf{1}) &= 6.8 \text{ min} \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = (6.8)(60) \text{ s} \\ &= \mathbf{408 \text{ s}} \end{aligned}$$

**EXAMPLE 1.18** Convert 0.24 m to centimeters.

**Solution:** The conversion factor is

$$1 \text{ m} = 100 \text{ cm}$$

Since the meter is to be removed as the unit of measurement, it must appear in the denominator of the (1) factor as follows:

$$\text{Step 1.} \quad \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) = \mathbf{1}$$

$$\text{Step 2.} \quad 0.24 \text{ m}(\mathbf{1}) = 0.24 \cancel{\text{ m}} \left( \frac{100 \text{ cm}}{1 \cancel{\text{ m}}} \right) = (0.24)(100) \text{ cm} \\ = \mathbf{24 \text{ cm}}$$

The products (1)(1) and (1)(1)(1) are still 1. Using this fact, we can perform a series of conversions in the same operation.

**EXAMPLE 1.19** Determine the number of minutes in half a day.

**Solution:** Working our way through from days to hours to minutes, always ensuring that the unit of measurement to be removed is in the denominator, results in the following sequence:

$$0.5 \text{ day} \left( \frac{3 \cancel{\text{ h}}}{1 \text{ day}} \right) \left( \frac{60 \text{ min}}{1 \cancel{\text{ h}}} \right) = (0.5)(24)(60) \text{ min} \\ = \mathbf{720 \text{ min}}$$

**EXAMPLE 1.20** Convert 2.2 yards to meters.

**Solution:** Working our way through from yards to feet to inches to meters results in the following:

$$2.2 \text{ yards} \left( \frac{3 \cancel{\text{ ft}}}{1 \text{ yard}} \right) \left( \frac{12 \cancel{\text{ in.}}}{1 \cancel{\text{ ft}}} \right) \left( \frac{1 \text{ m}}{39.37 \cancel{\text{ in.}}} \right) = \frac{(2.2)(3)(12)}{39.37} \text{ m} \\ = \mathbf{2.012 \text{ m}}$$

The following examples are variations of the above in practical situations.

**EXAMPLE 1.21** In Europe, Canada, and many countries, the speed limit is posted in kilometers per hour. How fast in miles per hour is 100 km/h?

**Solution:**

$$\left( \frac{100 \text{ km}}{\text{h}} \right) (\mathbf{1})(\mathbf{1})(\mathbf{1})(\mathbf{1}) \\ = \left( \frac{100 \cancel{\text{ km}}}{\text{h}} \right) \left( \frac{1000 \cancel{\text{ m}}}{1 \cancel{\text{ km}}} \right) \left( \frac{39.37 \cancel{\text{ in.}}}{1 \cancel{\text{ m}}} \right) \left( \frac{1 \cancel{\text{ ft}}}{12 \cancel{\text{ in.}}} \right) \left( \frac{1 \text{ mi}}{5280 \cancel{\text{ ft}}} \right) \\ = \frac{(100)(1000)(39.37)}{(12)(5280)} \frac{\text{mi}}{\text{h}} \\ = \mathbf{62.14 \text{ mph}}$$

**TABLE 1.3**

Symbol	Meaning
$\neq$	Not equal to $6.12 \neq 6.13$
$>$	Greater than $4.78 > 4.20$
$\gg$	Much greater than $840 \gg 16$
$<$	Less than $430 < 540$
$\ll$	Much less than $0.002 \ll 46$
$\geq$	Greater than or equal to $x \geq y$ is satisfied for $y = 3$ and $x > 3$ or $x = 3$
$\leq$	Less than or equal to $x \leq y$ is satisfied for $y = 3$ and $x < 3$ or $x = 3$
$\cong$	Approximately equal to $3.14159 \cong 3.14$
$\Sigma$	Sum of $\Sigma (4 + 6 + 8) = 18$
$\ $	Absolute magnitude of $ a  = 4$ , where $a = -4$ or $+4$
$\therefore$	Therefore $x = \sqrt{4} \therefore x = \pm 2$
$\equiv$	By definition Establishes a relationship between two or more quantities
$a:b$	Ratio defined by $\frac{a}{b}$
$a:b = c:d$	Proportion defined by $\frac{a}{b} = \frac{c}{d}$

Many travelers use 0.6 as a conversion factor to simplify the math involved; that is,

$$(100 \text{ km/h})(0.6) \cong 60 \text{ mph}$$

and

$$(60 \text{ km/h})(0.6) \cong 36 \text{ mph}$$

**EXAMPLE 1.22** Determine the speed in miles per hour of a competitor who can run a 4-min mile.

**Solution:** Inverting the factor 4 min/1 mi to 1 mi/4 min, we can proceed as follows:

$$\left(\frac{1 \text{ mi}}{4 \text{ min}}\right)\left(\frac{60 \text{ min}}{\text{h}}\right) = \frac{60}{4} \text{ mi/h} = \mathbf{15 \text{ mph}}$$

### 1.10 SYMBOLS

Throughout the text, various symbols will be employed that you may not have had occasion to use. Some are defined in Table 1.3, and others will be defined in the text as the need arises.

### 1.11 CONVERSION TABLES

Conversion tables such as those appearing in Appendix A can be very useful when time does not permit the application of methods described in this chapter. However, even though such tables appear easy to use, frequent errors occur because the operations appearing at the head of the table are not performed properly. In any case, when using such tables, try to establish mentally some order of magnitude for the quantity to be determined compared to the magnitude of the quantity in its original set of units. This simple operation should prevent several impossible results that may occur if the conversion operation is improperly applied.

For example, consider the following from such a conversion table:

To convert from	To	Multiply by
Miles	Meters	$1.609 \times 10^3$

A conversion of 2.5 mi to meters would require that we multiply 2.5 by the conversion factor; that is,

$$2.5 \text{ mi}(1.609 \times 10^3) = \mathbf{4.0225 \times 10^3 \text{ m}}$$

A conversion from 4000 m to miles would require a division process:

$$\frac{4000 \text{ m}}{1.609 \times 10^3} = 2486.02 \times 10^{-3} = \mathbf{2.486 \text{ mi}}$$

In each of the above, there should have been little difficulty realizing that 2.5 mi would convert to a few thousand meters and 4000 m would be only a few miles. As indicated above, this kind of anticipatory thinking will eliminate the possibility of ridiculous conversion results.